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Alina Cristina Viorel, Ioan-Adrian Viorel, Larisa Strete

On the Magnetic Flux Density Analytical Calculation in the Air-Gap of Electric Machines with Wide Open Slots

The magnetic flux density distribution in the air-gap of electric machines with wide open slots is discussed in the paper. The finite element methods (FEM) obtained results are compared with that calculated analytic estimations. The case of double sided slotted machines with large values of the slot to opening tooth pitch ratio is considered, interesting conclusions being draw.

Keywords: double slotted electric machines, air-gap flux density.

1. Introduction

The air-gap magnetic flux density variation and values versus circumferential coordinate provide valuable information in evaluating electric machines' torque (average, peak, cogging or ripple), back emf value and harmonic content, and the main inductances, which may be dependent on rotor position. The finite element method (FEM) has proved by now its accuracy and consequently it becomes a common method to calculate the air-gap flux density in electric machines. Since FEM calculation is still a time consuming method. The analytic estimation of the air-gap flux density variation represents an efficient and desirable tool.

Important results in the domain of synchronous and respectively induction machine air-gap magnetic field were published, to only the works of Weber [1], Heller [2] and Ozawa [3].

In the last years some results concerning the air-gap field of the synchronous permanent magnet (PM) machine [4] were also obtained. The calculation of the air-gap variable permeance of the double slotted electric machines was also done, [5], [6], for example.

A general case of a machine with slots on both sides of the air-gap is discussed in the paper, without any reference to the mmf distribution.

The 2D-FEM models, in linear layout, were constructed for all studied variants, considering different tooth pitch to air-gap length ratios and slot width to double air-gap length ratios and also for various mmf values.

Some the analytical models were presented as were the developments for Carter factor too. For the case of the double slotted air-gap topology, a comparison between analytical and 2D-FEM numerical computed values of Carter's factors is done, evincing the important difference in the case of large ratios of the pole pitch to air-gap length.

2. 2D-FEM Models

The 2D-FEM analysis was performed on two basic models. A structure with two slots, a coil on the stator and a salient rotor with the same topology as the stator were selected for the first model, Fig.1. For symmetry reasons the stator length is equal to two tooth pitches, which means that for the exterior stator teeth only half width is taken.

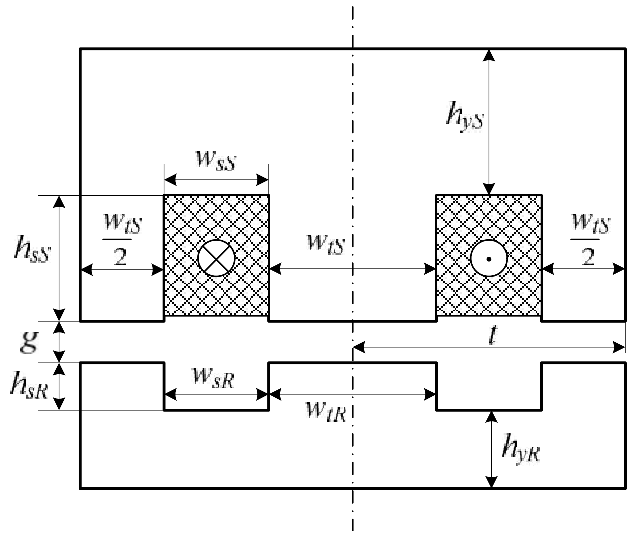

Figure 1. Double side identical saliency

The second structure, is similar to the first one but the rotor tooth pitch, tooth and slot width are different to that of the stator. No coil with current is placed in the rotor slots.

The notations are the usual ones: t_S, t_R – stator and rotor tooth pitch, w_{tS}, w_{tR} – stator and rotor tooth width, w_{sS}, w_{sR} – stator and rotor slot width, h_{sS}, h_{sR} – stator and rotor slot height, h_{yS}, h_{yR} – stator and rotor yoke height, g – air-gap length. All structures were parameterized in a way to make possible adequate variation of the most important dimensions, only the stator tooth pitch, which is in fact the circumferential length, was kept constant. A linear layout was considered, but it does not affect the generality of the results.

The air-gap flux density is calculated on a mesh line situated in the middle of the air-gap. Six equidistant air-gap layers were considered, in order to obtain a more homogenous distribution of the vector magnetic potential and to avoid the discontinuities due to the magnetic permeability difference between the iron core and respectively air-gap domain.

Based on 2D-FEM analysis obtained values, the Carter's factor is calculated as the ratio between peak air-gap flux density value in the stator tooth axis $B_{g\max}$, and the corresponding average value B_{gav}.

$$K_C = B_{g\max} / B_{gav} , \quad (1)$$

The average value of the air-gap flux density B_{gav} results:

$$B_{gav} = \frac{2}{t} \int_0^{t/2} B_g(x) dx , \quad (2)$$

where $B_g(x)$ are the air-gap flux density values obtained at equidistant points via 2D-FEM analysis and t is a general notation for the tooth pitch.

