ANALELE UNIVERSITĂȚII
"EFTIMIE MURGU" REȘIȚA
ANUL XXII, NR. 1, 2015, ISSN 1453 - 7397

Sergiu Ciprian Catinas

Applications of Rotational Stiffness in Numerical Methods for Reinforced Concrete Elements Subject to Horizontal Efforts

A detailed investigation of the RC elements subject to horizontal forces and the effect of the rotational stiffness were conducted in this paper by using numerical methods. Since the early '1960 this technique become used by engineers but only in the last decade, due to the development of the computing machines the method was used on a large scale. The paper deals with a technique. The paper deals with a summary of recent techniques in manipulating the stiffness matrix of the constitutive materials in efficiently solving problems related to connection between horizontal forces and rotational stiffness. The paper propose some practical methods deducted from theoretical formulation of the stiffness matrix and propose new formulation of the stiffness matrix for this completely applicable to the new technique. At last the paper analyses a three level RC structure using a FEM based computer soft with the method proposed and delivers the results.

Keywords: *manipulating of the stiffness matrix; rotational stiffness; numerical methods; horizontal forces; link between rotational stiffness and horizontal forces*

1. Introduction

The method described here is based on the theory initially developed by Hooke and then extended by Cauchy. This method implies a numerical method based on a set of equilibrium equations with respect to the kinematical admissible field conditions. Based on these equations is that needs to be solved is created this technique that takes in consideration the constitutive law of materials. The general form of strain-stress connection is expressed by Cauchy in a general form that involves 36 components that form the stiffness matrix to be consider in an numerical analyze.

However due to the symmetry that is in relation with the constitutive law of the most common materials only 21 of these constants are independent. More over for the reinforced concrete, a cohesive and frictional material with a complex behavior, the current practice admits some simplifying rules in order to reduce the number of the components for the stiffness matrix of the reinforced concrete elements. By applying this stiffness matrix for reinforced concrete elements in numerical methods it is possible to reduce the number of the components to 9. By convention the elements are called Young's modulii E_1, E_2, E3, the 3 Poisson's ratios v_{23}, v_{31}, v_{12}, and the 3

For modeling the tension stiffening energy there are several models proposed (e.g. liniar model proposed by Hilleborg [1]. or bilinear proposed by Roelfstra and Wittmann [2]) by the introduction of the fracture energy concept GF as introduced by Hillerborg as in figure 1. These models can bring accuracy to the calculus, especially in a long term analyses. The paper presents techniques to create RC elements that have better behaviour through dissipation of the energy by introducing rotational efforts.

The paper deals with the link between horizontal efforts and other efforts that are in the RC elements. The link becomes a source of inspiration on developing new types of RC elements. The new elements have a better performance when the horizontal force is applied. A new stiffness matrix is developed so that the elements can be analyzed. At last a theoretical 3 level RC structure with the new elements is created and compared.

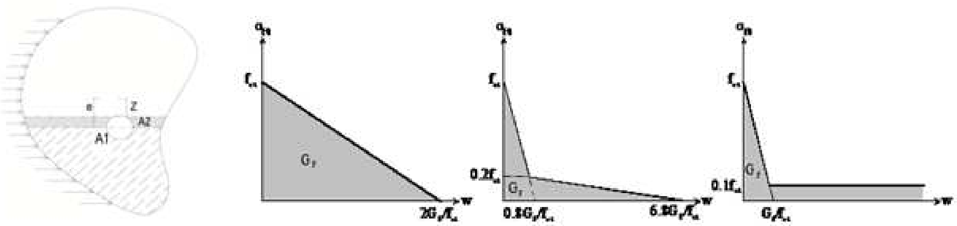

Figure 1. Schematic element section- friction energy

2. Research significance

The RC elements proposed in this paper are important in study over applied numerical methods in the overall knowledge repository. The research significance resides from the manipulation of a stiffness matrix according to the specific case involved. The paper deals with a triaxial effort and the manipulation of the stiffness matrix with respect to the linked among efforts. More over the paper creates a pattern suitable for implementation into computer computation programs. The other research significance aspects reside from the seismic protection system that can be created based on this technique and also leads to new areas of study.

3. Experimental investigation

For the purpose of showing the benefit of the technique proposed a theoretical RC structure was modeled using Abacus Cae. A 3 level structure was considered with 4 columns per floor. The beams considered have 4.5 meters and a cross section of 30x50 cm and the other beam is 5.6 meters long and a cross section. A system that allows rotation of the columns is implemented as seen in figure 2 theoretical and in figure 3-practical.

