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Optimization of Compressive Strength of Concrete made with Granite and Palm Kernel Shells as Coarse Aggregate

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Abstract The demand of concrete has led to the depletion of granite and gravel, this has made conventional aggregate expensive. Hence, sourcing for alternative local material for the construction of concrete becomes necessary. A mathematical model was developed to predict and optimize the compressive strength of Granite – palm kernel shells aggregate concrete using Henry Scheffe's regression theory. Concrete Cubes were cast consisting of thirty (30) different mix ratios. The first fifteen (15) mix ratios were used to determine the coefficients of the model and the second fifteen (15) mix ratios were used to validate the model. The results obtained from the mathematical model agreed favourably with the experimental data. Adequacy of the model was tested using fisher f-test at 95% confidence level and found to be adequate. A compressive strength of 23.86 N/mm^2 was predicted by the mathematical model with mix ratio of 0.546:1:1.67:2.14:1.6 (water : cement : sand : granite : palm kernel shells). A Java computer programme was coded to select the optimized compressive strength of granite – palm kernel shells aggregate concrete.

Keywords Coarse aggregate, palm kernel shells, concrete, compressive strength, optimization, mathematical model

1. Introduction

In construction, concrete has a high demand and this has led to a decrease in granite and gravel deposit [1]. The demand of concrete has made this aggregate expensive. Sometimes where the construction work is carried out is so far from where the aggregate is deposited; the cost of transporting the aggregate to the construction site could be much higher than the cost of buying the aggregate.

Aggregate takes a large volume of concrete and this means that it has influence in its behaviour or performance of the concrete. The stability of concrete is contributed by aggregate [2].

Oil palm shell is an end product of oil palm manufacturing process. Oil palm is a fruit of palm tree. The palm tree grows in area where the temperature is very high and rains a lot such as Nigeria. Oil palm fruit is made up of two major parts: (1) PULP. The pulp is a yellow fruit and when crushed, produces palm oil. (2) KERNEL: The kernel is bounded in the shell of the seed when kernel is crushed, it produces palm kernel oil.

In the process of palm oil a large quantity of by product result such as Empty Fruit Bunches (EFB), Palm Kernel Shells (PKS) or oil palm shells (OPS) and palm oil mill effluence (POME) which add to the Nation pollution problem. The waste material are stockpiled and dumped, such have caused storage problem in the vicinity of the factories as large quantities of these wastes are produced daily [2].

In developed countries, the construction industries have identified the use of waste natural material as alternative to conventional aggregates by reducing the size of the structural members. This has brought immense change in the development of high rise structures using light weight concrete (LWC).

For some time now, Nigerian government has been calling for the use of local materials in the construction industries to limit cost of construction. There has therefore been greater call for the sourcing and development of alternative, non conventional local construction materials [3].

Palm Kernel shells (pks) are waste product from palm oil industries and are organic by product from processed palm oil fruit. The palm kernel shells are usually kept in open places where they have no economic advantage. Palm kernel shell particles ranges from 0 – 5mm, 5 – 10mm and 10 – 15mm in size [4].

2. Statement of Problem

Nigerian government has been calling for the use of Local Materials for the construction Industries. There has therefore been greater call for the sourcing and development of alternative, nonconventional local construction material [3].

In the Construction Industry, the cost of concrete production is high, there is the need to source for local materials to reduce cost. High demand of concrete had led to depletion of granite and gravel deposit [1]. Therefore, the need to source for local material becomes necessary.

The cost of transportation of granite and gravel from where they are deposited to some construction sites sometimes could be high. For economical purpose sourcing of local material like palm kernel shells is necessary.

3.1. Materials and Method

Materials used in this research work are; palm kernel shells, granite, portland cement and sharp sand.

a. **Palm Kernel Shells**: Palm Kernel shells were sourced from Okposi Community in Ogba/Egbema/Ndoni Local Government Area, Rivers State, Nigeria. The palm kernel shells were washed to remove impurities that may have effect and sun dried.

b. **Granite**: Granite ranges in size from 5mm to 10mm were used for this research work. The granite was obtained from a building material market in Port Harcourt city, Rivers State, Nigeria.

c. **Portland Cement**: the cement used for this work was ordinary Portland cement from Dangote. This ordinary Portland cement conform to BS12 1996. It was got from Port Harcourt city, Rivers State, Nigeria where it was sold.

d. **Sand**: Sharp sand used for this research work was got from the market.

3.2. Use of H. Scheffe's Theory

According to Henry Scheffe's [5], a five component mixture like the Granite – Palm Kernel shells aggregate concrete, the proportion, X_i of the i^{th} component of the mixture most satisfy the following constraint:

$$X_i \geq 0 \qquad (1)$$

(i = 1, 2, 3, 4, 5)

$X_i \geq 0$ is the component proportion.

And the sum of all proportion of the constituents of the five components of Granite – Palm kernel shells aggregate concrete must be equal to unit,

i.e.

$$\sum_{i=1}^{q} X_i = 1 \qquad (2)$$

For the five – component Granite – Palm Kernel shells aggregate concrete,

$$1 = X_1 + X_2 + X_3 + X_4 + X_5 \qquad (3)$$

q = number of component in the mixture

Compressive strength equation of H. Scheffe's (5,2) simplex design for five components mixture like granite palm kernel shells concrete given by Obam [6] as;

$$Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{15} X_1 X_5 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{25} X_2 X_5 + \beta_{34} X_3 X_4 + \beta_{35} X_3 X_5 + \beta_{45} X_4 X_5 \qquad (4)$$

Where β_i and X_i are coefficients of response equations and pseudo components of the mix response.

The number (N) required for the mixture experiment is given by H. Scheffe's [5] as;

$$N = \frac{(q+m-1)!}{m!\,(q-1)!} \qquad (5)$$

N = Number of mix required

m = Degree of polynomial equation

q = Number of components in the mixture

q = 5, m = 2

$$N = \frac{(5+2-1)!}{2!(5-1)!} = \frac{6!}{2! \times 4!} = 15$$

15 is the number of mixture required in this experiment.

