CLOSED FORM FORMULAS FOR ELF INDUCTIVE COUPLING BETWEEN FINITE LENGTH WIRES WITH EARTH RETURN

Giovanni **LUCCA**
SIRTI S.p.A, Milano, Italy
G.Lucca@sirti.it

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Abstract: *This paper deals with the evaluation of the inductive coupling between an overhead finite length wire circuit and a buried wire circuit, both with earth return, in the range of Extremely Low Frequencies (ELF). In particular, the attention is focused on the key parameter for this kind of problems i.e. the per unit length (p.u.l.) mutual impedance between the two circuits. Starting from the exact formula involving a Sommerfeld's integral, two new approximated analytical formulas, having different levels of approximation, are derived; the advantage of these formulas is that, in such a way, the p.u.l. mutual impedance is expressed only by means of elementary functions so avoiding numerical calculations of the Sommerfeld's integral. Finally, a comparison between the two proposed formulas is made.*

1. INTRODUCTION

It is known that the issue of Extremely Low Frequency electromagnetic interference from power lines/electrified railway lines on pipelines is an important topic related to the electrical safety for staff getting in touch with accessible parts of the pipeline. Risks for safety exist whether the power line/railway line is both in faulty condition or is in normal operating condition; moreover, overvoltage and overcurrent induced on the pipeline may produce damages to the pipeline itself (e.g. insulating coating perforation) or to devices and apparatuses connected to it (e.g. cathodic protection devices). Finally, one has not to forget the AC corrosion risk that is related to the electromagnetic induction generated on the pipe-earth circuit under normal operating condition of the inducing line [1-2].

This paper focuses on the inductive interference generated on long buried metallic structures such as pipelines, ducts, telecommunication cables; the frequencies considered are the typical ones used for transmission and distribution of electrical power i.e. 50-60Hz.

The key parameter for the evaluation of the inductive coupling is the p.u.l. mutual impedance which is directly proportional to the inductive component of the electric field generated by the power line conductors. Such conductors, hung, between two consecutive towers, can be represented, in first approximation, as long horizontal and finite length wires parallel to the soil surface.

In literature, only few contributions, concerning the approach to this problem by means of the finite length wires model, exist; in fact, most of the works are focused on the infinite length wires model whose progenitor is the famous paper by Carson published in 1926 [3].

For a comparison between the two approaches (i.e. finite and infinite length wire models) one can have a look at [4].

It is also worth to mention alternative approaches involving the Finite Element Method [5-8], but they are out of the scope of the present paper.

Among the papers, dealing with mutual coupling between finite length wires with earth return, one has to mention [9-14] where the mutual impedance is expressed by means of the so called "Neumann formulas" involving integrals along the wires path. A different technique is adopted in [15] where the author, by considering an overhead linear horizontal antenna and by applying the "complex image theory", obtained approximated closed form formulas for electric and magnetic fields valid in the quasi-static range and in the upper semi-space. Nevertheless, no formulas were given for evaluating the field in the lower semi-space i.e. in the soil.

This paper proposes new closed form formulas, built up by means of elementary functions, for evaluating, the inductive component of the electric field in the soil in the ELF range. In particular, thanks to these formulas, one can avoid numerical integration of Sommerferld's integral.

2. DERIVATION OF THE FORMULAS

2.1. General

The starting point is evaluating the inductive component of the electric field generated by a finite length overhead wire with earth return carrying a harmonic constant current given by the phasor I_{pl}. The overhead wire, assumed having constant height H over the soil, is a representation of a generic power line conductor hung between two consecutive towers (See *fig.1*).

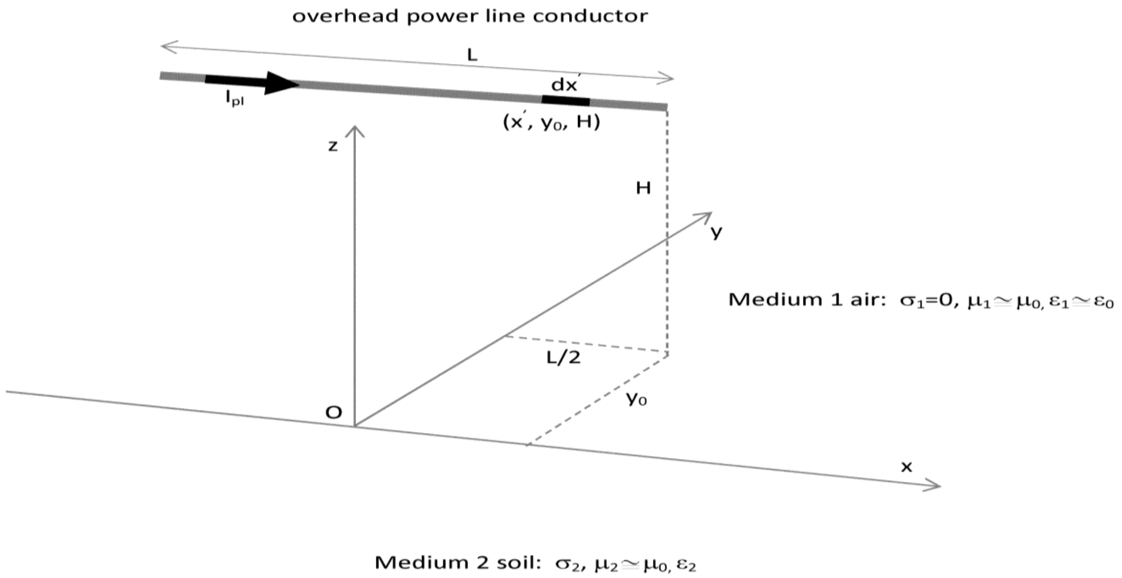

Fig. 1. Representation of a power line conductor as an overhead finite wire length

It is convenient to suppose the wire, having length *L*, as composed by infinitesimal horizontal electric dipoles each one of length *dx*'; thus, the field produced by the whole wire can be calculated as sum of all the contributions generated by the single dipoles.

According to [16], the inductive x-component of the electric field produced in the soil (medium 2, *z*<0) in the generic point *(x, y, z)* by a single dipole placed in *(x', y_0, H)* is proportional to the *x*-component of the electric Hertz potential *Π i.e.*:

$$dE_{2x}^{ind}(x,y,z) = k_2^2 \Pi_{2x}(x,y,z) =$$
$$= \frac{j\omega\mu_0 I dx'}{4\pi}\int_0^{\infty}\frac{2\lambda e^{-\sqrt{\lambda^2-k_1^2}H}e^{\sqrt{\lambda^2-k_2^2}z}}{\sqrt{\lambda^2-k_1^2}+\sqrt{\lambda^2-k_2^2}}J_0\left(\lambda\sqrt{(x-x')^2+(y'-y_0)^2}\right)d\lambda \qquad (1)$$

Note that (1) is expressed by means of a Sommerfeld's integral. In (1), *j* is the imaginary unit, J_0 is the Bessel function of the first kind and order 0 and k_1, k_2 are respectively given by:

$$k_1^2 = -j\omega\mu_0(j\omega\varepsilon_0) \qquad (2)$$

$$k_2^2 = -j\omega\mu_0(\sigma_2 + j\omega\varepsilon_2) \qquad (3)$$

In order to obtain the field produced by the wire, one has to integrate expression (1) over the whole wire length, i.e.:

$$E_{2x}^{ind}(x,y,z)$$
$$= \int_{-\frac{L}{2}}^{\frac{L}{2}}\left[\frac{j\omega\mu_0 I}{4\pi}\int_0^{\infty}\frac{2\lambda e^{-\sqrt{\lambda^2-k_1^2}H}e^{\sqrt{\lambda^2-k_2^2}z}}{\sqrt{\lambda^2-k_1^2}+\sqrt{\lambda^2-k_2^2}}J_0\left(\lambda\sqrt{(x-x')^2+(y'-y_0)^2}\right)d\lambda\right]dx' \qquad (4)$$

2.2. Approximated formulas

By introducing suitable hypotheses, it is possible to simplify (1), so that the field $E_{2x}^{ind}(x,y,z)$ can be expressed by means of an analytical expression much simpler than (4).