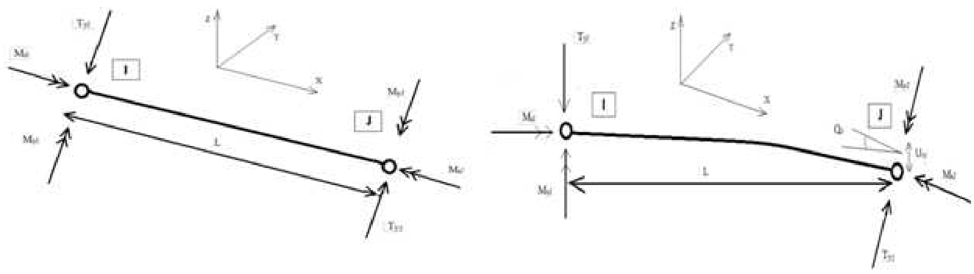

Figure 2. El. with M_x,$M_{y,}T_{y-}$ undeformed 1-b El. with M_x,$M_{y,}T_{y-}$deformed

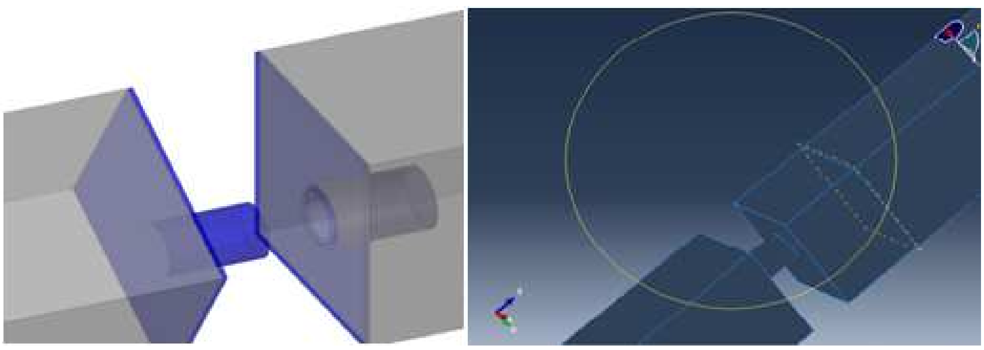

Figure 3. Column solution to allow rotation b Column solution to allow rotation

The RC structure is subject to horizontal efforts similar with a seismic force. The results are compared with the same structure but with no rotational movements of the columns allowed. The comparative results underline the benefit of the system. The rotational pins have an eccentricity of 3 cm of the center of the cross section along axis X and Y and the columns are 4.0 meters and a cross section of 30 cm by 30 cm. To mention that a system that allows rotation of the cross section of the columns can be created in a different manner.

4. Materials

The materials used in Abaqus simulation are ordinary materials used in RC structures For columns is used concrete described as elastic material with a modulus of elasticity of 210 and a density of 2000 As for the metal used the modulus of elasticity is 210…and a density of 3000. For the purpose of the research it is considered that no reinforcement bars are introduced, as Neil did [3]. As for description of contact surfaces that interact is considered 2 types: with or without penalty.

5. Items of investigation

A theoretical RC structure was analyzed using Abacus CAE, as in figure 4 and the focus of the analysis was for the total movement of the structure compared with the same structure using the same materials but with no rotational movement for the columns.

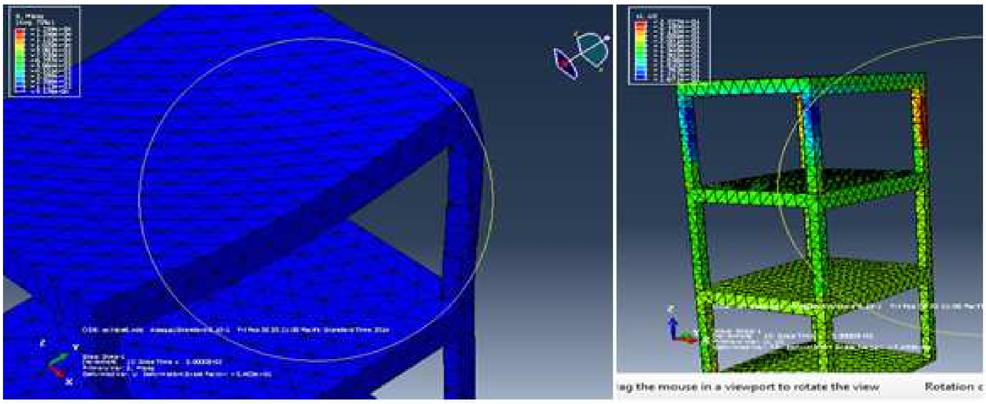

Figure 4.a. Structure analyzed-column rotated **4.b.**Structure analyzed

The fracture energy was analyzed to see the improvement brought by the introduction of rotational movement in columns, using O'Hara [4]. The columns studied are 30 cm by 30 cm and the metal pin is 25cm long with a radius of 5 cm , made by plain steel with a modulus of elasticity of 210 Mpa.