The coefficients in terms of pseudo components, xi and laboratory response of the first fifteen ratios gave the following relations as regression model equation of H. Scheffe's (5,2) simplex design [7];

$$Y(x) = n_1 x_1 (2x_1 - 1) + n_2 X_2 (2x_2 - 1) + n_3 X_3 (2x_3 - 1) + n_4 X_4 (2X_4 - 1) + n_5 X_5 (2X_5 - 1) + 4n_{12} X_1 X_2 + 4n_{13} X_1 X_3 + 4n_{14} X_1 X_4 + 4n_{15} X_1 X_5 + 4n_{23} X_2 X_3 + 4n_{24} X_2 X_4 + 4n_{25} X_2 X_5 + 4n_{34} X_3 X_4 + 4n_{35} X_3 X_5 + 4n_{45} X_4 X_5 \qquad (6)$$

Equation (6) is the response function for optimization of Granite Palm Kernel shells aggregate concrete consisting of five components.

n_i and n_{ij} are the response (compressive strengths) at the point i and ij. The values of these responses are determined by carrying out compressive test on cubes using Granite – palm kernel shells aggregate concrete.

3.3 Simplex Design

Simplex is a polygon. The simplest simplex is a straight line. Granite – Palm Kernel shells aggregate concrete is a five – component mix consisting of water, cement, sand, Granite and palm kernel shells, according to Henry Scheffe this can be analyzed using a four dimensional simplex lattice as shown below;

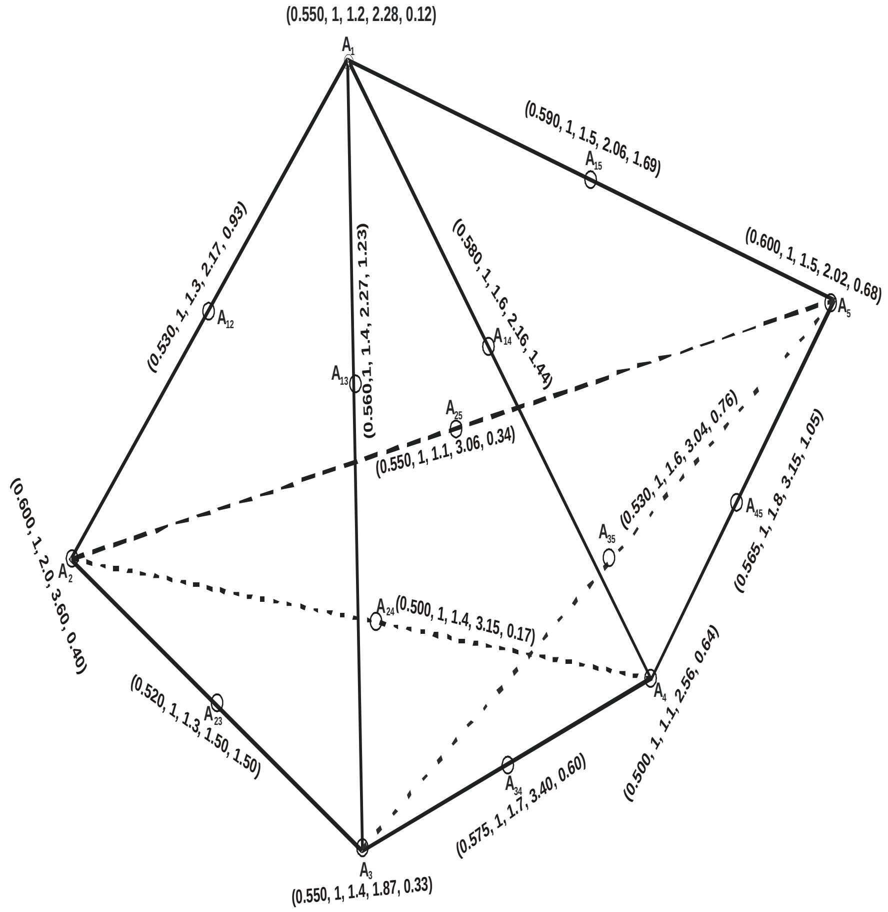

Figure 1: Simplex Lattice for (5,2) factor space

Osadebe et al [8] the actual mix ratios relate with Pseudo mix ratio by the equation below;

$$\{S\} = [A]\{X\} \tag{7}$$

$$\begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{pmatrix} = \begin{pmatrix} 0.550 & 0.600 & 0.550 & 0.500 & 0.600 \\ 1 & 1 & 1 & 1 & 1 \\ 1.2 & 1.1 & 1.4 & 1.1 & 1.5 \\ 2.28 & 1.80 & 1.87 & 2.56 & 2.02 \\ 0.12 & 0.20 & 0.33 & 0.64 & 0.68 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix} \tag{8}$$

3.2. Concrete Mix Ratios

Five mix ratios (real and pseudo) that fine the vertices of pentahedron simplex lattice used in the study are shown in Table 1 below;

Table 1: First five mix ratio (actual and pseudo) obtained from Scheffe's (5, 2) factor space

Points	Real Mix Ratios					Pseudo Mix Ratios				
	Water S_1	Cement S_2	Sand S_3	Granite S_4	PKS S_5	Water X_1	Cement X_2	Sand X_3	Granite X_4	PKS X_5
N_1	0.550	1	1.2	2.28	0.12	1.0	0.0	0.0	0.0	0.0
N_2	0.600	1	1.1	1.80	0.20	0.0	1.0	0.0	0.0	0.0
N_3	0.550	1	1.4	1.87	0.33	0.0	0.0	1.0	0.0	0.0
N_4	0.500	1	1.1	2.56	0.64	0.0	0.0	0.0	1.0	0.0
N_5	0.600	1	1.5	2.02	0.68	0.0	0.0	0.0	0.0	1.0

Legend: PKS = Palm Kernel Shells

Table 1 shows the five real and pseudo mix ratios corresponding to the points of observation N_1, N_2, N_3, N_4, N_5 located at the five vertices of the four – dimensional simplex lattice. For a (5, 2) simplex design, ten (10) other observation are needed to get a total of 15 observations needed for the development of the compressive strength equation. The remaining ten (10) points are located at the mid points of the lines joining the five (5) vertices. On successive substitution on these ten (10) pseudo mix ratios into Equation (7), the real mix ratios corresponding to the pseudo ones were obtained. Their values are show in Table 2.