The simplifying assumptions can be listed as follows:

- Neglecting displacement currents i.e.: $k_1 \cong 0$ (in the air) and $k_2^2 = -j\omega\mu_0\sigma_2$ (in the earth)
- Due to the extremely low frequencies considered (50-60Hz), to the typical range of values for the soil conductivity σ_2 (i.e.: $[10^{-4}, 10^{-1}]$S/m) and to the typical range of values for the burial depth of pipes (i.e.: [0.5, 2]m), it is possible to write with good approximation:

$$e^{\sqrt{\lambda^2-k_2^2}z} \cong e^{\lambda z} \tag{5}$$

Therefore, by defining $\gamma^2 = k_2^2$, formula (1) can be simplified as follows:

$$dE_{2x}^{ind}(x,y,z) = \frac{j\omega\mu_0 I}{4\pi}\int_0^\infty \frac{2\lambda e^{-\lambda H}e^{\lambda z}}{\lambda+\sqrt{\lambda^2+\gamma^2}} J_0\left(\lambda\sqrt{(x-x')^2+(y'-y_0)^2}\right)d\lambda \tag{6}$$

In (6), according to [17] a further simplification can be introduced:

$$\frac{2\lambda}{\lambda+\sqrt{\lambda^2+\gamma^2}} = 1 - e^{2\frac{\lambda}{\gamma}}\left[1+\frac{1}{3}\left(\frac{\lambda}{\gamma}\right)^3+\cdots\right] \tag{7}$$

By substituting (7) into (6) and after some algebric steps, that have been omitted for brevity, one obtains, by the help of certain identities, [18]:

$$dE_{2x}^{ind}(x,y,z) = \frac{j\omega\mu_0 I dx'}{4\pi}\left\{\begin{array}{c} \frac{1}{\sqrt{(x-x')^2+(y-y_0)^2+(|z|+H)^2}} + \\ + \frac{-1}{\sqrt{(x-x')^2+(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2}} + \\ -\frac{(-1)^3}{\gamma^3}\frac{3\left(|z|+H+\frac{2}{\gamma}\right)\left[(x-x')^2+(y-y_0)^2\right]-2\left(|z|+H+\frac{2}{\gamma}\right)^3}{\left(\sqrt{(x-x')^2+(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2}\right)^7} \end{array}\right\} \tag{8}$$

In particular, the following identity has been used:

$$\frac{1}{3\gamma^3}\int_0^\infty \lambda^3 e^{-\left(|z|+H+\frac{2}{\gamma}\right)\lambda} J_0\left(\lambda\sqrt{(x-x')^2+(y'-y_0)^2}\right)d\lambda =$$

$$\frac{(-1)^3}{3\gamma^3}\frac{d^3}{d\left(|z|+H+\frac{2}{\gamma}\right)^3}\left(\frac{1}{\sqrt{(x-x')^2+(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2}}\right) \tag{9}$$

In order to obtain the total electric field produced by the overhead wire, one has to integrate expression (8), over whole wire length so obtaining:

$$E_{2x}^{ind}(x,y,z) = \frac{j\omega\mu_0 I}{4\pi}\left\{\begin{array}{c} \ln\left(\dfrac{\frac{L}{2}-x+\sqrt{\left(\frac{L}{2}-x\right)^2+(y-y_0)^2+(|z|+H)^2}}{-\left(\frac{L}{2}+x\right)+\sqrt{\left(\frac{L}{2}+x\right)^2+(y-y_0)^2+(|z|+H)^2}}\right)+ \\ -\ln\left(\dfrac{\frac{L}{2}-x+\sqrt{\left(\frac{L}{2}-x\right)^2+(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2}}{-\left(\frac{L}{2}+x\right)+\sqrt{\left(\frac{L}{2}+x\right)^2+(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2}}\right)+ \\ +\dfrac{(-1)^3}{\gamma^3}\dfrac{3\left(|z|+H+\frac{2}{\gamma}\right)}{\left[(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2\right]^2}\sum_{k=0}^{1}\dfrac{(-1)^k}{2k+3}\binom{1}{k}\dfrac{(x'-x)^{2k+3}}{\left[\sqrt{(x-x')^2+(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2}\right]^{2k+3}}\Bigg|_{x'=\frac{-L}{2}}^{x'=\frac{L}{2}}+ \\ +\dfrac{(-1)^3}{\gamma^3}\dfrac{\left[3\left(|z|+H+\frac{2}{\gamma}\right)(y-y_0)^2-2\left(|z|+H+\frac{2}{\gamma}\right)^3\right]}{\left[(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2\right]^3}\sum_{k=0}^{2}\dfrac{(-1)^k}{2k+1}\binom{2}{k}\dfrac{(x'-x)^{2k+1}}{\left[\sqrt{(x-x')^2+(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2}\right]^{2k+1}}\Bigg|_{x'=\frac{-L}{2}}^{x=\frac{L}{2}} \end{array}\right\} \tag{10}$$

Also in this case, all the mathematical passages leading from (8) to (10) have been omitted for brevity.

The p.u.l. mutual impedance between the overhead wire with earth return and a buried parallel wire with earth return evaluated in the generic point *(x, y, z)* belonging to the wire axis is defined by:

$$Z_m(x,y,z) = \frac{E_{2x}^{ind}(x,y,z)}{I} = \frac{j\omega\mu_0}{4\pi}\left\{\begin{array}{c} \ln\left(\dfrac{\frac{L}{2}-x+\sqrt{\left(\frac{L}{2}-x\right)^2+(y-y_0)^2+(|z|+H)^2}}{-\left(\frac{L}{2}+x\right)+\sqrt{\left(\frac{L}{2}+x\right)^2+(y-y_0)^2+(|z|+H)^2}}\right)+ \\ +\ln\left(\dfrac{\frac{L}{2}-x+\sqrt{\left(\frac{L}{2}-x\right)^2+(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2}}{-\left(\frac{L}{2}+x\right)+\sqrt{\left(\frac{L}{2}+x\right)^2+(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2}}\right)+ \\ +\dfrac{(-1)^3}{\gamma^3}\dfrac{3\left(|z|+H+\frac{2}{\gamma}\right)}{\left[(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2\right]^2}\sum_{k=0}^{1}\dfrac{(-1)^k}{2k+3}\binom{1}{k}\dfrac{(x'-x)^{2k+3}}{\left[\sqrt{(x-x')^2+(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2}\right]^{2k+3}}\Bigg|_{x'=\frac{-L}{2}}^{x'=\frac{L}{2}}+ \\ +\dfrac{(-1)^3}{\gamma^3}\dfrac{\left[3\left(|z|+H+\frac{2}{\gamma}\right)(y-y_0)^2-2\left(|z|+H+\frac{2}{\gamma}\right)^3\right]}{\left[(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2\right]^3}\sum_{k=0}^{2}\dfrac{(-1)^k}{2k+1}\binom{2}{k}\dfrac{(x'-x)^{2k+1}}{\left[\sqrt{(x-x')^2+(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2}\right]^{2k+1}}\Bigg|_{x'=\frac{-L}{2}}^{x=\frac{L}{2}} \end{array}\right\} \tag{11}$$

Thus, from (11) one gets a function $Z_m=Z_m(x, y, z)$ that it is composed by four addends; the first and second addends represent the basic contribution (logarithmic terms) while the third and fourth ones may be considered as additional corrective terms; in particular, it can be interesting to estimate their influence on the results.

