ANALELE UNIVERSITĂȚII
“EFTIMIE MURGU” REȘIȚA
ANUL XXII, NR. 1, 2015, ISSN 1453 - 7397

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Experimental Generation of the Intersection Curve of Two Cylinders: An Algorithm Based on a New Paradigm

Computational Geometry is currently using approximation techniques based on convex polygons, with good results in some topics, but with severe limitation of applicability. In this paper we present the first step of a larger project, aiming to lead us to the plane projection of a 3D surface obtained by intersecting two cylinders (a very frequent problem in obtaining the stencils for welded ensembles). As a first step, we present an algorithm for experimentally obtaining the intersection curve of two cylinders. The new paradigm and also the fundamentally new aspect of our project is that we construct our algorithm by simulation of the analytical relations describing the curve, and not by approximation using convex polygons.

Keywords*: Computational Geometry, experimental generation, cylinders intersection, SolidWorks*

1. A brief state of the art in 3D shape modeling in Computational Geometry

Computational Geometry combines algorithms from analytical geometry, classical projective representation techniques and the calculus facilities offered by the new computer generations. With a group of well studied topics in the field of triangulation and associated aspects [1], [2], computational geometry is now at the very beginning of a new area of preoccupations, in the field of shape modeling.

We say ‘the very beginning’ because the state of the art [3], [4] is more an inspiring manifesto for the future generation of engineers and architects: the discussion is practically limited at very impressive colored figures generated using computer facilities, and almost nothing about equations modeling. Also, it is clear that, due to the triangulation results based on convex polygons approximations, the trend stays on using finite element methods.

So, we arrive at our project: we intend to develop algorithms for 3D corps intersection based on simulating analytical relations. We do not use finite element type approximation, but directly generating the corps and, after computing the intersections, represent it on the figure. In this first step, we generate the intersection curve of two cylinders. For the simulation part, we use SolidWorks [5].

2. The double nature of a Computational Geometry approach

Berg ([6]) resumes very well the double nature of a Computational Geometry approach: ‘Good solutions to algorithmic problems of a geometric nature are mostly based on two ingredients. One is throug understanding of the geometric properties of the problem, the other is a proper application of algorithmic techniques and data structures. If you don’t understand the geometry of the problem, all the algorithms of the world won’t help you to solve it efficiently. On the other hand, even if you perfectly understand the geometry of the problem, it is hard to solve it effectively if you don’t know the right algorithmic techniques.’ So, we will present our work in this manner: the problem, the way we geometrically understood it and the algorithm.

2. The problem: a stencil problem with two cylinders

The initial problem is illustrated in figure 1:

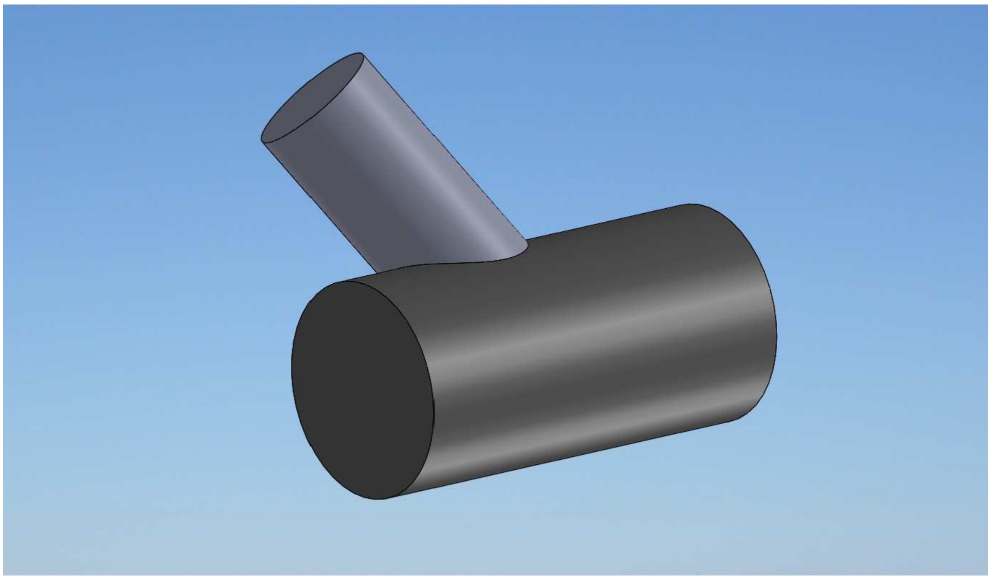

Figure 1. The initial problem: obtaining the stencil for the upper cylinder

given the two cylinders, we need the stencil of the sidewise surface of the upper cylinder (a problem very frequent in welded ensembles engineering). The problem is represented in more technical manner in figure 2.