Table 2: Additional ten mix ratios (real and pseudo) for formulation of the optimization function

Points	Real Mix Ratios					Pseudo Mix Ratios				
	Water S_1	Cement S_2	Sand S_3	Granite S_4	PKS S_5	Water X_1	Cement X_2	Sand X_3	Granite X_4	PKS X_5
N_{12}	0.530	1	1.3	2.17	0.93	0.50	0.50	0.0	0.0	0.0
N_{13}	0.560	1	1.4	2.27	1.23	0.50	0.0	0.50	0.0	0.0
N_{14}	0.580	1	1.6	2.16	1.44	0.50	0.0	0.0	0.50	0.0
N_{15}	0.590	1	1.5	2.06	1.69	0.50	0.0	0.0	0.0	0.50
N_{23}	0.520	1	1.3	1.50	1.50	0.0	0.50	0.50	0.0	0.0
N_{24}	0.500	1	1.4	3.15	0.17	0.0	0.50	0.0	0.50	0.0
N_{25}	0.550	1	1.1	3.06	0.34	0.0	0.50	0.0	0.0	0.50
N_{34}	0.575	1	1.7	3.40	0.60	0.0	0.0	0.50	0.50	0.0
N_{35}	0.530	1	1.6	3.04	0.76	0.0	0.0	0.50	0.0	0.50
N_{45}	0.565	1	1.8	3.15	1.05	0.0	0.0	0.0	0.50	0.50

Legend: PKS = Palm Kernel Shells

In order to validate the model, extra fifteen points (C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14}, and C_{15}) of observations were used. These observations served as control mix ratios of the concrete mixes in this research.

Table 3: Actual and pseudo components of fifteen (15) control points observation

Points	Real Mix Ratios					Pseudo Mix Ratios				
	Water S_1	Cement S_2	Sand S_3	Granite S_4	PKS S_5	Water X_1	Cement X_2	Sand X_3	Granite X_4	PKS X_5
C_1	0.541	1	1.600	3.70	0.20	0.333	0.333	0.333	0	0
C_2	0.574	1	1.700	3.78	0.42	0.333	0	0.333	0.333	0
C_3	0.590	1	1.801	3.74	0.66	0.333	0	0	0.333	0.333
C_4	3.553	1	1.700	3.20	0.80	0.25	0.25	0.25	0.25	0
C_5	0.600	1	1.602	2.77	0.93	0.25	0	0.25	0.25	0.25
C_6	0.555	1	1.400	2.38	1.02	0.25	0.25	0.25	0	0.25
C_7	0.546	1	1.670	2.14	1.16	0.50	0.25	0.25	0	0
C_8	0.581	1	1.490	2.18	1.46	0.25	0	0.25	0	0.5
C_9	0.586	1	1.630	2.20	1.80	0.40	0.20	0.20	0.20	0
C_{10}	0.571	1	1.440	1.45	1.45	0.20	0.20	0.20	0.20	0.20
C_{11}	0.563	1	1.600	2.72	0.14	0.30	0.10	0.20	0.20	0.20
C_{12}	0.545	1	1.350	2.47	0.27	0.10	0.20	0.20	0.30	0.20
C_{13}	0.561	1	1.400	2.41	0.43	0.35	0.15	0.25	0	0.25
C_{14}	0.556	1	1.390	2.26	0.57	0.25	0.20	0.15	0.20	0.20
C_{15}	0.542	1	1.340	2.05	0.68	0.45	0.05	0	0.20	0.30

Legend: PKS = Palm Kernel shells

3.3. Compressive Strength Test

Compressive strength test was carried out using compressive testing machine in order to determine the compressive strength needed to formulate and validate the optimization function. Granite – Palm Kernel shells aggregate concrete specimens' cubes with size 150 X 150 x 150 mm^3 were cast. Total number of ninety cubes were cast for the thirty (30) mix ratios in Tables 1, 2 and 3. Three cubes from each were produced. The first 45 cubes made following Tables 1 and 3.2 were used for the formulation of the optimization model, the second set of 45 cubes made following table 3 were used to validate the optimization model. The concrete cubes were cured in a water container for 28 days and were crushed to obtain their compressive strength using the equation below.

Table 4: Compressive strength in N/mm^2 at 28 days old concrete cubes of granite – palm kernel shells aggregate concrete

Point	Replicate 1	Replicate 2	Replicate 3	Mean value
N_1	30.58	30.62	30.58	30.59
N_2	28.76	28.67	28.87	28.75
N_3	28.00	28.00	28.42	28.14
N_4	27.24	27.28	27.28	27.25
N_5	26.98	26.98	28.89	26.95
N_{12}	25.51	25.51	25.55	25.52
N_{13}	25.11	25.11	25.11	25.11
N_{14}	24.00	24.00	24.03	24.01
N_{15}	23.60	23.60	24.04	23.75
N_{23}	22.80	22.53	23.53	23.62

Table 5: Compressive strength in N/mm2 at 28 days old concrete cubes for control experiment of granite – palm kernel shells aggregate concrete

Point	Replicate 1	Replicate 2	Replicate 3	Mean value
C_1	23.02	24.00	22.31	23.11
C_2	22.31	22.31	22.22	22.28
C_3	21.64	21.69	21.73	21.69
C_4	21.07	21.07	21.07	21.07
C_5	20.05	20.05	20.05	20.05
C_{12}	19.60	19.96	19.77	19.77
C_{13}	19.47	19.29	19.36	19.37
C_{14}	17.91	19.07	18.58	18.52
C_{15}	18.27	18.54	18.62	18.48

C_{23}	16.89	18.76	18.00	17.88
C_{25}	20.00	20.09	20.00	20.03
C_{34}	19.07	19.07	19.04	19.06
C_{35}	18.27	17.32	19.11	18.40
C_{24}	21.70	21.70	21.70	21.70
C_{45}	16.98	16.93	16.98	16.96

Three (3) concrete cubes were cast for each mix ratio. Each of the three cubes from a mix ratio is referred to as 'Replicate'.

4. Results and Discussion

4.1. Compressive strength of granite- palm kernel shells aggregate concrete

The compressive strength of granite – palm kernel shells aggregate concrete decrease with increase in percentage replacement of granite with palm kernel shells. As quantity of palm kernel shells increases, the specific area increases, this requires more cement paste to bond properly with the kernel shells. Strength requires good bonding of the aggregates and cement. This is in agreement in the work of Daniel et al [9]. Thus, as bonding reduces with increase in replacement of palm kernel shell, compressive strength reduces.

4.2. Optimization function for predicting the Compressive Strength of the Concrete

Results of the compressive strength from 30^{th} main experiments and control experiment are given in tables 4 and 5.