6. Analytical investigation

Current practice involves a stiffness matrix developed in the last decade and implemented in computer based calculations programs that allows determining the state of stress and strain in a cross section. These techniques are based on a common stiffness matrix as expressed bellow and are suitable for analyses with complex properties like concrete with reinforced polypropylene fibbers or concrete with fibber carbon and the results are satisfactory. The main equation is expressed in (1).

$$\{F_e\} = \{F_e\} \cdot \{U_e\}, \tag{1}$$

Where the stiffness matrix is as in (2). However, the most common analyses for these numerical methods involve a stiffness matrix as see in (2). Please notice that this matrix involves the connections between strain and stress that are most interconnected. The connectivity among pure stress and pure rotation along axis X, if we consider the axis x as the longest axis is neglected. More over the rotational moment computation can be done only through finite element analyses to a various cross section shape, but can be estimated for the most common cross section shapes.

$$\{K_i\} = \begin{bmatrix}
\frac{ES}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{ES}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{-6EI_z}{L^2} & 0 & \frac{-12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\
0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{-12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^2} & 0 \\
0 & 0 & 0 & \frac{GI_t}{L} & 0 & 0 & 0 & 0 & 0 & \frac{-GI_t}{L} & 0 & 0 \\
0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 \\
0 & \frac{-6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} & 0 & \frac{-6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} \\
-\frac{ES}{L} & 0 & 0 & 0 & 0 & 0 & \frac{ES}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\
0 & 0 & \frac{-12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^2} & 0 \\
0 & 0 & 0 & \frac{-GI_t}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GI_t}{L} & 0 & 0 \\
0 & 0 & \frac{-6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 & 0 & 0 & \frac{-6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} & 0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L}
\end{bmatrix} \tag{2}$$

The note above leads to the idea of practical manners like simply observation that are easy to understand and implement. It is easy to notice the lack of connection among the rotational stress/strain, according to Rafueneau [5] and the rest of the strains/stresses involved. This lack of connectivity is due to the low level of influence of the rotational moment over the rest of the matrix, and also due to a rare frequency of this kind of effort in current civil practice. However the main idea resides from this notice and involves a transfer of the efforts among components of the stiffness matrix of the most common reinforced concrete elements.

$$\begin{Bmatrix} \frac{12EI_{y/z}}{L^3} & \alpha_1 & \frac{-6EI_{y/z}}{L^2} & \frac{-12EI_{y/z}}{L^3} & \alpha_2 & \frac{-6EI_{y/z}}{L^2} \\ \alpha_3 & \frac{GK_T}{L} & \alpha_4 & \alpha_5 & \frac{GK_T}{L} & \alpha_6 \\ \frac{12EI_{y/z}}{L^3} & \alpha_7 & \frac{4EI_{y/z}}{L} & \frac{6EI_{y/z}}{L^2} & \alpha_8 & \frac{2EI_{y/z}}{L} \\ \frac{12EI_{y/z}}{L^3} & \alpha_9 & \frac{6EI_{y/z}}{L^2} & \frac{12EI_{y/z}}{L^3} & \alpha_{10} & \frac{6EI_{y/z}}{L^2} \\ \alpha_{11} & \frac{GK_T}{L} & \alpha_{12} & \alpha_{13} & \frac{GK_T}{L} & \alpha_{14} \\ \frac{-6EI_{y/z}}{L^2} & \alpha_{15} & \frac{2EI_{y/z}}{L} & \frac{6EI_{y/z}}{L^2} & \alpha_{16} & \frac{4EI_{y/z}}{L} \end{Bmatrix} \begin{Bmatrix} U_{y/z1} \\ \theta_{x1} \\ \theta_{y/z1} \\ U_{y/z2} \\ \theta_{x2} \\ \theta_{y/z2} \end{Bmatrix} = \begin{Bmatrix} F_{y/z1} \\ R_{x1} \\ M_{y/z1} \\ F_{y/z2} \\ R_{x2} \\ M_{y/z2} \end{Bmatrix} \quad (3)$$

From the theoretical idea to practical idea is only one step. This step involves the RC element with a unique property of letting the element with a limited rotational degree of freedom. For this purpose a stiffness matrix showing the connection between horizontal efforts and rotational efforts will be considered as in (3).
The unknowns in (3) are factors α_i . In order to find out the unknowns a system with one element as expressed in figure 1-a and 1-b is considered. By considering α_1 as known and considering (4) , due to finite element approach and angles expressed in Radians.

$$\theta_{yj} = L \cdot U_{yj} \quad (4)$$

By analyzing cases from figure 1-a and 1-b with one node of the element considered blocked and the other free we can create link formulas among α_i factors with the property of expressing all of the α_i factors with respect to α_1 as described by (12).Case with node I encastre and node J allowed moving with all degree of freedom and considering the force along axis Y as the only effort, the rotational effort, as equals in value but of contrary sign. As for the first node all movements are considered 0 .By writing the equations from the stiffness matrix that is to calculate T_I and then T_J:

$$\frac{-12EI_{y/z}}{L^3} U_j + \alpha_1 \theta_{yj} + \frac{6EI_y}{L^2} \theta_{yj} = T_I \quad (5)$$

$$\frac{12EI_{y/z}}{L^3} U_j + \alpha_2 \theta_{yj} - \frac{6EI_y}{L^2} \theta_{yj} = T_j \quad (6)$$

By considering the other effort it is easy to deduct the symmetry of the stiffness matrix for all α_i factors. As for loads combined conveniently and considering the transition of efforts from one end to another we can deduct al α_i factors with respect to α_i .Case with node J having moment Mz/y blocked and rotational moment M_{XI} blocked and node I allowed to move along all degrees of freedom and applying an effort T_I. That means that the movements θ_{Rj} and θ_{yj} are 0.Therefore the equations can be written as follows in (7) and (8).

$$\frac{12EI_y}{L^3}U_i - \frac{12EI_z}{L^3}U_j - \alpha_2\theta_{Rj} + \frac{6EI_y}{L^2}\theta_{yj} = T_I \tag{7}$$

$$\frac{12EI_z}{L^3}U_i - \frac{12EI_y}{L^3}U_j - \alpha_1\theta_{RI} + \frac{6EI_z}{L^2}\theta_{zj} = T_J \tag{8}$$

Doing similar cases as above and consider the symmetry of the stiffness matrix along diagonal of the stiffness matrix proposed with a connection between efforts becomes as in (9) with all α_1 depending on α_i .