3. Analytical approximations

Many analytical approximations of the air-gap flux density variation function of circumferential coordinate were proposed up to now, some of the basic ones being the following:

i) Webber's approximation [1]

$$B_{gav} = \frac{2}{t} \int_0^{t/2} B_g(x) dx ,$$

$$= 0.5\left(1 - 1/\sqrt{1 + (w_s / 2g)^2}\right)$$

$$a = \frac{t - w_s}{w_s} , \; x \in [0, t/2] \quad (3)$$

ii) Heller's modified approximation [2]

$$B_g(x) = B_{g\max,} , \; x \in [0, \; w_t) \quad ,$$

$$B_g(x) = B_{g\max}(1- \quad + \quad \cos y)\ , \quad x \in [\quad w_t, t/2]$$

$$y = \frac{\text{®}x}{\frac{t}{2} - \quad w_t} - \frac{\text{®} \; w_t}{\frac{t}{2} - \quad w_t} \tag{4}$$

iii) Equivalent air-gap permeance based approximation [2], [5], [6]

$$B_g(x) = B_{g\max,} \cdot g \cdot P(x)$$

$$P(x) = \frac{1}{g}\frac{1}{K_C}\left(1 + p_r \cos(\frac{2x}{t}\text{®})\right)$$

$$x \in [0, t/2] \tag{5}$$

The signification of the notations is: x – circumferential coordinate, t, w_t, w_s – tooth pitch, tooth width and slot opening, g – air-gap length, K_C, p_r – Carter's factor respectively the coefficient of the air-gap variable equivalent permeance considering slots only on one air-gap side.

Some other air-gap flux density variation approximation as a nonsinusoidal variation, considering a simplified flux lines topology in the air-gap and an exponential approximation through a curve fitting procedure can be considered. [7]

There are many equations, mostly obtained via conformal mapping, that allow for Carter's factor calculation, anyone could be as good as the following one [2, 7]:

$$k_C = \left[1 - \frac{1}{t/w_s(5g/w_s + 1)}\right]^{-1} \tag{6}$$

4. Numerical results

The double sided slotted air-gap topology is covered usually [2, 5, 6] by introducing an air-gap equivalent permeance, which is for an immobile rotor,

$$P(x) = g\,\frac{1}{gK_{CS}}\left(1 + p_{rS}\cos(\frac{2x}{t_S}\text{®})\right) \cdot \frac{1}{gK_{CR}}\left(1 + p_{rR}\cos(\frac{2x}{t_R}\text{®} + \quad _R)\right) \tag{7}$$

where K_{CS}, K_{CR}, p_{rS}, p_{rR} are the Carter's factors and permeance coefficients calculated by considering slots only on the stator, respectively rotor, and β_R is the initial rotor tooth axis displacement against the stator tooth axis (if the initial position is the aligned one than β_R = 0).

Any other approximation can be used to calculate the resulting air-gap equivalent permeance in a similar way.

It is clear that (7) will correctly describe the air-gap equivalent permeance in a double sided topology if the Carter's factor is equal with the product of the elemental Carter's factors.

In the case of double sided slotted air-gap topology two variants were considered:

i) The stator and the rotor have the same tooth pitch, tooth width and slot opening, the current flowing only trough the stator coil.

i) The stator and rotor have different tooth pitches, tooth widths and slot openings, the current flowing only trough the stator coil.

In Tables 1 and 2 the peak and average values of the air-gap flux density and respectively Carter's factor values are given, all calculated via 2D-FEM, for two combinations of t/g and w_s/2g ratios. To obtain accurate results, several Carter's factor calculation has been made for different values of coil magneto-motive force F presented in the first two tables. In the paper, T represents the stator tooth pitch when there are no slots on the rotor and G is the air-gap length.

Table 1. FEM obtained results T/G=150; W_s/2G=50

F	A	600	800	1000	1200	
B_{gmax}	T	0.9217	1.222	1.503	1.675	Single slotted
B_{gav}	T	0.339	0.449	0.55	0.612	
K_{C1}	-	2.718	2.722	2.729	2.739	
B_{gmax}	T	0.946	1.246	1.51	1.687	Aligned double-slotted
B_{gav}	T	0.34	0.447	0.541	0.599	
K_{C2al}	-	2.78	2.78	2.79	2.797	
B_{gmax}	T	0.0838	0.112	0.14	0.168	Unaligned double-slotted
B_{gav}	T	0.0288	0.038	0.048	0.057	
K_{C2un}	-	2.906	2.912	2.912	2.912	
K_{C2av}	-	2.843	2.846	2.851	2.855	

Usually the resulting Carter's factor in the case of double sided slotted air-gap topology is calculated as a product of two elementary Carter's factors K_{CS} and K_{CR} [2].

$$K_C = K_{CS} K_{CR} \tag{8}$$

In the examples presented in Tables 1 and 2 the elementary Carter's factors are equal, then

$$K_{C2} = K_{C1}^2 \tag{9}$$

where K_{C1} and K_{C2} are the Carter's factors value calculated considering slots-teeth topology only on one side, respectively on both sides of the air-gap.