The compressive strength of each cube was determined using the equation below.

$$\text{Compressive strength} = \frac{\text{Crush Load (N)}}{\text{Effective Area}(\text{mm}^2)}$$

From Table 4, the coefficients of the equation (6) are obtained as follows;

N_1 =30.59
N_2 =28.75
N_3 =28.14
N_4 =27.25
N_5 =27.95
N_{12} =4(25.52) =102.08
N_{13} =4(25.11) =100.44
N_{14} =4(24.01) =96.04
N_{15} =4(23.75) =95
N_{23} =4(22.62) =90.48
N_{24} =4(22.15) =88.60
N_{25} =4(21.33) =85.32

N_{34} =4(20.55) =82.20
N_{35} =4(20.05) =80.20
N_{45} =4(19.36) =76

Putting the above obtained values of coefficients into Equation (6) above gives;

$$Y = 30.59x_1(2x_1-1) + 28.75x_2(2x_2-1)+28.14x_3(2x_3-1) +27.25x_4(2x_4-1)+26.95x_5(2x_5-1)+ 102.08x_1x_2 +100.44x_1x_3 +96.04x_1x_4 + 95x_1x_5 + 90.48x_2x_3+88.60x_2x_4+85.32x_2x_5 +82.20x_3x4+80.20x_3x_5 + 76x_4x_5 \quad (9)$$

Equation (9) is the mathematical model for predicting and optimization of the compressive strength of Granite-palm kernel shells aggregate concrete based on 28 days strength.

Substituting mix ratios of points C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}, C_{14} and C_{15} into the equation (9) give the model response (compressive strength) as in Table 4.

Table 6: Model and Experimental results of compressive strength (N/mm2) of granite – palm kernel shells aggregate concrete

Points	Replicate 1	Replicate 2	Replicate 3	Mean Experimental Value	Model Prediction
N_1	30.58	30.62	30.57	30.59	30.59
N_2	28.75	28.70	28.80	28.75	28.75
N_3	28.10	28.15	28.17	28.14	28.14
N_4	27.28	27.27	27.20	27.25	27.25
N_5	26.97	26.97	26.91	26.95	26.95
N_{12}	25.51	26.05	25.00	25.52	25.52
N_{13}	25.14	25.01	25.18	25.11	25.11
N_{14}	24.33	23.00	24.20	24.01	24.01
N_{15}	23.18	23.65	23.70	23.75	23.75
N_{23}	22.52	22.66	22.68	22.62	22.62
N_{24}	22.16	22.10	22.19	22.15	22.15
N_{25}	21.40	21.42	21.16	2 1.33	21.33
N_{34}	20.59	20.50	20.56	20.55	20.55
N_{35}	20.02	20.10	20.03	20.05	20.05
N_{45}	19.86	18.30	19.92	19.36	19.36
C_1	23.00	24.01	22.32	23.11	22.75
C_2	22.30	22.34	22.20	22.28	21.34
C_3	21.65	21.70	21.72	21.69	20.18
C_4	21.21	20.90	21.10	21.07	20.65
C_5	20.00	20.10	20.06	20.05	19.00
C_{12}	18.97	20.60	19.80	19.77	20.29
C_{13}	19.48	19.30	19.40	19.37	23.86
C_{14}	17.94	19.05	18.57	18.52	20.84
C_{15}	18.24	18.58	18.62	18.48	21.79
C_{23}	16.89	18.77	17.98	17.85	18.85
C_{24}	20.10	20.90	21.10	20.70	19.52
C_{25}	20.02	20.07	20.00	20.03	18.10
C_{34}	19.05	19.10	19.03	19.06	20.95
C_{35}	18.30	17.80	19.10	18.40	19.23
C_{45}	16.99	16.93	16.96	16.96	21.26

4.3. Testing the adequacy of the mathematical model

Testing the adequacy of the mathematical model was done using fisher test at 95% confidence level on the compression strength at the control points C_1, C_2, C_3, C_4, C_5, C_{12}, C_{13}, C_{14}, C_{15}, C_{23}, C_{24}, C_{25}, C_{34}, C_{35} and C_{45}. Two hypothesis were established, which are;

1. Null Hypothesis

There is no significant difference between the model's results and experimental results.

2. Alternative Hypothesis

There is a significant difference between the model's results and experimental results.

The assumption of 95% accuracy implies that 5% or below of the model's results will be incorrect.

The Null Hypothesis will be accepted if: $1/F_{(table)} < S1^2 / S2^2 < F_{table}$.

If the condition is not satisfied then the alternative Hypothesis will be accepted.

The test was carried out using the table 7 below.

Table 7: Fisher-Statistical Test Computation for the Model

Points	Ye	Ym	Y_e-mean	Y_m-mean	$(Y_e\text{-mean})^2$	$(Y_m\text{-mean})^2$
C_1	23.11	22.75	3.22	2.19	10.37	4.80
C_2	22.28	21.34	2.39	0.77	5.71	0.59
C_3	21.69	20.18	1.79	-0.39	3.20	0.15
C_4	21.07	20.65	1.18	0.08	1.39	0.01
C_5	20.05	19.00	-0.16	-1.57	0.03	2.46
C_6	19.77	20.29	-0.12	-0.28	0.01	0.08
C_7	19.37	23.86	-0.52	3.29	0.27	10.82

C_8	18.52	20.84	-1.37	0.27	1.88	0.07
C_9	18.48	21.79	-1.41	1.22	1.99	1.49
C_{10}	17.85	18.85	-2.04	-1.72	4.16	2.96
C_{11}	20.70	19.52	1.81	-1.05	3.28	1.10
C_{12}	20.03	18.10	0.14	-2.47	0.02	6.10
C_{13}	19.06	20.95	-0.83	0.38	0.69	0.14
C_{14}	18.40	19.23	-1.49	-1.34	2.22	1.80
C_{15}	16.95	21.26	-2.94	0.69	8.64	0.48
Total	298.33	308.62			43.86	33.05
Mean	19.89	20.54				

LEGEND:

Ye = Compressive Strength from control experiment.

Ym = Compressive Strength from second degree polynomial equation.