Please note that a formulation as in (17) is suitable for the seismic protection system proposed only. A formulation as (9) is suitable for any system that involves the element to execute free rotations, according to Priestley [6]. Therefore the intend to be in a more of a general case, brings the formulation as in (9) with the desire of allow the design of the structure to change the stiffness properties of the element according to the needs that are vast and are related to the height of the structure, seismic zone, the shape of the RC structure and the architectural needs involved. That is the purpose of expressing all of the α_i unknowns with respect to α_1 with FEM methods with respect to loads, because loads and neutral axis can vary among columns, and the final cross section shape of the columns.

$$\begin{Bmatrix} \frac{12EI_{y/z}}{L^3} & \alpha_1 & \frac{-6EI_{y/z}}{L^2} & \frac{-12EI_{y/z}}{L^3} & -\alpha_1 & \frac{-6EI_{y/z}}{L^2} \\ \alpha_1 & \frac{GK_T}{L} & \alpha_2 & -\alpha_1 & \frac{GK_T}{L} & \alpha_3 \\ \frac{12EI_{y/z}}{L^3} & -\alpha_1 & \frac{4EI_{y/z}}{L} & \frac{6EI_{y/z}}{L^2} & \alpha_1 & \frac{2EI_{y/z}}{L} \\ \frac{12EI_{y/z}}{L^3} & -\alpha_1 & \frac{6EI_{y/z}}{L^2} & \frac{12EI_{y/z}}{L^3} & \alpha_1 & \frac{6EI_{y/z}}{L^2} \\ -\alpha_1 & \frac{GK_T}{L} & \alpha_4 & \alpha_1 & \frac{GK_T}{L} & \alpha_5 \\ \frac{-6EI_{y/z}}{L^2} & \alpha_3 & \frac{2EI_{y/z}}{L} & \frac{6EI_{y/z}}{L^2} & \alpha_5 & \frac{4EI_{y/z}}{L} \end{Bmatrix} = K_e \tag{9}$$

With

$$\alpha_2 = \frac{GI_T}{L} - \alpha_1 L \tag{10}$$

$$\alpha_3 = \frac{GI_T}{L} + \alpha_1 L \tag{11}$$

$$\alpha_4 = \frac{-6GI_T}{L} - 3\alpha_1 L \tag{12}$$

$$\alpha_5 = \frac{-2GI_T}{L} - \alpha_1 L \tag{13}$$

As for α_1 can be estimated as in (16) due to formulation of expression of rotational moment expressed in (14).

$$M_R = \mu M_1 - \mu M_2 - \mu M_3 \tag{14}$$

Where $M_{1,2,3}$ represents the moment induced from the three surfaces of contact among elements and μ is the frictional coefficient from the friction force, considered equal for all 3 surfaces, according to Silwerbrand [7]. The 3 surfaces of contact are considered in this case: bottom rectangular plate with the cross area of the pin excluded, exterior face of cylindrical pin with no top and bottom and the top of the metallic pin. Also the rotational moments can be expressed analytical as in (15).

$$M_R = \int_{A1} (\sigma\mu\sqrt{x^2 + y^2})dxdy - \int_{A2} (\sigma\mu\sqrt{x^2 + y^2})dxdy - \int_{A1} (\mu\tau\pi R_{cyl}^2)dxdy \tag{15}$$

Considered M_R as known due to FEM methods α_1 can be expressed as in (16).

$$\alpha_1 = \frac{LM_R}{GI_T} \tag{16}$$

Considering the stiffness matrix as expressed originally in (2) and considering the stiffness matrix expressed in (9) the final stiffness matrix for columns with the rotational degree of freedom becomes as seen in (17).