To this aim, it is convenient to define the p.u.l. mutual impedance $Z_m'=Z_m'(x, y, z)$ without additional terms, i.e.:

$$Z'_m(x,y,z) = \frac{j\omega\mu_0 I}{4\pi}\left\{\begin{array}{l} \ln\left(\dfrac{\frac{L}{2}-x+\sqrt{\left(\frac{L}{2}-x\right)^2+(y-y_0)^2+(|z|+H)^2}}{-\left(\frac{L}{2}+x\right)+\sqrt{\left(\frac{L}{2}+x\right)^2+(y-y_0)^2+(|z|+H)^2}}\right)+ \\ -\ln\left(\dfrac{\frac{L}{2}-x+\sqrt{\left(\frac{L}{2}-x\right)^2+(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2}}{-\left(\frac{L}{2}+x\right)+\sqrt{\left(\frac{L}{2}+x\right)^2+(y-y_0)^2+\left(|z|+H+\frac{2}{\gamma}\right)^2}}\right)\end{array}\right\} \quad (12)$$

Note that formula (12) is obtained by considering only the first addend, inside square brackets, in formula (7).

3. COMPARISON BETWEEN THE MUTUAL IMPEDANCE FORMULAS

In order to calculate the influence of the corrective terms, it is useful to define the following quantities:

$$\Delta Z_{m\,\%}(x,y,z) = \frac{|Z_m(x,y,z)|-|Z'_m(x,y,z)|}{|Z_m(x,y,z)|}100 \quad (13)$$

$$\Delta\phi(x,y,z) = arg\big(Z_m(x,y,z)\big) - arg\big(Z'_m(x,y,z)\big) \quad (14)$$

Being $\Delta Z_{m\%}$ the per cent relative difference between the modulus of Z_m and Z_m' while $\Delta\phi$ is the difference between their arguments.

It is convenient to express the results by means of polar plots being the polar radius $R=R(x, y)$ given by the formula:

$$R(x,y) = \sqrt{x^2+(y-y_0)^2} \quad (15)$$

The polar plots that follow have been obtained for $y_0 = 0$, $H = 20$m, and $h_{pipe} = -1.5$m (typical burial depth of a pipeline), $L = 250$m (typical distance between two consecutive towers for a High Voltage power line) and the polar angle is counter-clockwise measured starting from the positive x semi-axis. They have been drawn for different values of soil conductivity and in correspondence of different values for the polar radius R, i.e.: $\delta/100$, $\delta/10$, $\delta/2$, δ, 2δ, 3δ, 5δ. Being δ the skin depth in the earth at the frequency f = 50Hz.

For convenience, the formula for the skin depth in the earth is reported:

$$\delta = \frac{1}{\sqrt{\pi f \mu_2 \sigma_2}} \tag{16}$$

Figures 2-4 show the polar plots of $\Delta Z_{m\%}$ and $\Delta\phi$ evaluated for different values of soil conductivity and it can be noticed that:

- The per cent relative difference $\Delta Z_{m\%}$ ranges approximatively in the interval [-20%, 25%]
- The difference $\Delta\phi$ ranges approximatively in the interval [-3^0, 13^0]

These differences occur essentially in the range [$\delta/2$, 3δ] for the polar radius while, on the contrary they are negligible outside that range. Therefore, one has:

- if $R \ll \delta$ or $R \gg \delta$, then the use of Z_m or Z_m' is essentially equivalent
- on the contrary, when $R \sim \delta$, a certain not negligible difference between Z_m and Z_m' can be noticed.

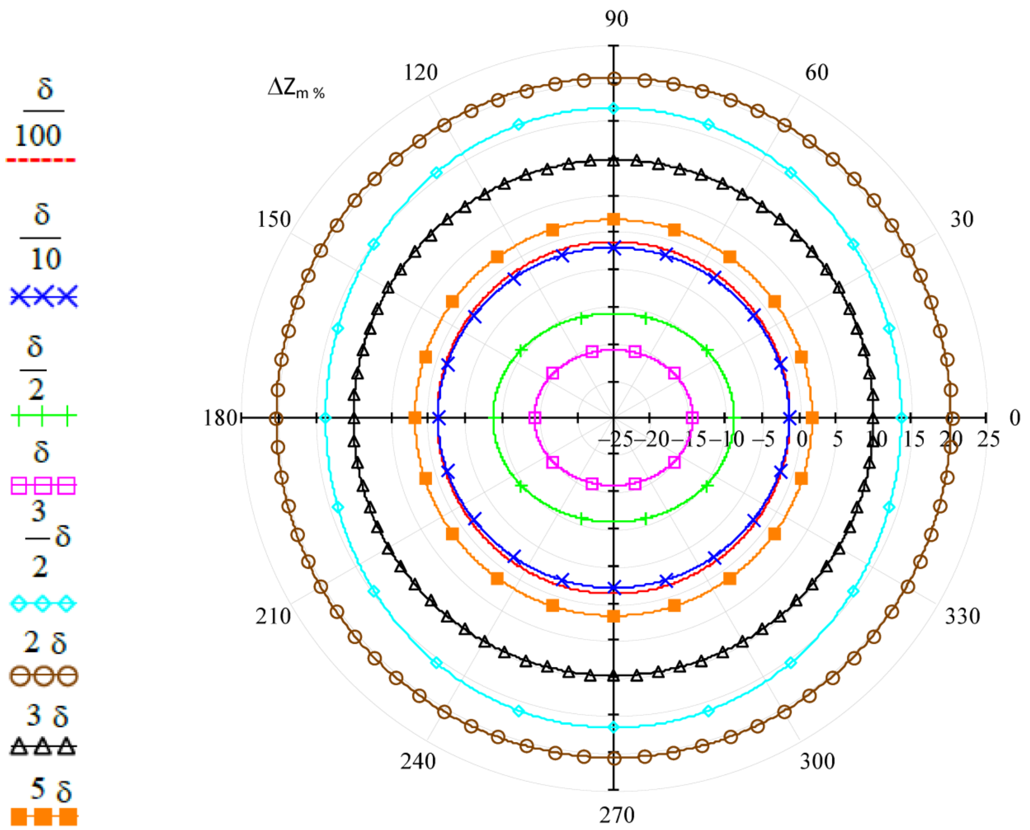

Fig. 2a. Polar plot of $\Delta Z_{m\%}$ for different values of polar radius; $\sigma_2 = 2\cdot10^{-2}$S/m