N = 15

$$Se^2 = \sum \frac{(\text{Ye} - \text{ mean})2}{\text{N} - 1} = \frac{43.86}{14} = 3.13$$

$$Sm^2 = \sum \frac{(\text{Ym} - \text{ mean})2}{\text{N} - 1} = \frac{33.05}{14} = 2.36$$

$$\text{F}_{\text{calculated}} = \frac{S1^2}{S1^2}$$

Where $S1^2$ is the greater of Se^2 and Sm^2, while $S2^2$ is the smaller of the two.

Here, $S1^2 = Se^2 = 3.13$ and $S2^2 = Sm^2 = 2.36$

$$\text{F}_{\text{calculated}} = \frac{3.13}{2.36} = 1.32627$$

The mathematical model is acceptable at 95% confidence level if;

$$\frac{1}{\text{F}_{\alpha(v1,v2)}} < \frac{S1^2}{S1^2} < \text{F}_{\alpha(v1,v2)}$$

Significant level $\alpha = 1\text{-}0.95 = 0.05$; Degree of Freedom, $v = \text{N} - 1 = 14$

Therefore, $F\ \alpha\ (\text{v1,v2}) = F_{0.05\ (14,\ 14)}$

From the standard F – Statistical Table,

$F_{0.05\ (14,\ 14)} = 2.443$

$$\frac{1}{\text{F}_{\alpha(v1,v2)}} = \frac{1}{2.443} = 0.4093$$

Hence, the condition that;

$$\frac{1}{\text{F}_{\alpha(v1,v2)}} < \frac{S1^2}{S1^2} < \text{F}_{\alpha(v1,v2)}$$

which is $0.409 < 1.32627 < 2.443$ is satisfied

Therefore the Null Hypothesis that, "there is no significant difference between the experimental and the model expected results is accepted. This implies that the mathematical model equation is adequate.

4.4. Java Computer Programming for Testing of Model

Java computer programming language was written for the application of the proposed mathematical model. A step by step description of the Application procedure is presented as shown in the appendix. The programming was written to optimize the compressive strength of granite–palm kernel shell aggregate concrete.

5. Conclusions and Recommendations

5.1. Conclusions

- Palm kernel shells can be use as aggregate for the production of light weight concrete.
- There exist limited cost of construction of concrete using palm kernel shells as partial replacement of granite
- The use of palm kernel shells as aggregate for concrete as partial replacement of granite assist in preventing depletion of granite and gravel.

- The use of palm kernel shells as aggregate for concrete is a means for waste disposal of the by-product to the areas where it is produced.
- As the percentage replacement of palm kernel shells with granite increases, there is reduction in workability, density and compressive strength of the Granite-palm kernel shell aggregate concrete.
- Following the result of the experiment conducted, palm kernel shells aggregate can be use for the production of concrete grade 25 for light Wight concrete as normal mix of 1:1:2.
- The batch with the point (N_1) with 5% replacement of palm kernel shells has the highest compressive strength of 28.59 N/mm^2, which batch with the point (N_{43}), mix ratio 1:2:4 with strength of 17.48N/mm$^{2.}$
- The use of Henry Scheffe's (5,2) polynomial equation, the mathematical model for the mix design of a five component of Granite-palm kernel shells aggregate concrete was formulated. predictions were tested at 95% accuracy with the use of Fisher Test and found to be adequate. With known mix ratios, the mathematical model can predict the response (compressive strength) of Granite-palm kernels shells aggregate concrete or vice versa.

5.2. Recommendations

- Due to unavailability of fund for the production of concrete, the use of palm kernel shells should be encouraged where the needed materials are not available.
- An admixture (plasticizer) is necessary to be used in Granite-palm kernel shells aggregate concrete in other to increase the workability of the mix such that the water content can be low and the concrete strength can be increased.
- It is recommended that, while palm kernel shells can be use as aggregate for concrete production, further study should be carried out on long term length of palm kernel concrete.