$$\{K_i\} = \begin{bmatrix}
\frac{ES}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{ES}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI_z}{L^3} & 0 & \alpha_1 & 0 & \frac{-6EI_z}{L^2} & 0 & \frac{-12EI_z}{L^3} & 0 & -\alpha_1 & 0 & \frac{6EI_z}{L^2} \\
0 & 0 & \frac{12EI_y}{L^3} & \alpha_{11} & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{-12EI_y}{L^3} & -\alpha_{11} & \frac{-6EI_y}{L^2} & 0 \\
0 & \alpha_1 & \alpha_{11} & \frac{GI_t}{L} & \alpha_{12} & \alpha_2 & 0 & -\alpha_1 & -\alpha_{11} & \frac{-GI_t}{L} & \alpha_{13} & \alpha_3 \\
0 & 0 & \frac{6EI_y}{L^2} & \alpha_{12} & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & \alpha_{14} & \frac{2EI_y}{L} & 0 \\
0 & \frac{-6EI_z}{L^2} & 0 & \alpha_2 & 0 & \frac{4EI_z}{L} & 0 & \frac{-6EI_z}{L^2} & 0 & \alpha_4 & 0 & \frac{2EI_z}{L} \\
-\frac{ES}{L} & 0 & 0 & 0 & 0 & 0 & \frac{ES}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-12EI_z}{L^3} & 0 & -\alpha_1 & 0 & \frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & 0 & \alpha_1 & 0 & \frac{6EI_z}{L^2} \\
0 & 0 & \frac{-12EI_y}{L^3} & -\alpha_{11} & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{12EI_y}{L^3} & \alpha_{11} & \frac{-6EI_y}{L^2} & 0 \\
0 & -\alpha_1 & -\alpha_{11} & \frac{-GI_t}{L} & \alpha_{14} & \alpha_4 & 0 & \alpha_1 & \alpha_{11} & \frac{GI_t}{L} & \alpha_{15} & \alpha_5 \\
0 & 0 & \frac{-6EI_y}{L^2} & \alpha_{13} & \frac{2EI_y}{L} & 0 & 0 & 0 & \frac{-6EI_y}{L^2} & \alpha_{15} & \frac{4EI_y}{L} & 0 \\
0 & \frac{6EI_z}{L^2} & 0 & \alpha_3 & 0 & \frac{2EI_z}{L} & 0 & \frac{6EI_z}{L^2} & 0 & \alpha_5 & 0 & \frac{4EI_z}{L}
\end{bmatrix} \tag{17}$$

Please note that a small reduction of the cross section area occurs and that can be expressed in the stiffness matrix by introducing a connection among rotational effort and the first effort expressed in the stiffness matrix, according to Shigeru [8]. However, due to small influence it can be introduced simply by changing the cross section area to a new cross section area considered the smallest when a rotation occurs with respect to originally cross section area and the eccentricity. For a square cross section shape this can be expressed as in (18).

$$S_1 = S - exa \tag{18}$$

As for the RC beams, as in figure 5 the first important notice is that beams becomes more subject to wrapping as the rotation of the beam occurs due to the angle of the loads applied with respect to usual cross section. More over is hard to create a beam that has the capability of rotation due to challenges expressed further more. Some challenges and observations of the theoretical analyses are due. First for the columns the wrapping effect is not considered due to the low influence of wrapping, but the effect or the reduction of the cross section has to be considered as a factor of safety, β_S. This factor is introduced due to the movement of the cross sections of the end of the elements. β_S is estimated as

$$\beta_S = \beta_{SN} \cdot \lambda \tag{18}$$

A major challenge for transferring the efforts towards rotational effort is the modeling of the tension stiffening. The tension between the primary cracks can increase sensitively (up to 3 times) the stiffness of a structural concrete member and can be very important especially in Serviceability Limit States design, but neglected for the Ultimate Limit States design. Never the less the modeling of the tension stiffening can bring great deal of accuracy to calculus, usually by dividing the crack width to the average crack spacing, usually 2/3 of the element height.The creep represents one of the challenges in transfer of the efforts. The creep is modeling is described by a procedure that provides good accuracy with fast results.

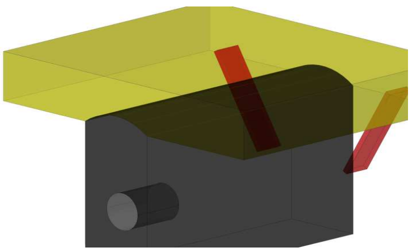

Figure 5. Element proposed for beam allowed to execute rotation

Creep is obtained by scaling the uniaxial stress-strain curve with the factor (1+φ), where φ is the creep coefficient. There is a slightly overestimation of the deformation that is involved by this procedure. Next a stiffness matrix for the beams with rotation capabilities is created. First a connection among rotational moment and wrapping moment is introduced by Rafueneau [3] in (18).

$$K_{TOR} = \begin{Bmatrix} -\psi 1 & -\psi 2 & \psi 1 & -\psi 2 \\ -\psi 2 & \psi 3 & \psi 2 & \psi 4 \\ \psi 1 & \psi 2 & -\psi 1 & \psi 2 \\ -\psi 2 & \psi 4 & \psi 2 & \psi 3 \end{Bmatrix} \tag{18}$$

With

$$\psi 1 = GK_T \frac{\lambda \sinh \lambda L}{2(\cosh \lambda - 1) - \lambda L \sinh \lambda L} \tag{19}$$

$$\psi 2 = GK_T \frac{\cosh \lambda L - 1}{2(\cosh \lambda - 1) - \lambda L \sinh \lambda L} \tag{20}$$

$$\psi 3 = GK_T \frac{\sinh \lambda L - \lambda L \cosh \lambda L}{2(\cosh \lambda - 1) - \lambda L \sinh \lambda L} \tag{21}$$

$$\psi 4 = GK_T \frac{\lambda L - \sinh \lambda L}{2(\cosh \lambda - 1) - \lambda L \sinh \lambda L} \tag{22}$$

$$\lambda^2 = \frac{GK_T}{EI_W} \tag{23}$$

By introducing this matrix in the stiffness matrix obtain the stiffness matrix as (24).