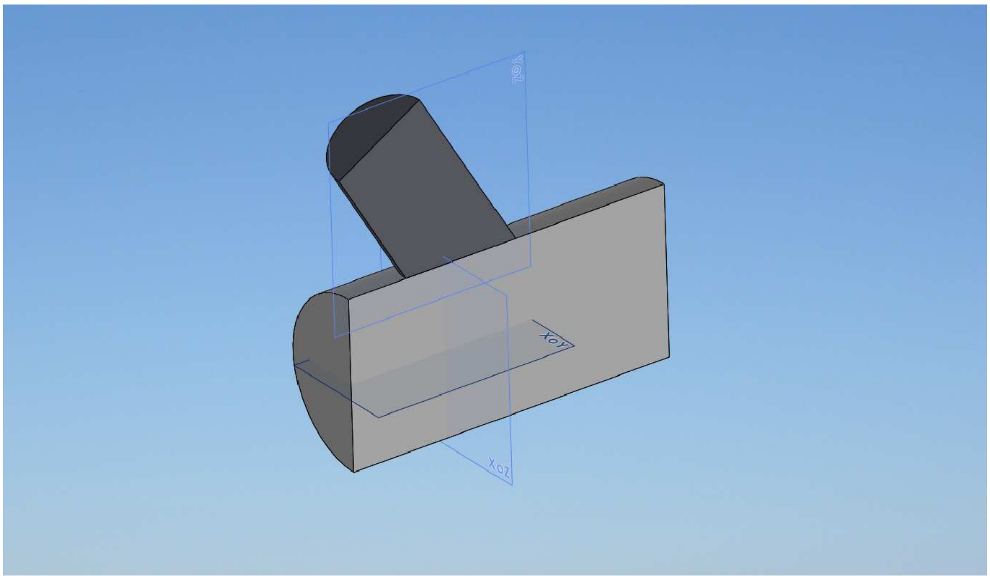

Figure 2. The cylinders presented in section, with reference axes

For the ease of presentation, we presented the two cylinders one having the Ox axe as symmetry axe, and the other having the symmetry axe in the yOz plane. In general, since the translation and rotation does not modify the dimensionality of a corpse, we can reduce any other concrete problem at this case.

3. Understanding of the geometric properties of the problem

At the first analysis of the geometrical properties, we considered simply to intersect the two cylinders in analytical form, meaning to find the parametric solution of a system of two cylinder-type equations:

$$F\left(P_1(x,y,z),P_2(x,y,z)\right) \quad (1)$$

leading us to solve a too complicate system of equations.

At a second analysis, we realized that we can simplify the problem: since we are searching an algorithm for a graphically representation of the solution, we can rephrase the problem in two simpler steps:

step 1: noticing that any point from the intersection curve appears geometrically as an intersection of two generators, we reduce the problem at finding the intersection of two generators lines of the cylinders, one from each of them; mathematically, we reduce second degree equations at first degree equations an we reduce a lot the calculus effort;

step 2: having found one point of the curve, we iterate the point into the range of admissible values established from the concrete dataset.

4. The algorithm and the result

We consider the standard parametric representation of a line in 3D space

$$x = x_0 + tl,$$
$$y = y_0 + tm$$
$$z = z_0 + tn,\ t \in R \tag{2}$$

and replace the parameters x_0, l, y_0, m, z_0, n twice: once for a generator line g_1 of the first cylinder and then for a generator line g_2 of the other cylinder. The intersection of the g_1 and g_2 lines gives us the point I, belonging to the intersection curve.

We illustrated in figure 3 (bellow) the algorithm and the result: the generator lines g_1 and g_2, their intersection point I and the complete curve obtained by iterating the point in the whole range of admissible values of parameter $t \in R$.

We have to mention that initially we intended to wrote, for the calculus part, a calculus program in C an then export it in SolidWork for the graphical part. But, we found in [3] some basic code about intersecting cylinders (only for cylinders having the symmetry axes on the on the Ox and Oz of the coordinates system) and we adapted using SolidWorks calculus functions; then, we modified it, by adding parameters for generating other type of cylinders and the intersection calculus.

Note: in paragraphs 3 and 4 we used adaptations of formulas from Vrănceanu [7]).