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APPENDIX

JAVA COMPUTER PROGRAMME WRITTEN TO OPTIMIZE COMPRESSIVE STRENGTH OF GRANITE – PALM KERNEL SHELLS AGGREGATE CONCRETE.

```
import java.util.Scanner;

public class ControlPoints {
static double c1, c2, c3, c4, c5, c6, c7, c8, c9, c10, c11, c12, c13, c14, c15;
static double x1, x2, x3, x4, x5, y;
static Scanner sc = new Scanner(System.in);

public static void main (String [] args){
//First value of C
System.out.println("Input the first value of water (x1):");
x1 = sc.nextDouble();

System.out.println("Input the first value of cement (x2):");
x2 = sc.nextDouble();

System.out.println("Input the first value of sand (x3):");
x3 = sc.nextDouble();

System.out.println("Input the first value of Granite (x4):");
x4 = sc.nextDouble();

System.out.println("Input the first value of PKS (x5):");
x5 = sc.nextDouble();

 c1 = (30.59 * x 1 * ((2*x1)-1)) + (28.75 * x2 *((2*x2)-1)) + (28.14 * x3 *((2 * x3)-1)) + 27.25 * x4
*((2 * x4)-1))+ (26.95 * x5 *((2 * x5)-1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 * x1 *
x4) + (95 * x1 * x5) + (90.48 * x2 * x3) + (88.60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 * x4) +
(80.20 * x3 * x5) + (76 * x4 * x5);

System.out.println();
System.out.println();

//Second value of C
System.out.println("Input the secnd value of water (x1):");
x1 = sc.nextDouble();

System.out.println("Input the second value of cement (x2):");
x2 = sc.nextDouble();

System.out.println("Input the second value of sand (x3):");
```

```
x3 = sc.nextDouble();

System.out.println("Input the second value of Granite (x4):");
x4 = sc.nextDouble();

System.out.println("Input the second value of PKS (x5):");
x5 = sc.nextDouble();

c2 = (30.59 * x1 * ((2*x1)-1)) + (28.75 * x2 *((2*x2)-1)) + (28.14 * x3 *((2 * x3)-1)) + 27.25 * x4
*((2 * x4)-1)) + (26.95 * x5 *((2 * x5)-1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 * x1 *
x4) + (95 * x1 * x5) + (90.48 * x2 * x3) + (88.60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 * x4) +
(80.20 * x3 * x5) + (76 * x4 * x5);

System.out.println();
System.out.println();
//Third value of C
System.out.println("Input the third value of water (x1):");
x1 = sc.nextDouble();

System.out.println("Input the third value of cement (x2):");
x2 = sc.nextDouble();

System.out.println("Input the third value of sand (x3):");
x3 = sc.nextDouble();

System.out.println("Input the third value of Granite (x4):");
x4 = sc.nextDouble();

System.out.println("Input the third value of PKS (x5):");
x5 = sc.nextDouble();

c3 = (30.59 * x1 * ((2*x1)-1)) + (28.75 * x2 *((2*x2)-1)) + (28.14 * x3 *((2 * x3)-1)) +(27.25 * x4
*((2 * x4)-1)) + (26.95 * x5 *((2 * x5)-1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 * x1 *
x4) + (95 * x1 * x5) + (90.48 * x2 * x3) + (88.60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 * x4) +
(80.20 * x3 * x5) + (76 * x4 * x5);

System.out.println();
System.out.println();

//fourth value of C
System.out.println("Input the fourth value of water (x1):");
x1 = sc.nextDouble();

System.out.println("Input the fourth value of cement (x2):");
x2 = sc.nextDouble();
System.out.println("Input the fourth value of sand (x3):");
```

```
x3 = sc.nextDouble();

System.out.println("Input the fourth value of Granite (x4):");
x4 = sc.nextDouble();

System.out.println("Input the fourth value of PKS (x5):");
x5 = sc.nextDouble();

c4 = (30.59 * x1 * ((2*x1)-1)) + (28.75 * x2 *((2*x2)-1)) + (28.14 * x3 *((2 * x3)-1)) + (27.25 * x4
*((2 * x4)-1)) + (26.95 * x5 *((2 * x5)-1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 * x1 *
x4) + (95 * x1 * x5) + (90.48 *x2 * x3) + (88.60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 * x4) +
(80.20 * x3 * x5) + (76 * x4 * x5);

System.out.println();
System.out.println();

//fifth value of C
System.out.println("Input the fifth value of water (x1):");
x1 = sc.nextDouble();

System.out.println("Input the fifth value of cement (x2):");
x2 = sc.nextDouble();

System.out.println("Input the fifth value of sand (x3):");
X3 = sc.nextDoublte();

System.out.println("Input the thirteenth value of Granite (x4):");
x4 = sc.nextDouble();
System.out.println("Input the thirteenth value of PKS (x5):");
x5 = sc.nextDouble();

c5 = (30.59 * x1 * ((2*x1)-1)) + (28.75 * x2 *((2*x2)-1)) + (28.14 * x3 *((2 * x3)-1)) + (27.25 * x4
*((2 * x4)-1)) + (26.95 * x5 *((2 * x5)-1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 * x1 *
x4) + (95 * x1 * x5) + (90.48 * x2 * x3) + (88.60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 * x4) +
(80.20 * x3 * x5) + (76 * x4 * x5);

System.out.println();
System.out.println();

//sixth value of C
System.out.println("Input the sixth value of water (x1):");
x1 = sc.nextDouble();

System.out.println("Input the sixth value of cement (x2):");
x2 = sc.nextDouble();
```

```
System.out.println("Input the sixth value of sand (x3):");
x3 = sc.nextDouble();

System.out.println("Input the sixth value of Granite (x4):");
x4 = sc.nextDouble();

System.out.println("Input the sixth value of PKS (x5):");
x5 = sc.nextDouble();

c6 = (30.59 * x1 * ((2*x1)-1)) + (28.75 * x2 *((2*x2)-1)) + (28.14 * x3 *((2 * x3)-1)) + (27.25 * x4
*((2 * x4) -1)) + (26.95 * x5 *((2 * x5) -1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 * x1 *
x4) + (95 * x1 * x5) + (90.48 * x2 * x3) + (88.60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 * x4) +
(80.20 * x3 * x5) + (76 * x4 * x5);

System.out.println();
System.out.println();

//seventh value of C
System.out.println("Input the seventh value of water (x1):");
x1 = sc.nextDouble();

System.out.println("Input the seventh value of cement (x2):");
x2 = sc.nextDouble();

System.out.println("Input the seventh value of sand (x3):");
x3 = sc.nextDouble();

System.out.println("Input the seventh value of Granite (x4):");