$$\{K_i\} = \begin{bmatrix}
\frac{ES}{L} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-ES}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & 0 & \frac{6EI_z}{L^2} & 0 & \frac{-12EI_z}{L^3} & 0 & 0 & 0 & 0 & \frac{-6EI_z}{L^2} \\
0 & 0 & \frac{12EI_y}{L^3} & \beta_i & \beta_i & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{-12EI_y}{L^3} & \beta_i & \beta_i & \frac{-6EI_y}{L^2} & 0 \\
0 & 0 & \beta_i & -\psi_1 & -\psi_2 & \beta_i & 0 & 0 & 0 & \beta_i & \psi_1 & \psi_2 & \beta_i & 0 \\
0 & 0 & \beta_i & -\psi_2 & -\psi_3 & \beta_i & 0 & 0 & 0 & \beta_i & \psi_2 & \psi_4 & \beta_i & 0 \\
0 & 0 & \frac{6EI_y}{L^2} & \beta_i & \beta_i & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & \beta_i & \beta_i & \frac{2EI_y}{L} & 0 \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & 0 & \frac{4EI_z}{L} & 0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & 0 & \frac{2EI_z}{L} \\
\frac{-ES}{L} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{ES}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-12EI_z}{L^3} & 0 & 0 & 0 & 0 & \frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\
0 & 0 & \frac{-12EI_y}{L^3} & \beta_i & \beta_i & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{12EI_y}{L^3} & \beta_i & \beta_i & \frac{6EI_y}{L^2} & 0 \\
0 & 0 & \beta_i & \psi_1 & \psi_2 & \beta_i & 0 & 0 & 0 & \beta_i & \psi_1 & \psi_2 & \beta_i & 0 \\
0 & 0 & \beta_i & \psi_2 & \psi_4 & \beta_i & 0 & 0 & 0 & \beta_i & \psi_2 & \psi_3 & \beta_i & 0 \\
0 & 0 & \frac{6EI_y}{L^2} & \beta_i & \beta_i & \frac{2EI_y}{L} & 0 & 0 & 0 & \frac{-6EI_y}{L} & \beta_i & \beta_i & \frac{4EI_y}{L} & 0 \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & 0 & \frac{2EI_z}{L} & 0 & \frac{6EI_z}{L} & 0 & 0 & 0 & 0 & \frac{4EI_z}{L}
\end{bmatrix} \tag{24}$$

The resolving of the unknown in (24), called βi is done in the same manner as considerer for the columns by considering one element with this stiffness matrix and one node of the element with some degree of freedom blocked and the other end with degree of freedom permitted in a convenient manner and considering the connections among the efforts.

In the stiffness matrix the unknowns are β_i. By solving the unknown through convenient cases with end conditions there is possible to express the unknown as related to β_1. The final stiffness matrix for the beams becomes as seen in 27
With a note that in a similar manner can be treated the moment along axis Z and β_i are expressed with respect to β_1 that is expressed as in (25) the stiffness matrix is as in (26).

$$\beta_1 = \int_{A1} (\tau \sqrt{x^2 + y^2}) dxdy - \int_{A2} (\sigma \mu \sqrt{x^2 + y^2}) dxd - \int_{A1} (\mu \sigma \pi R_{Cyl}^2) dxd \qquad (25)$$

$$\{K_i\} = \begin{bmatrix}
\frac{ES}{L} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-ES}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & 0 & \frac{6EI_z}{L^2} & 0 & \frac{-12EI_z}{L^3} & 0 & 0 & 0 & 0 & \frac{-6EI_z}{L^2} \\
0 & 0 & \frac{12EI_y}{L^3} & \beta_1 & \beta_2 & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{-12EI_y}{L^3} & \beta_1 & \beta_2 & \frac{-6EI_y}{L^2} & 0 \\
0 & 0 & \beta_1 & -\psi_1 & -\psi_2 & \beta_3 & 0 & 0 & 0 & -\beta_1 & \psi_1 & \psi_2 & \beta_5 & 0 \\
0 & 0 & \beta_2 & -\psi_2 & -\psi_3 & \beta_4 & 0 & 0 & 0 & -\beta_2 & \psi_2 & \psi_4 & \beta_6 & 0 \\
0 & 0 & \frac{6EI_y}{L^2} & \beta_3 & \beta_4 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & \beta_7 & \beta_8 & \frac{2EI_y}{L} & 0 \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & 0 & \frac{4EI_z}{L} & 0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & 0 & \frac{2EI_z}{L} \\
\frac{-ES}{L} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{ES}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-12EI_z}{L^3} & 0 & 0 & 0 & 0 & \frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\
0 & 0 & \frac{-12EI_y}{L^3} & \beta_1 & \beta_2 & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{12EI_y}{L^3} & \beta_1 & \beta_2 & \frac{6EI_y}{L^2} & 0 \\
0 & 0 & -\beta_1 & \psi_1 & \psi_2 & \beta_7 & 0 & 0 & 0 & \beta_1 & \psi_1 & \psi_2 & \beta_9 & 0 \\
0 & 0 & -\beta_2 & \psi_2 & \psi_4 & \beta 8 & 0 & 0 & 0 & \beta_2 & \psi_2 & \psi_3 & \beta_{10} & 0 \\
0 & 0 & \frac{6EI_y}{L^2} & \beta_5 & \beta_6 & \frac{2EI_y}{L} & 0 & 0 & 0 & \frac{-6EI_y}{L} & \beta_9 & \beta 10 & \frac{4EI_y}{L} & 0 \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & 0 & \frac{2EI_z}{L} & 0 & \frac{6EI_z}{L} & 0 & 0 & 0 & 0 & \frac{4EI_z}{L}
\end{bmatrix} \qquad (26)$$