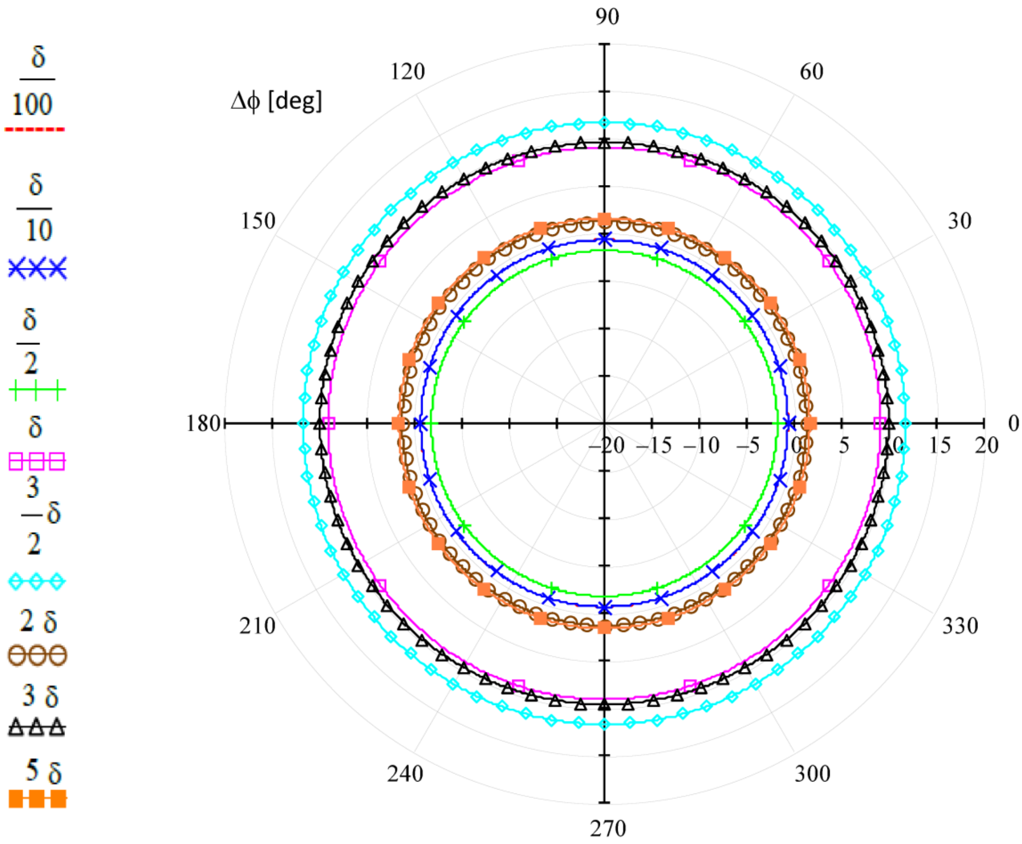

Fig. 2b. Polar plot of $\Delta\phi$ for different values of polar radius; $\sigma_2=2\cdot10^{-2}$S/m

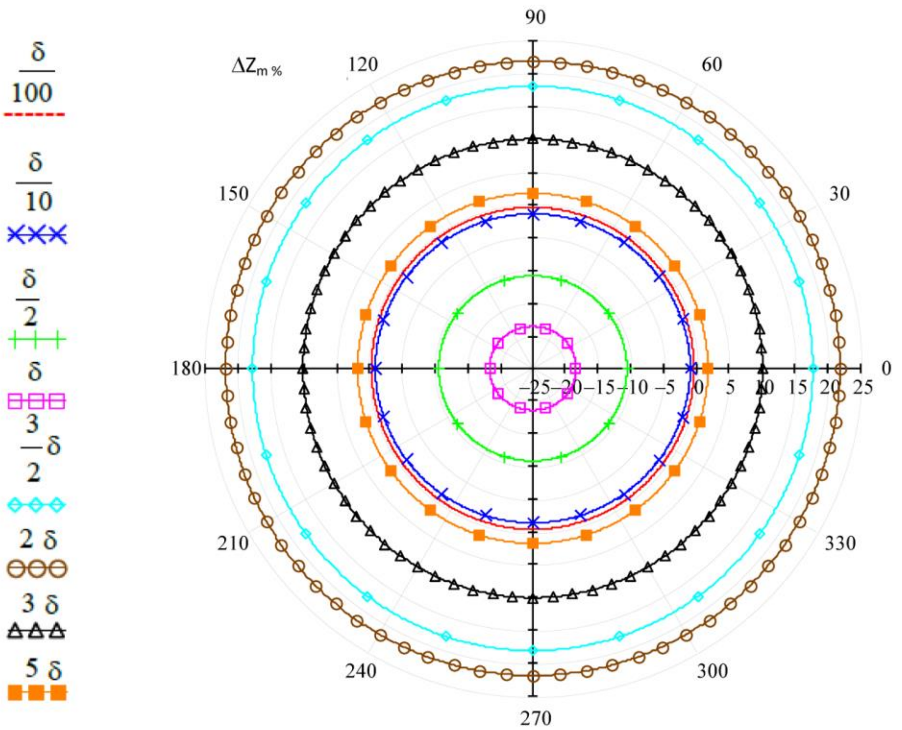

Fig. 3a. Polar plot of $\Delta Z_{m\%}$ for different values of polar radius; $\sigma_2=2\cdot10^{-3}$S/m

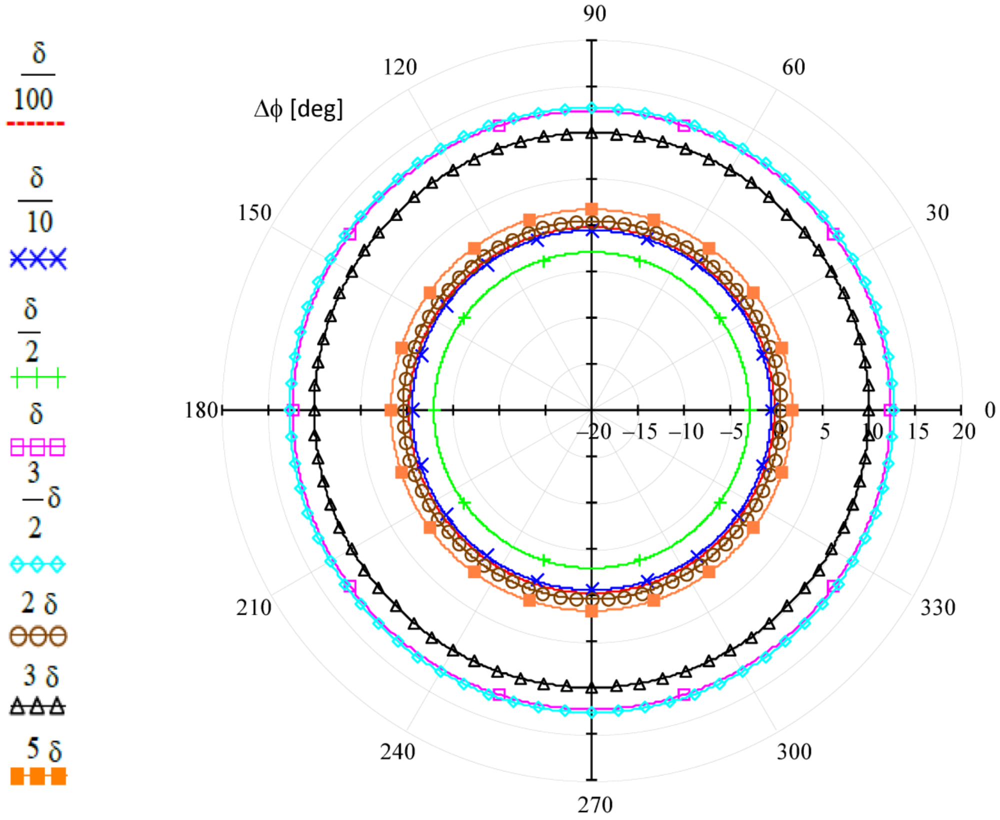

Fig. 3b. Polar plot of $\Delta\phi$ for different values of polar radius; $\sigma_2=2\cdot10^{-3}$S/m