x4 = sc.nextDouble();

System.out.println("Input the seventh value of PKS (x5):");
x5 = sc.nextDouble();

c7 = (30.59 * x1 * ((2*x1)-1)) + (28.75 * x2 *((2*x2)-1)) + (28.14 * x3 *((2 * x3)-1)) + (27.25 * x4
*((2 * x4)-1)) + (26.95 * x5 *((2 * x5)-1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 * x1 *
x4) + (95 * x1 * x5) + (90.48 * x2 * x3) + (88.60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 * x4) +
(80.20 * x3 * x5) + (76 * x4 * x5);

System.out.println();
System.out.println();

//eight value of C
System.out.println("Input the eight value of water (x1):");
x1 = sc.nextDouble();
System.out.println("Input the eight value of cement (x2):");
x2 = sc.nextDouble();
```

```
System.out.println("Input the eight value of sand (x3):");
x3 = sc.nextDouble();

System.out.println("Input the eight value of Granite (x4):");
x4 = sc.nextDouble();

System.out.println("Input the eight value of PKS (x5):");
x5 = sc.nextDouble();

c8 = (30.59 * x1 * ((2*x1)-1)) + (28.75 * x2 *((2*x2)-1)) + (28.14 * x3 *((2 * x3)-1)) + (27.25 * x 4
*((2 * x4) -1)) + (26.95 * x5 *((2 * x5) - 1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 * x1 *
x4) + (95 * x1 * x5) + (90.48 * x2 * x3) + (88.60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 * x4) +
(80.20 * x3 * x5) + (76 * x4 * x5);

System.out.println();
System.out.println();

//ninth value of C
System.out.println("Input the ninth value of water (x1):");
x1 = sc.nextDouble();

System.out.println("Input the ninth value of cement (x2):");
x2 = sc.nextDouble();

System.out.println("Input the ninth value of sand (x3):");
x3 = sc.nextDouble();

System.out.println("Input the ninth value of Granite (x4):");
x4 = sc.nextDouble();

System.out.println("Input the ninth value of PKS (x5):");
x5 = sc.nextDouble();

c9 = (30.59 * x1 * ((2*x1) - 1)) + (28.75 * x2 *((2*x2) - 1)) + (28.14 * x3 *((2 * x3) - 1)) + (27.25 *
x4 *((2 * x4) - 1)) + (26.95 * x5 *((2 * x5) - 1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 *
x1 * x4) + (95 * x1 * x5) + (90.48 * x2 * x3) + (88.60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 *
x4) + (80.20 * x3 * x5) + (76 * x4 * x5);

System.out.println();
System.out.println();

//tenth value of C
System.out.println("Input the tenth value of water (x1):");
x1 = sc.nextDouble();
```

```
System.out.println("Input the tenth value of cement (x2):");
x2 = sc.nextDouble();

System.out.println("Input the tenth value of sand (x3):");
x3= sc.nextDouble();

System.out.println("Input the tenth value of Granite (x4):");
x4 = sc.nextDouble();

System.out.println("Input the tenth value of PKS (x5):");
x5 = sc.nextDouble();

c10 = (30.59 * x1 * ((2*x1) - 1)) + (28.75 * x2 *((2*x2) - 1)) + (28.14 * x3 *((2 * x3) - 1)) + (27.25 *
x4 *((2 * x4) - 1)) + (26.95 * x5 *((2 * x5) - 1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 *
x1 * x4) + (95 * x1 * x5) + (90.48 * x2 * x3) + (88. 60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 *
x4) + (80.20 * x3 * x5) + (76 * x4 * x5);

System.out.println();
System.out.println();

// eleventh value of C
System.out.println("Input the eleventh value of water (x1):");
x1 = sc.nextDouble();

System.out.println("Input the eleventh value of cement (x2):");
x2 = sc.nextDouble();

System.out.println("Input the eleventh value of sand (x3):");
x3 = sc.nextDouble();

System.out.println("Input
the eleventh value of Granite (x4):");
x4 = sc.nextDouble();

System.out.println("Input the eleventh value of PKS (x5):");
x5 = sc.nextDouble();

c11 = (30.59 * x1 * ((2*x1) - 1)) + (28.75 * x2 *((2*x2) - 1)) + (28.14 * x3 *((2 * x3) - 1)) + (27.25 *
x4 *((2 * x4) - 1)) + (26.95 * x5 *((2 * x5) - 1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 *
x1 * x4) + (95 * x1 * x5) + (90.48 * x2 * x3) + (88.60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 *
x4) + (80.20 * x3 * x5) + (76 * x4* x5);

System.out.println();
System.out.println();
//twelvth value of C
System.out.println("Input the twelvth value of water (x1):");
```

```
x1 = sc.nextDouble();

System.out.println("Input the twelvth value of cement (x2):");
x2 = sc.nextDouble();

System.out.println("Input the twelvth value of sand (x3):");
x3 = sc.nextDouble();

System.out.println("Input the twelvth value of Granite (x4):");
x4 = sc.nextDouble();

System.out.println("Input the twelvth value of PKS (x5):");
x5 = sc.nextDouble();

c12 = (30.59 * x1 * ((2*x1) - 1)) + (28.75 * x2 *((2*x2) - 1)) + (28.14 * x3 *((2 * x3) - 1)) + (27.25 * x4 *((2 * x4) - 1)) + (26.95 * x5 *((2 * x5) - 1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 * x1 * x4) + (95 * x1 * x5) + (90.48 * x2 * x3) + (88.60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 * x4) + (80.20 * x3 * x5) + (76 * x4 * x5);

System.out.println();
System.out.println();

//thirteenth value of C
System.out.println("Input the thirteenth value of water (x1):");
x1 = sc.nextDouble();

System.out.println("Input the thirteenth value of cement (x2):");
x2 = sc.nextDouble();

System.out.println("Input the thirteenth value of sand (x3):");
x3 = sc.nextDouble();

System.out.println("Input the thirteenth value of Granite (x4):");
x4 = sc.nextDouble();

System.out.println("Input the thirteenth value of PKS (x5):");
x5 = sc.nextDouble();

c13 = (30.59 * x1 * ((2*x1) - 1)) + (28.75 * x2 *((2*x2) - 1)) + (28.14 * x3 *((2 * x3) - 1)) + (27.25 * x4 *((2 * x4) - 1)) + (26.95 * x5 *((2 * x5) - 1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 * x1 * x4) + (95 * x1 * x5) + (90.48 * x2 * x3) + (88.60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 * x4) + (80.20 * x3 * x5) + (76 * x4 * x5);

System.out.println();
System.out.println();
```

```
//fourteenth value of C
System.out.println("Input the fourteenth value of water (x1):");
x1 = sc.nextDouble();