with

$$\beta_2 = \frac{-\beta_1}{\sinh \lambda} \qquad (27)$$

$$\beta_3 = \frac{-\beta_1 \cdot L + \psi 1 - \psi 2}{\sinh \lambda} \qquad (28)$$

$$\beta_4 = \frac{\beta_1 \cdot L + \psi 2 + \psi 3}{\sinh \lambda} \qquad (29)$$

$$\beta_4 = \frac{\beta_1 \cdot L - \psi 2 + \psi 2}{\sinh \lambda} \qquad (30)$$

$$\beta_6 = \frac{\beta_1}{\sinh \lambda} \qquad (31)$$

$$\beta_7 = \frac{-\beta_1 \cdot L - \psi 1 + \psi 2}{\sinh \lambda} \quad (32)$$

$$\beta_8 = \frac{-\beta_1 \cdot L - \psi 2 + \psi 3}{\sinh \lambda} \quad (33)$$

$$\beta_9 = \frac{-\beta_1 \cdot L - \psi 1 + \psi 4}{\sinh \lambda} \quad (34)$$

$$\beta_{10} = \frac{\beta_1 \cdot L + \psi 1 - \psi 2}{\sinh \lambda} \quad (35)$$

For RC columns the major codes of practice introduced after 2000 introduced reliable models. However confinement caused by the Poisson lateral effect acts only at high values loads as seen in figure 8. That implies that the technique is most valuable for Ultimate Limit State design, according to Hyunhoon [9]. In terms of energy it is easy to observe the extra potential energy in case of seismic moves that takes place, similar with the concept of obtaining plastic articulations in the event of the seism so well known by current engineering practice.

Due to this notes and observations the conclusion leads to the idea of implementing this system to the columns first and observe the results.

7. Comparison of predictions and experimental results

As expressed in analytical investigation the idea of creating RC beams with capability of rotation is hard to implement. As for the columns the idea seems more suitable. So a system that allows columns to execute rotation is created as seen in fig 65 with a non variable eccentricity, first along axis X and then along axis Y. Please note that other systems can be created as well. As expected improvements are shown in the final movements of the third floor and also of the energy.

As for columns subject to horizontal impact that simulates a seismic movement the results in terms of energy are expressed in table 1. Obviously the rotation of columns along their longest axis involves a dissipation of energy through frictional force this dissipation of energy is benefice for the structure, but also brings a disadvantage due to reduction of cross section area.

Therefore the solution brings limited benefits in reducing the total energy accumulated in the structure. If considered a total energy assimilated in the structure according to formula 36, then the total dissipation of energy is 35% in this case.

8. Experimental results and discussion

Due to the seismic force estimated as Choi [10] as in (36) it is easy to see the impact of the system applied to the columns of a RC structure. As expected the total energy is reduced with a percentage that can reach 35%.This would benefit the structure and make the structure more resistant to horizontal forces, especially the seismic impact. A seism impact was considered for the theoretical experiment with tabular amplitude. Please note that this reduction of energy is seen in the reduction of energy at the level of the total strain energy as well.

$$E_i = E_k + E_D + E_S + E_H \quad (36)$$

With the significance that Kinetic Energy +Dumping Energy +Strain Energy +Hysteric Irrecoverable Energy equals Input Energy.

Please note that the system implemented to RC structure's columns bring some disadvantage as well like connection among pins and the column and that can be solved through additional stirrups. Other note resides from the fact that this system that allows columns to execute rotations needs repairs after any seismic force.

	Time	Case 1	Case 2	Case 3	Case 4
External work	0.01	0.21	0.08	0.08	0.08
	0.02	0.35	0.21	0.21	0.21
	0.03	0.84	0.42	0.42	0.42
	0.04	1.32	0.78	0.78	0.78
	0.05	2.6	1.18	1.18	1.18
Frictional dissipation	0.01	0	0	0	0
	0.02	0	0	2.2	2.6
	0.03	0	0	18.4	19.4
	0.04	0	0.9	40.4	45.6
	0.05	0	8.4	84.8	93.9
Internal energy	0.01	0.21	0.08	0.08	0.08
	0.02	0.35	0.21	0.21	0.21
	0.03	0.84	0.42	0.42	0.42
	0.04	1.32	0.78	0.78	0.78
	0.05	2.6	1.18	1.18	1.18
Static stabilization	0.01	0.11	1.12	1.18	1.19
	0.02	0.22	3.5	3.4	3.3
	0.03	0.28	5.1	5.2	5.3
	0.04	0.34	7.2	7.4	7.6
	0.05	0.38	9.1	9.1	9.2
Strain Energy	0.01	0.21	0.05	0.08	0.08
	0.02	0.35	0.18	0.21	0.21
	0.03	0.84	0.42	0.42	0.42
	0.04	1.32	0.78	0.78	0.78
	0.05	2.6	1.18	1.18	1.18
Total energy	0.01	-0.39	-0.01	-0.04	-0.05
	0.02	-0.40	-0.19	-0.09	-0.11
	0.03	-0.54	-0.28	-0.14	-0.16
	0.04	-0.68	-0.06	-0.23	-0.27
	0.05	0	0	0	0
Viscous energy	0.01	0	109	109	109
	0.02	0	110	110	110
	0.03	0	110	110	110
	0.04	0	110	110	110
	0.05	0	110	110	110