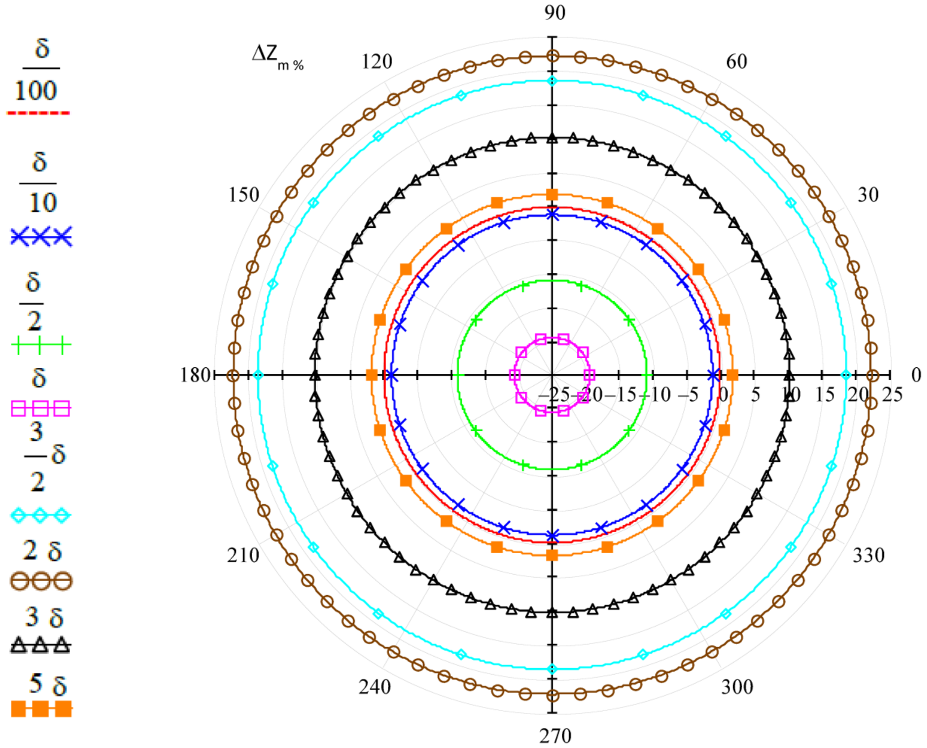

Fig. 4a. Polar plot of $\Delta Z_{m\%}$ for different values of polar radius; $\sigma_2=2\cdot10^{-4}$S/m

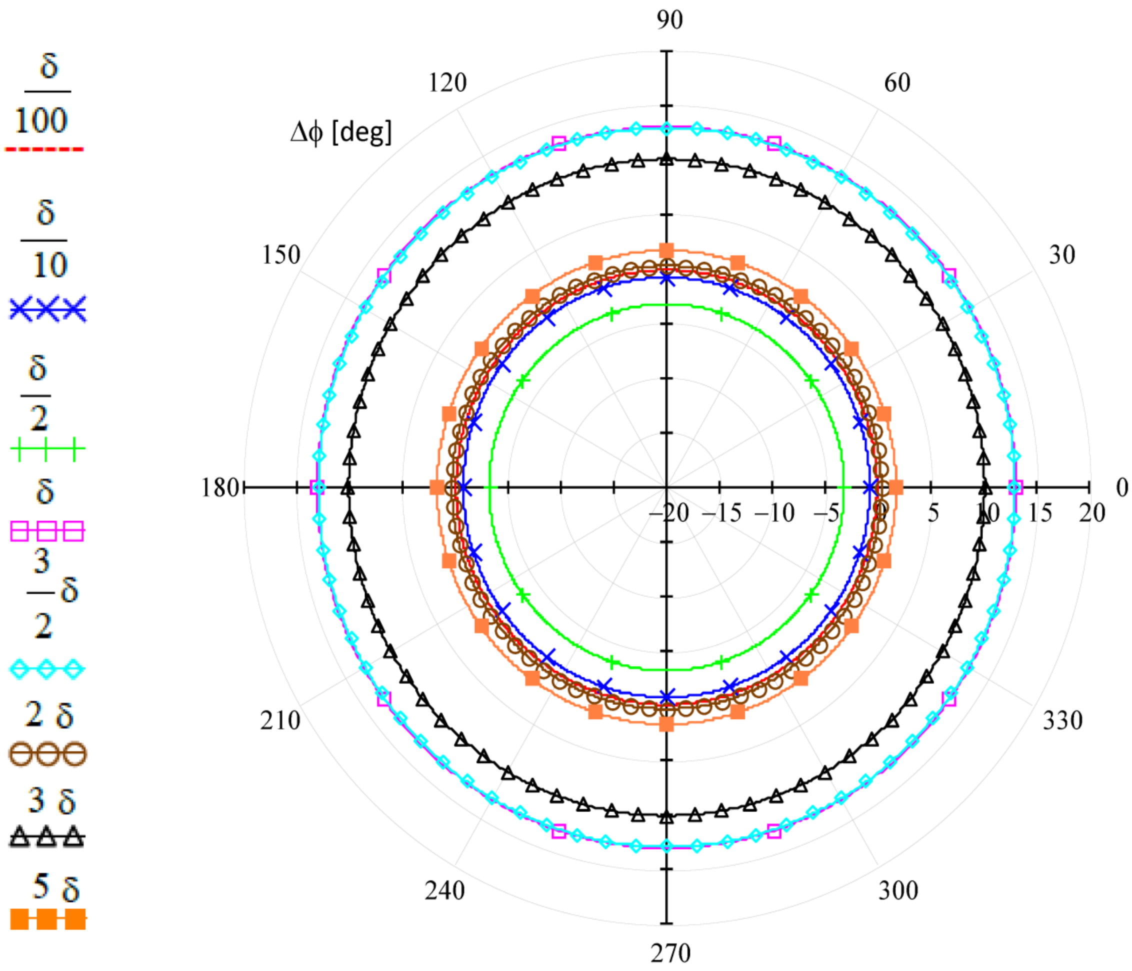

Fig. 4b. Polar plot of $\Delta\phi$ for different values of polar radius; $\sigma_2=2\cdot10^{-4}$S/m

4. INFLUENCE OF ADDITIONAL TERMS IN INTERFERENCE CALCULATIONS

A typical and straightforward application of the proposed formulas consists in evaluating the p.u.l. electromotive force (emf) induced on a pipeline-earth circuit under the inductive influence of nearby power lines and then calculating the induced voltage and current on the circuit itself.

Thus, it is interesting to check the influence in using $Z_m{}'$ instead of Z_m when calculating induced voltage and current on a buried pipeline subjected to the 50Hz inductive coupling from a power line. To this purpose, a simple example consisting of a parallelism between an overhead power line conductor 10km long and a pipeline 5km long is considered. See *fig.5* for a simple sketch. The power line conductor-earth circuit carries a constant inducing current I_{pl} while the pipe-earth circuit is closed at its boundaries on the characteristic impedance.

By treating the pipeline-earth circuit as a transmission line circuit, the influence of the electromagnetic field produced by the power line is modelled by a p.u.l. distributed emf generator applied along the pipeline-earth circuit. In the present example, being inducing and induced line parallel, such a generator is related to the above defined p.u.l. mutual impedances respectively by:

$$e(x) = Z_m(x,D)I_{pl} \qquad\qquad e'(x) = Z'_m(x,D)I_{pl} \tag{17}$$

The theory and the formulas for evaluating induced voltage and current on a transmission line circuit under the influence of external electromagnetic fields can be found in [19] and to it one can refer for details.