System.out.println("Input the fourteenth value of cement (x2):");
x2 = sc.nextDouble();

System.out.println("Input the fourteenth value of sand (x3):");
x3 = sc.nextDouble();

System.out.println("Input the fourteenth value of Granite (x4):");
x4 = sc.nextDouble();

System.out.println("Input the fourteenth value of PKS (x5):");
x5 = sc.nextDouble();

c14 = (30.59 * x1 * ((2*x1) - 1)) + (28.75 * x2 *((2*x2) - 1)) + (28.14 * x3 *((2 * x3) - 1)) + (27.25 *
x4 *((2 * x4) - 1)) + (26.95 * x5 *((2 * x5) - 1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 *
x1 * x4) + (95 * x1 * x5) +  (90.48 * x2 * x3) + (88.60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 *
x4) + (80.20 * x3 * x5) + (76 * x4 * x5);

System.out.println();
System.out.println();

//fifteenth value of C
System.out.println("Input the fifteenth value of water (x1):");
x1 = sc.nextDouble();

System.out.println("Input the fifteenth value of cement (x2):");
x2 = sc.nextDouble();

System.out.println("Input the fifteenth value of sand (x3):");
x3 = sc.nextDouble();

System.out.println("Input the fifteenth value of Granite (x4):");
x4 = sc.nextDouble();

System.out.println("Input the fifteenth value of PKS (x5):");
x5 = sc.nextDouble();

c15 = (30.59 * x1 * ((2*x1) - 1)) + (28.75 * x2 *((2*x2) - 1)) + (28.14 * x3 * ((2 * x3) - 1)) + (27.25 *
x4 *((2 * x4) - 1)) + (26.95 * x5 *((2 * x5) - 1)) + (102.08 * x1 * x2) + (100.44 * x1 * x3) + (96.04 *
x1 * x4) + (95 * x1 * x5) + (90.48 * x2 * x3) + (88.60 * x2 * x4) + (85.32 * x2 * x5) + (82.20 * x3 *
x4) + (80.20 * x3 * x5) + (76 * x4 * x5);

String a1 = String.format("%.02f", c1);
```

```
String a2 = String.format("%.02f", c2);
String a3 = String.format("%.02f", c3);
String a4 = String.format("%.02f", c4);
String a5 = String.format("%.02f", c5);
String a6 = String.format("%.02f", c6);
String a7 = String.format("%.02f", c7);
String a8 = String.format("%.02f", c8);
String a9 = String.format("%.02f", c9);
String a10 = String.format("%.02f", c10);
String a11 = String.format("%.02f", c11);
String a12 = String.format("%.02f", c12);
String a13 = String.format("%.02f", c13);
String a14 = String.format("%.02f", c14);
String a15 = String.format("%.02f", c15);
System.out.println("C1 = " + a1);
System.out.println("C2 = " + a2);
System.out.println("C3 = " + a3);
System.out.println("C4 = " + a4);
System.out.println("C5 = " + a5);
System.out.println("C6 = " + a6);
System.out.println("C7 = " + a7);
System.out.println("C8 = " + a8);
System.out.println("C9 = " + a9);
System.out.println("C10 = " + a10);
System.out.println("C11 = " + a11);
System.out.println("C12 = " + a12);
System.out.println("C13 = " + a13);
System.out.println("C14 = " + a14);
System.out.println("C15 = " + a15);

System.out.println();
System.out.println();
System.out.println();

if(c1 > c2 & c2 > c3 & c3 >c4 & c4>c5 & c5>c6 & c6>c7 & c7>c8 & c8>c9 & c9>c10 & c10 > c11
& c11>c12 & c12>c13 &c13>c14 & c14>c15){

   System.out.println("The highest value of C is " + c1);
}
if(c2 > c3 & c3 >c4 & c4>c5 & c5>c6 & c6>c7 & c7>c8 & c8>c9 & c9>c10 & c10 > c11 & c11>c12
& c12>c13 &c13>c14 & c14>c15 & c15>c1){

   System.out.println("The highest value of C is " + c2);
}
if(c1 > c2 & c3 >c4 & c4>c5 & c5>c6 & c6>c7 & c7>c8 & c8>c9 & c9>c10 & c10 > c11 & c11>c12
& c12>c13 &c13>c14 & c14>c15 & c15 > c1){
```

```
    System.out.println("The highest value of C is " + c3);
}
if(c1 > c2 & c2 > c3 & c4>c5 & c5>c6 & c6>c7 & c7>c8 & c8>c9 & c9>c10 & c10 > c11 & c11>c12
& c12>c13 &c13>c14 & c14>c15 & c15 > c1){

    System.out.println("The highest value of C is " + c4);
}
if(c1 > c2 & c2 > c3 & c3 >c4 & c5>c6 & c6>c7 & c7>c8 & c8>c9 & c9>c10 & c10 > c11 &
c11>c12 & c12>c13 &c13>c14 & c14>c15 & c15>c1){

    System.out.println("The highest value of C
is " + c5);
}
if(c1 > c2 & c2 > c3 & c3 >c4 & c4>c5 & c6>c7 & c7>c8 & c8>c9 & c9>c10 & c10 > c11 &
c11>c12 & c12>c13 &c13>c14 & c14>c15 & c15>c1){

    System.out.println("The highest value of C is " + c6);
}
if(c1 > c2 & c2 > c3 & c3 >c4 & c4>c5 & c5>c6 & c7>c8 & c8>c9 & c9>c10 & c10 > c11 &
c11>c12 & c12>c13 &c13>c14 & c14>c15 & c15>c1){

   System.out.println("The highest value of C is " + c7);
}
if(c1 > c2 & c2 > c3 & c3 >c4 & c4>c5 & c5>c6 & c6>c7 & c9>c10 & c10 > c11 & c11>c12 &
c12>c13 &c13>c14 & c14>c15 & c15>c1){
    System.out.println("The highest value of C is " + c8);
}
if(c1 > c2 & c2 > c3 & c3 >c4 & c4>c5 & c5>c6 & c6>c7 & c7>c8 & c9>c10 & c10 > c11 &
c11>c12 & c12>c13  &c13>c14 & c14>c15  & c15>c1){

    System.out.println("The highest value of C is " + c9);
}
    System.out.println("The highest value of C is " + c7 + "N/mm²");
if(c1 > c2 & c2 > c3 & c3 >c4 & c4>c5 & c5>c6 & c6>c7 & c7>c8 & c8>c9 & c10 > c11 & c11>c12
& c12>c13 &c13>c14 & c14>c15 & c15>c1){

    System.out.println("The highest value of C is " + c10);
}
if(c1 > c2 & c2 > c3 & c3 >c4 & c4>c5 & c5>c6 & c6>c7 & c7>c8 & c8>c9 & c9>c10 & c11>c12 &
c12>c13 & c13>c14 &c14>c15 & c15>c1){

    System.out.println("The highest value of C is " + c11);
}
if(c1 > c2 & c2 > c3 & c3 >c4 & c4>c5 & c5>c6 & c6>c7 & c7>c8 & c8>c9 & c9>c10 & c10 > c11
& c12>c13 &  c13>c14& c14>c15 & c15>c1){
```

```
    System.out.println("The highest value of C is " + c12);
}
if(c1 > c2 & c2 > c3 & c3 >c4 & c4>c5 & c5>c6 & c6>c7 & c7>c8 & c8>c9 & c9>c10 & c10 > c11
& c11>c12 & c13>c14 & c14>c15 & c15>c1){

    System.out.println("The highest value of C is " + c13);
}
if(c1 > c2 & c2 > c3 & c3 >c4 & c4>c5 & c5>c6 & c6>c7 & c7>c8 & c8>c9 & c9>c10 & c10 > c11
& c11>c12 & c12>c13 & c14>c15 & c15>c1){

    System.out.println("The highest value of C is " + c14);
}
}
}
```

Result:

C1 = 22.76
C2 = 21.34
C3 = 20.18
C4 = 20.65
C5 = 19.00
C6 = 20.29
C7 = 23.86
C8 = 20.84
C9 = 21.79
C10 = 18.85
C11 = 19.52
C12 = 18.10
C13 = 20.95
C14 = 19.27
C15 = 21.32
The highest value of C is 23.85875N/mm^2