It is clear, from the results given in Table 1, that the conventional theory, applied usually to induction motor [2, 5] cannot be extended when the ratio tooth of pitch to air-gap length and respectively the slot width to the double air-gap length ratio have large values. As can be seen, Table 1, K_{C1} values are very close to K_{C2} values calculated in aligned or unaligned position.

Table 2. FEM obtained results T/G=37.5; W_s/2G=6.25

F	A	2200	3000	3800	4600	
B_{gmax}	T	0.855	1.16	1.43	1.526	Single slotted
B_{gav}	T	0.628	0.85	1.047	1.113	
K_{C1}	-	1.36	1.362	1.368	1.372	
B_{gmax}	T	0.924	1.255	1.548	1.641	Aligned double-slotted
B_{gav}	T	0.65	0.88	1.083	1.145	
K_{C2al}	-	1.421	1.423	1.429	1.433	
B_{gmax}	T	1.164	1.595	1.822	1.913	Unaligned double-slotted
B_{gav}	T	0.578	0.772	0.895	0.982	
K_{C2un}	-	2.013	2.066	2.036	1.948	
K_{C2av}	-	1.717	1.745	1.733	1.691	Double slotted
K_{C1}^2	-	1.850	1.855	1.871	1.882	

Carter's factor calculated for double-sided identically slotted air-gap topology in aligned position K_{C2al} and in unaligned position K_{C2un} differ with less than 10%, which is not the case when t/g and w_s/2g ratios are much smaller. In Table 2 are given the values of K_{C1}^2 and K_{C2av} factors, the last one being:

$$K_{C2av} = 0.5(K_{C2al} + K_{C2un}) \tag{10}$$

As seen from the results given in Table 2 when t/g and w_s/2g ratios are quite small, the values of the resulting Carter's factor K_{C1}^2 are close to the values of average Carter's factor K_{C2av}. Consequently the equation (7) gives adequate results for equivalent air-gap calculation.

Table 3. Carter's factor calculation, double side slotted machine

$\frac{t_S}{g}$	$\frac{t_R}{g}$	$\frac{w_{sS}}{2g}$	$\frac{w_{sR}}{2g}$	B_{gmax}	B_{gav}	K_{C2FEM}	K_{CSA}	K_{CRA}	K_{C2A}
75	93.75	25	31.25	1.597	0.477	3.505	2.52	2.58	6.502
			23.43	1.621	0.493	3.284		1.82	4.586
			15.62	1.633	0.501	3.257		1.4	3.546
75	56.25	18.75	18.75	1.595	0.527	3.023	1.79	2.435	4.364
			14.06	1.688	0.743	2.272		1.751	3.138
			9.375	1.676	0.903	1.856		1.372	2.458
75	56.25	12.5	18.75	1.604	0.391	4.106	1.39	2.435	3.392
			14.06	1.7	0.552	3.079		1.751	2.439
			9.37	1.754	0.713	2.459		1.372	1.911
75	93.75	12.5	31.25	1.728	0.637	2.712	1.39	2.587	3.604
			23.43	1.778	0.891	1.994		1.820	2.536
			15.62	1.727	0.905	1.909		1.407	1.96

In Table 3 some results concerning the Carter's factor calculation in the case of a double-sided slotted non identical slot-teeth air-gap topology are given. The stator tooth pitch and slot openings are the same as in previously presented cases and two rotor tooth pitch are taken. The ratios of tooth pitch to slot opening are the same for stator and rotor, which means 2/3, 1/2 and 1/3, values considered in all the cases when the air-gap magnetic field is calculated via 2D-FEM models. In Table 3, K_{CSA}, K_{CRA} and K_{C2A} are analytically calculated values of the Carter's factor.

4. Conclusion

The magnetic flux density distribution in the air-gap of double-sided slotted electric machines is analyzed in the paper. Simple models of the slotted air-gap topology in linear layout are built and the 2D-FEM is applied to calculate the air-gap flux density distribution for different values of mmf and ratios of tooth pitch and slot opening to air-gap length. The Carter's factors are calculated analytical and based on the 2D-FEM analysis resulted values.

The most usual approximations which describe the air-gap flux density variation function of circumferential coordinate were given. The results obtained via analytical approximations were compared with the ones calculated by using 2D-FEM analysis.

In the case of double-sided slotted air-gap topology, it is shown that the resulting Carter's factor can be approximate by the product of the elemental Carter's factors calculated considering slots only on one side if the slot opening to air-gap length ratio is smaller than 25, otherwise the errors are quite important. For larger values of the ratio some different methods should be developed.

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Addresses:

- Lect. Dr. Eng. Alina Cristina Viorel, Lucian Blaga University of Sibiu, Blv. Victoriei, nr.10, 550024, Sibiu, alina.viorel@ulbsibiu.ro.
- Prof. Dr. Eng. Ioan-Adrian Viorel, Technical University of Cluj Napoca, str. Memorandumului, nr. 28, 400114, Cluj Napoca, ioan.adrian.viorel@mae.utcluj.ro
- Dr.Eng. Larisa Strete, Emerson Cluj Napoca, larsa.strete@mae.utcluj.ro