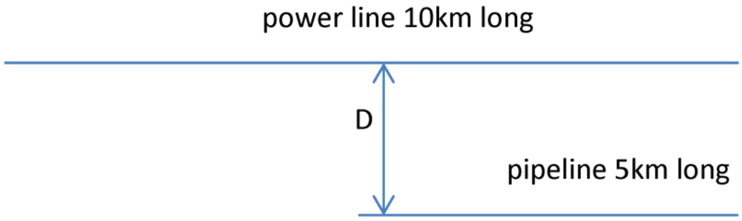

Fig. 5. Sketch of power line and pipeline layouts

In order to compare the results, it is worthwhile to define the following quantities:

- *U(x)* and *I(x)* induced voltage and current obtained when applying the generator *e(x)* to the pipeline-earth circuit
- *U'(x)* and *I'(x)* induced voltage and current obtained when applying the generator *e'(x)* to the pipeline-earth circuit
- *ΔU%(x)* the per cent relative difference relevant to the voltage that is:

$$\Delta U_{\%}(x) = \frac{|U(x)| - |U'(x)|}{|U(x)|}100 \tag{18}$$

- *ΔI%(x)* the per cent relative difference relevant to the current that is:

$$\Delta I_{\%}(x) = \frac{|I(x)| - |I'(x)|}{|I(x)|}100 \tag{19}$$

In *figures 6-8* the quantities $\Delta U_{\%}(x)$ and $\Delta I_{\%}(x)$ have been plotted versus pipeline progressive for different values of soil conductivity and in correspondence of different values for the lateral distance *D* (see *fig. 5*) between inducing and induced circuit i.e.: $\delta/100$, $\delta/10$, $\delta/2$, δ, 2δ, 3δ, 5δ.

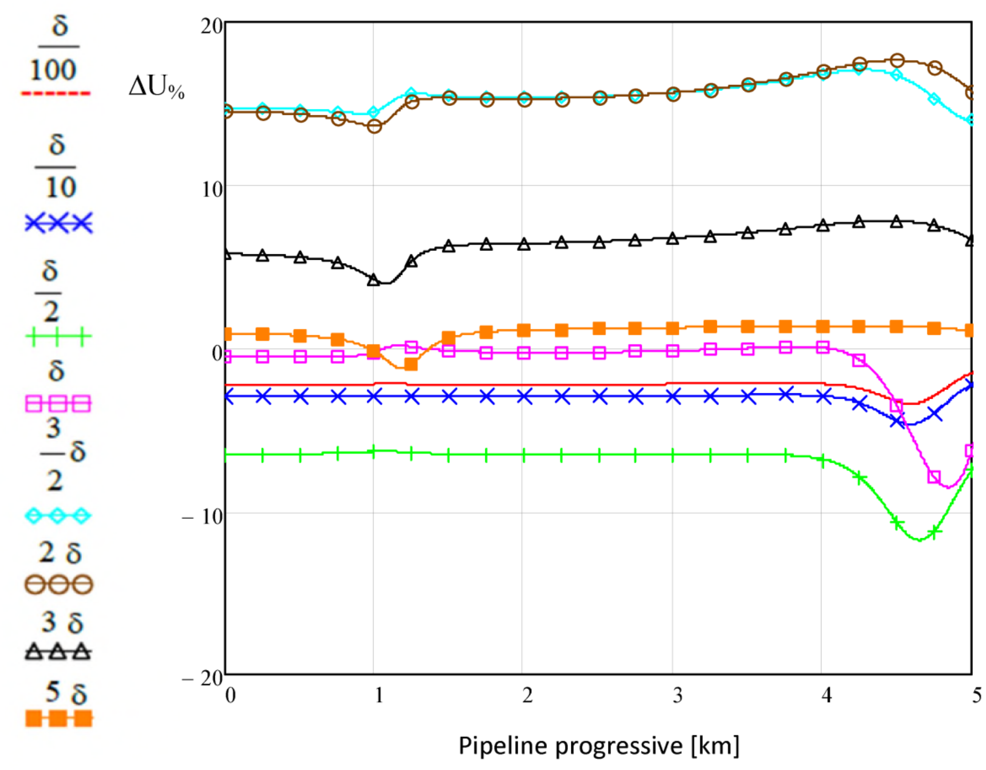

Fig. 6a. $\Delta U_{\%}$ versus pipeline progressive for different values of D; $\sigma_2=2\cdot10^{-2}$S/m

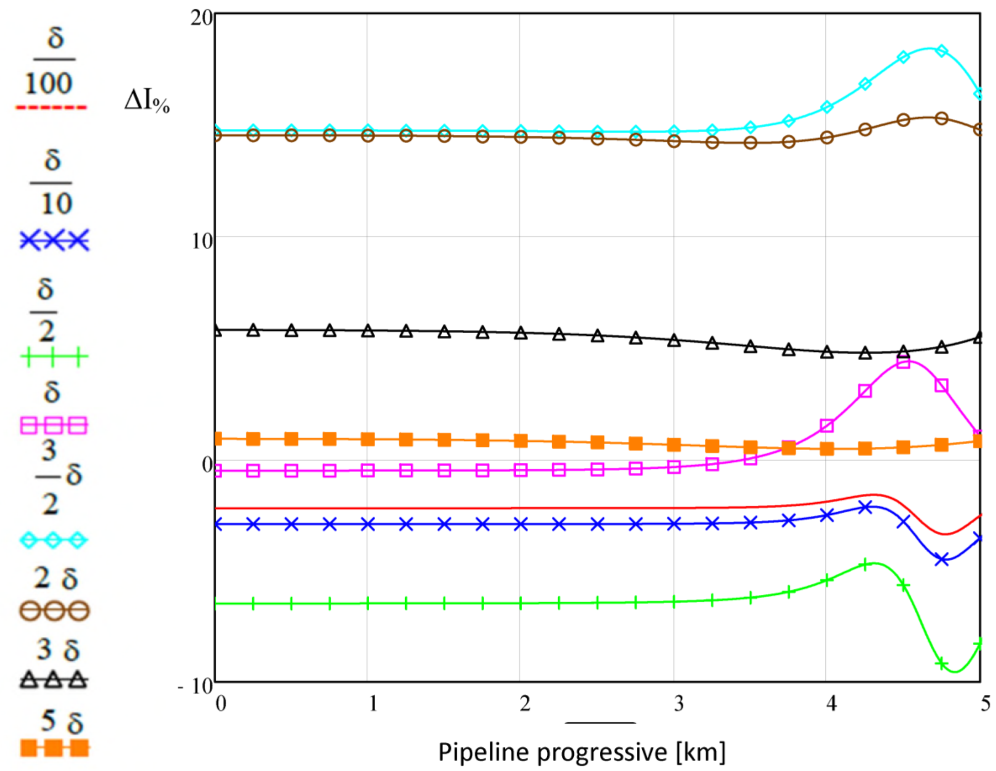

Fig. 6b. $\Delta I_{\%}$ versus pipeline progressive for different values of D; $\sigma_2=2\cdot10^{-2}$S/m