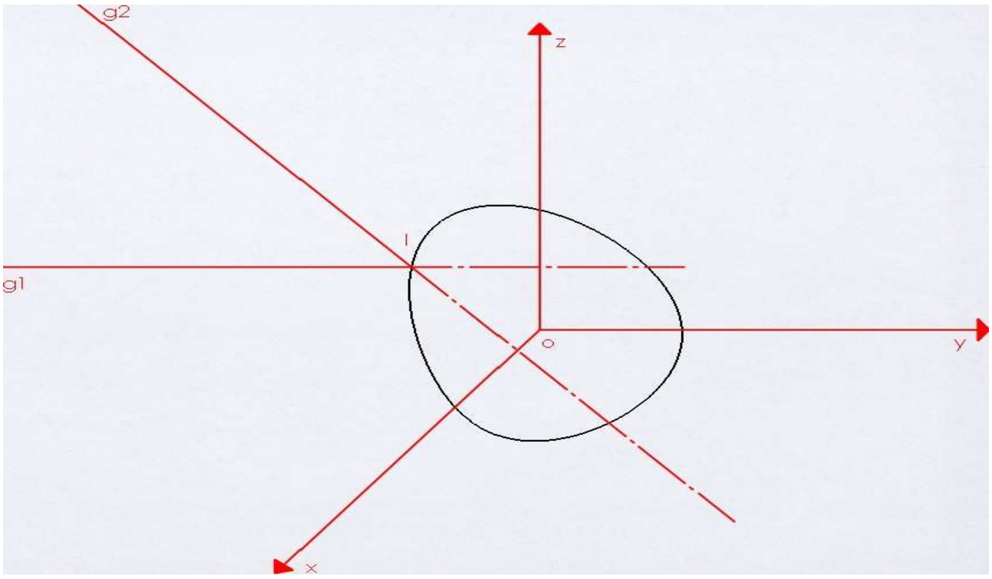

Figure 3. The curve resulted from the intersection of the two cylinders

Having access to display facilities, let's add another view, this one generated directly in SolidWorks (from the initial test, before starting calculus). Very easy to obtain, it gives an initial useless impression to our work! But, we arrive to the complicated part: by standard products, we cannot obtain the sidewise projection.

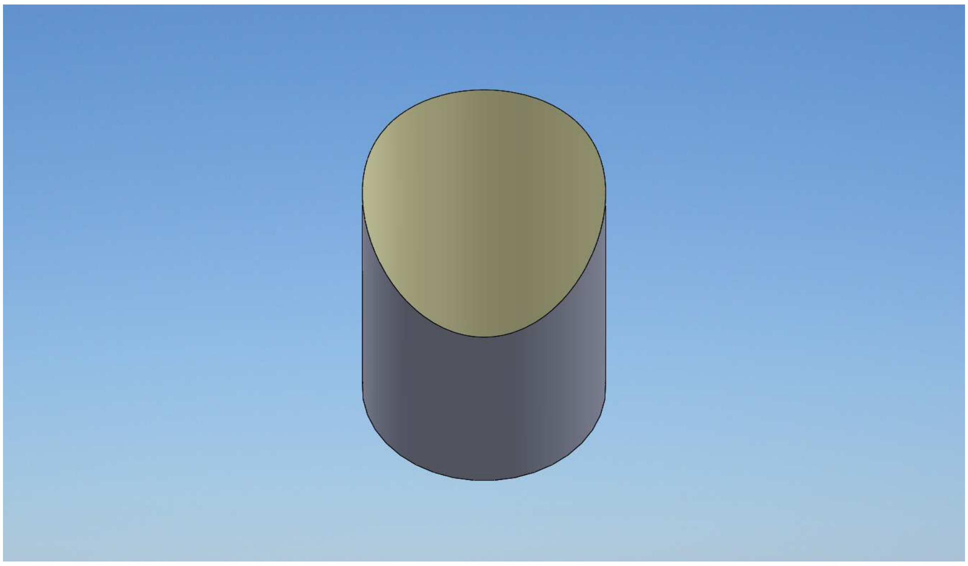

Figure 4. View of the top cylinder and the intersection

5. Conclusions and further research

The main idea of our work is a new parading for Computational Geometry projects: replacing the 'by convex polygons approximations' techniques with algorithms based on analytical geometry.

As part of a larger project aiming to obtain the stencil of the sidewise surface of an intersected cylinder we presented the first step: obtaining the 3D curve resulted from the intersection of the cylinders. The next step is obtaining the stencil of the sidewise surface. For this next step, we have to find a way of expressing analytically the axonometric approximation techniques now current in engineering and architecture.

Since all the usual geometrical corps has classical parametric descriptions, a further development of the project will be the extension of this approach to intersections of other corps (spheres, cones, etc).

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