Table 1 – Results of the Theoretical models analyzed.

*Case 1 is the structure build in a classic mode

*Case 2 is the structure with columns proposed and friction penalty 0,05

*Case 3 is a structure with columns proposed and a friction penalty 0.3

*Case 4 is the same structure with columns proposed and a friction penalty of 0.3 plus an extra load of 0.2 0f the self weight.

Please note as well that the friction coefficient is not a major factor for the energy, but as seen in fig. 7 and in table 1 can influence the timing of the friction energy dissipated.

9. Further research

A solution for RC structure subject to strong seismic forces is desired through this study. Therefore a part of the solution can represent a system with columns allowed to execute rotational movements. This study considers a case of a theoretical structure only, but does not consider any general solutions in terms of any RC structure. An algorithm that is suitable for any RC structure can be developed based on this structural solution. This algorithm can be implemented in a computer based computational program and can lead to better protected RC structures to the seismic movement.

Moreover, the rotational movement of the beams and other RC elements can be studied and improved based on the changing of the stiffness matrix in a manner that connects the rotational movement with other efforts.

9. Conclusions

Based on the results expressed in this paper and the theoretical study some conclusions can be drawn:

1. Rotational movement on RC columns can be calculated and estimated by using an enriched stiffness matrix with satisfactory results;
2. Using RC columns that allows rotational movement can bring an important benefit for the resistance of a RC structure to seismic efforts.
3. The idea of allowing some elements to perform controlled rotational movements can be a subject to study for other structural members.

References

[1] Hillerborg A., *The theoretical basis of method to determine the fracture energy G_F of Concrete*, Materials and Structures, 1985, pp 23-78.

[2] Roelfstra R.E., Wittmann F.H., *Numerical Method to Link Strain Softening with Fracture in Concrete,* Fracture Toughness and Fracture Energy in Concrete, Elsevier Science, 1986, pp 12-132.
[3] Neil M.H., Lin I., Veda T., *Anchorage of Reinforced Bars for Seismic Force,* ACI Structural Journal, V. 14, No. 3, Title no. 84-S43 Sept.-Oct. 1987, pp. 407-417.
[4] O'Hara S., Kent D.B.J., *ARE Review Manual* ,Dec. 8 2010, Chapter 6, pp 21-26.
[5] Rafueneau F., Casaux G., Mazars, J., *Nonlinear Torsion and Wrapping for Multifiber Beam Elements*, Finite Element of RC Structures 313, Title no.SP-237-7, Aug. 2006, pp 99-108.
[6] Nigel Priestley, Sieble M.J., F., Xiao, Y., Ravindra V., *Steel Jacket Retrofilling of Reinforced Concrete Bridge Columns for Enhanced Shear Strenght*-Part 1:Theoretical Considerations and Test Design, *ACI Structural Journal*, V. 13, No. 3,Title no. 91-S39, Jul.-Aug. 1994, pp 394-405.
[7] Silwerbrand J., *Stress and Strain in Composite Concrete Beams Subject to Differential Shrinkage*, ACI Structural Journal, V. 16, No. 3, Title no. 94-S31, Jul.-Aug. 1997, pp 347-353.
[8] Shigeru H., Park R., Tanaka H., *Seismic Load Tests on Interior and Exterior Beam-Column Joints with Substandard Reinforcing Details*, ACI Structural Journal, V. 76, No. 2, Title no. 97-S02, Jan.-Feb., 2000, pp 11-25.
[9] Hyunhoon C., Jinkoo K.H., *Evaluation of seismic energy demand and its application on design of buckling-restrained braced frames*, Structural Engineering and Mechanics, Vol. 31, No. 1, 2009, pp 93-112.
[10] Choi H., Erochko J., Christopoulos C., Tremblay R., *Comparison of the Seismic Response of Steel Buildings Incorporating Self-Centering Energy Dissipative Braces, Buckling Restrained Braced and Moment Resisting Frames* , The 14th World Conference on Earthquake Engineering October 12-17, 2008, Beijing, China, pp 212-221.

Address:

- Assist. Dr. Eng. Sergiu Cătinaş, "Technical University of Cluj Napoca", Str.Memorandumului , nr. 28, 400114, Cluj Napoca, catinassergiu@dst.utcluj.ro.