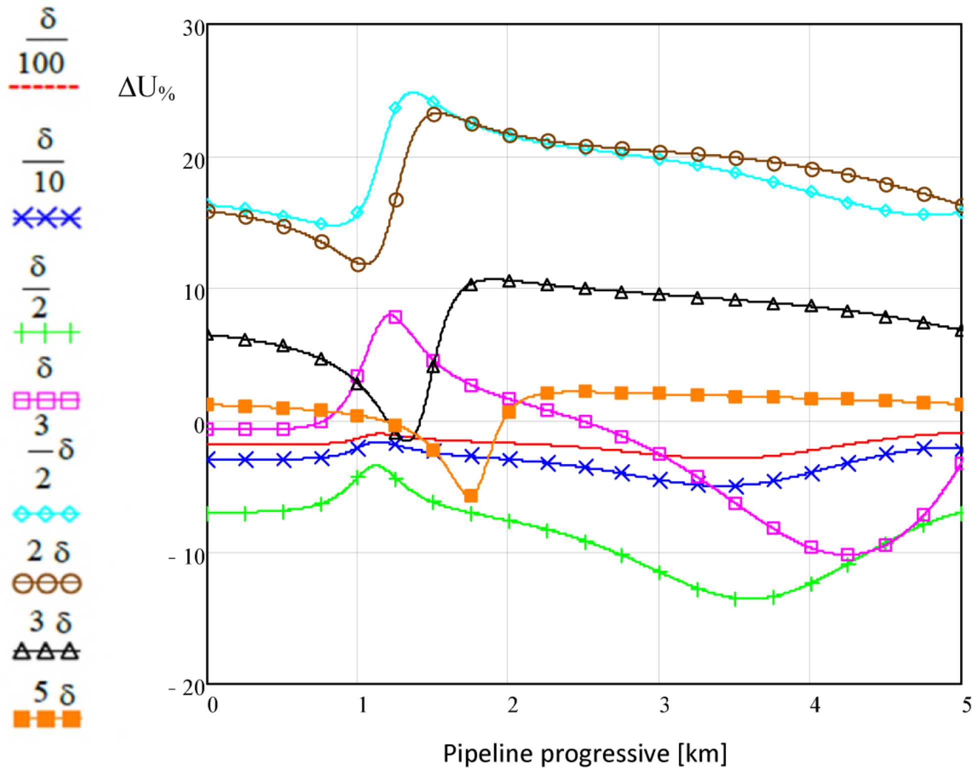

Fig. 7a. $\Delta U_{\%}$ versus pipeline progressive for different values of D; $\sigma_2=2\cdot10^{-3}$S/m

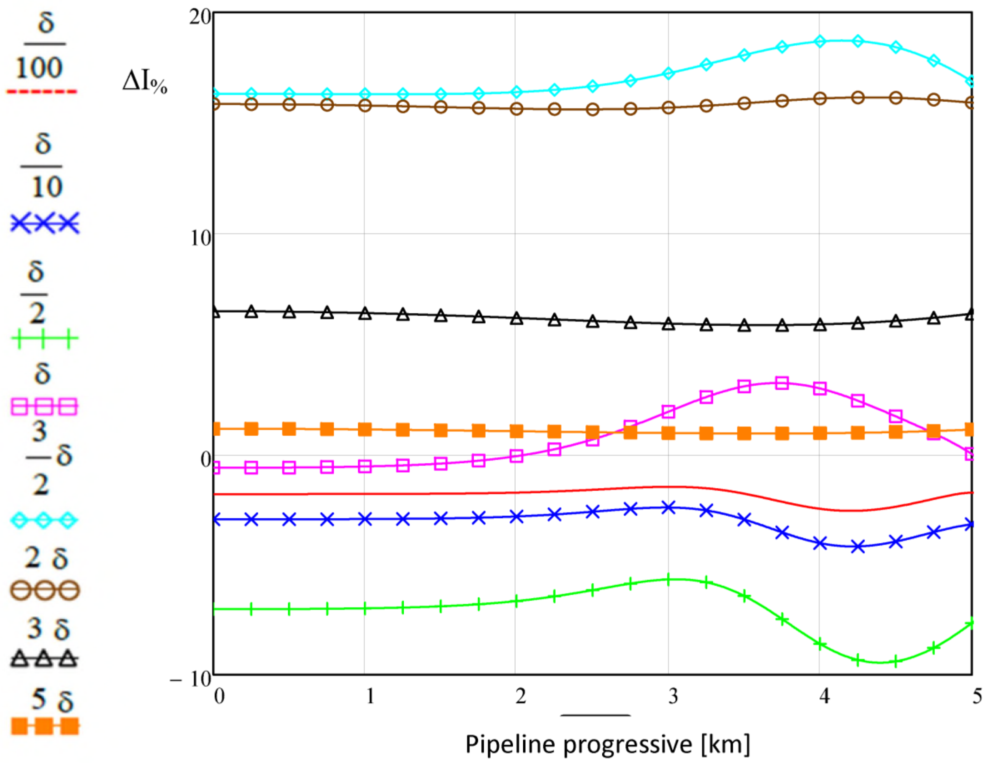

Fig. 7b. $\Delta I_{\%}$ versus pipeline progressive for different values of D; $\sigma_2=2\cdot10^{-3}$S/m

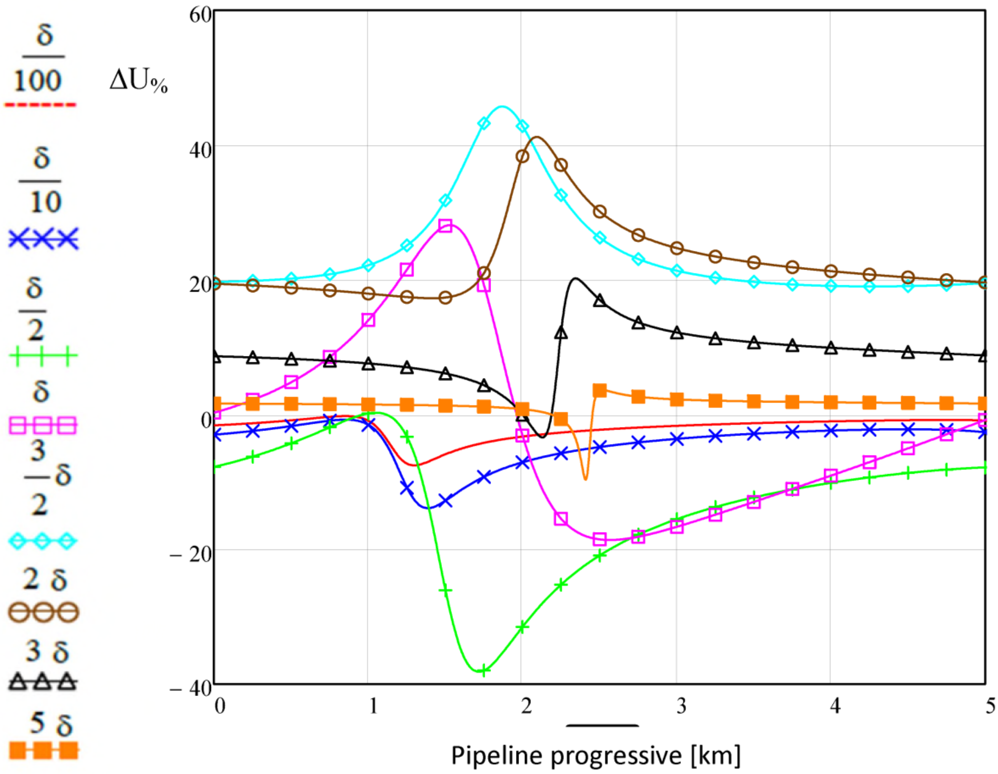

Fig. 8a. $\Delta U_{\%}$ versus pipeline progressive for different values of D; $\sigma_2=2\cdot10^{-4}$S/m

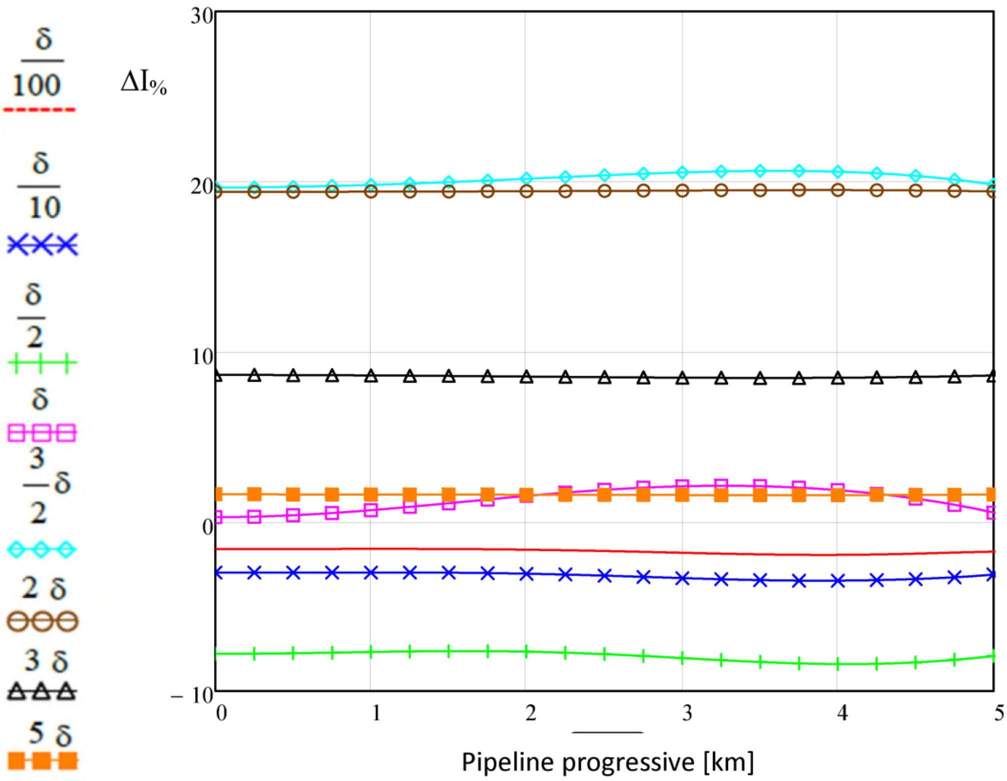

Fig. 8b. $\Delta I_{\%}$ versus pipeline progressive for different values of D; $\sigma_2=2\cdot10^{-4}$S/m

Figures 6-8 show that, in case of parallelism between power line and pipeline, depending on the lateral distance D and on the earth conductivity, the relative per cent difference in evaluating the interference results by using formula (10) or (11) can range approximatively:

- in the interval [-40%, 40%] for the induced voltage
- in the interval [-10%, 20%] for the induced current

Hence, in certain specific cases, depending on the lateral distance between power line and pipeline and especially with soil having low-medium conductivity, not negligible differences in using Z_m' instead of Z_m can be found when calculating induced voltage and current.

Furthermore, it is useful to notice that in [4] it is shown that the results obtained by using Z_m are consistent with the ones coming from the application of the formulas reported in [9] and [11].

The main advantage in using formulas (11) or (12) instead of the ones given in References [9-13] which are based on Neumann integrals, consists in their greater simplicity and in an easier evaluation of the induced emf on the pipeline-earth circuit.

5. CONCLUSIONS

In this paper two new and convenient closed form formulas for evaluating, in the ELF range, the p.u.l. mutual impedance between an overhead finite length wire and a buried wire, both with earth return, are proposed. The two different formulas yield values having per cent relative differences in the range [-20%, 25%].

Moreover, when evaluating the interference on the victim circuit (i.e. induced voltage and current), the use of the formula without corrective terms instead of the one with corrective terms may lead, in specific cases, to not negligible differences in the results. Thus, in applications, it is preferable to use the formula (12) instead of (11).

REFERENCES

[1] CIGRE, *Guide on the influence of High Voltage AC Power Systems on Metallic Pipelines,* Publication n. 95, 1995.

[2] CIGRE, *AC Corrosion on metallic pipelines due to interference froma AC power lines-Phenomenon modelling and countermeasures,* Publication n. 290, 2006.

[3] J. R. Carson, *Wave Propagation in Overhead Wires with Ground Return*, Bell System Technical Journal, Vol.5, p. 539-554, 1926.

[4] G. Lucca, *Different approaches in calculating AC inductive interference from power lines on*

pipelines, IET Science, Measurements & Technology, vol. 12, 2018.

[5] A. Cristofolini, A. Popoli, L. Sandrolini, *A comparison between Carson's formulae and a 2D FEM approach for the evaluation of AC interference caused by overhead power lines on buried metallic pipelines*, Prog. Electromagn. Res. C, vol.79, p.39-48, 2017.

[6] G. C. Christoforidis, D. P. Labridis, P. S. Dokopoulos: *A Hybrid Method for Calculating the Inductive Interference Caused by Faulted Power Lines to Nearby Buried Pipelines*, IEEE Trans. on Power Deliv., vol. 20, pp.1465-1473, 2005.

[7] D. D Micu, L. Czumbil, G. Christoforidis , A. Ceclan: *Layer Recurrent Neural Network Solution for an Electromagnetic Interference Problem* , IEEE Trans on Magnetics, vol. **47**, pp. 1410-1413, 2011.

[8] D. D Micu, L. Czumbil, G. Christoforidis : *AC interference on pipelines due to double circuit power lines: A detailed study*, Electric Power Systems Research, Vol. 103, pp. 1-8, 2013.

[9] R. M. Foster, *Mutual Impedance of Grounded Wires Lying on the Surface of the Earth*, Bell System Technical Journal, Vol.10, p. 408-419, 1931.

[10] R. M. Foster, *Mutual Impedance of Grounded Wires Lying on or Above the Surface of the Earth*, Bell System Technical Journal, Vol.12, p. 264-287, 1933.

[11] E. D. Sunde, *Earth Conduction Effects in Transmission System*, D. Van Nostrand Company Inc., 1949.

[12] L. J. Lacey, *The mutual impedance of earth-return circuits*, Proc. of the I.E.E., Vol. 99, Part IV, p. 156-167, 1952.

[13] M. V. Kostenko, K. E. Mihailov, E. L. Portinov, L. G. Pozdnjakov, V. B. Sokolov, I. M. Zarkhi, *Resistive and Inductive Interference on Communication Lines Entering Large Power Plants*, CIGRE Session 36-02, Paris, France, 1970.

[14] ITU, *Directives concerning the protection of telecommunication lines against harmful effects from electric power and electrified railway lines*, Vol. II: *Calculating induced voltages and currents in practical cases*, ITU, Geneva, 1999.

[15] P. R. Bannister, *Summary of Image Theory Expressions for the Quasi-Static Fields of Antennas at or Above the Earth's Surface*, Proceedings of the IEEE, vol.67, No. 7, p. 1001-1008, July 1979.

[16] J. R. Wait , *The electromagnetic fields of a horizontal dipole in the presence of a conducting half-space*, Canadian Journal of Physics, vol. 39, p. 1017-1028, 1961.

[17] J. R. Wait, K. P. Spies, *On the image representation of the quasi-static fields of a line current source above the ground*, Canadian Journal of Physics, vol. 47, p. 2731-2733, 1969.

[18] I. S. Gradshteyn, I. M. Ryzhik, *Table of Integrals, Series, and Products* Seventh Edition, Elsevier, 2007.

[19] ITU, *Directives concerning the protection of telecommunication lines against harmful effects from electric power and electrified railway lines*, Vol. III: *Capacitive, inductive and conductive coupling: physical theory and calculation methods*, ITU, Geneva, 1989.