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Effects of Soret and Dusty Fluid on MHD Flow Past over Inclined Porous Plate

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This paper deals with the effects of Soret and dusty fluid on unsteady MHD flow past over porous plate. The flow is along over inclined plate with constant wall temperature and mass diffusion. The external uniform magnetic field is applied perpendicular to the inclined surface. The surface of plate is considered to be porous and fluid taken is a viscous, incompressible, an electrically conducting. The MHD flow model is consisting equations of momentum, diffusion equation and energy equation. The governing MHD flow model of partial differential equations is transformed to dimensionless equations by using dimensionless variables. To examined the solution of the MHD flow model, useful sets of the values of the parameters have been considered. The governing equations involved analytically. The results obtained have been examined with the help of graphs drawn for various parameters.

Keywords: *MHD flow, Soret effect, dusty fluid, porous plate.*

1. Introduction

The influence of Soret on unsteady flow of dusty fluid along with heat and mass transfer have its importance in many applications such as power plant piping, combustion and petroleum transport and also this type of flow has uses in nuclear reactors, geothermal systems and filtration etc. Unsteady hydromagnetic flows of a dusty viscous fluid between two oscillating plates was analyzed by Debnath and Ghosh [6]. Datta et al. [5] have considered unsteady heat transfer to pulsatile flow of a dusty viscous incompressible fluid in a channel. Unsteady MHD flow and heat transfer of dusty fluid between parallel plates with variable physical properties was studied by Attia[2]. Further Attia [1] has work on unsteady flow of a dusty conducting fluid between parallel porous plates with temperature dependent viscosity. Gireesha et al. [7] have examined Heat transfer in MHD flow of a dusty fluid over a stretching sheet with viscous dissipation. Soret Effect on an oscillatory

MHD mixed convective mass transfer flow past an infinite vertical porous plate with variable suction was investigated by Ahmed and Sinha [4]. Anuradha and Priyadharshini [3] have analyzed heat and mass transfer on unsteady MHD free convective flow pass a semi-infinite vertical plate with Soret effect. Uwanta and Halima [11] have work on the influence of Soret and Dufour effects on MHD free convective heat and mass transfer flow over a vertical channel with constant suction and viscous dissipation. Influences of chemical reaction and wall properties on MHD Peristaltic transport of a dusty fluid with heat and mass transfer was examined by Muthuraj et al. [8]. Manjunatha along with Gireesha [9] have considered effects of variable viscosity and thermal conductivity on MHD flow and heat transfer of a dusty fluid. Soret effect on unsteady MHD flow past an Impulsively Started inclined oscillating plate with variable temperature and mass diffusion was study by us along with Rajput [10]. Further, we [12] studied chemical reaction effect on unsteady MHD flow past an exponentially accelerated inclined plate with variable temperature and mass diffusion in the presence of Hall current. The present study is carried out to investigate the combined effects of Soret and dusty fluid on unsteady MHD flow past over inclined porous plate with constant wall temperature and mass diffusion. The problem is solved by analytically. A useful set of graphical results illustrating the effects of different parameters included in the MHD flow model are presented and discussed.

2. Mathematics Analysis.

Consider an unsteady MHD flow of a viscous, incompressible, electrically conducting fluid past an inclined porous plate. The x axis is taken along the vertical plane and y axis is normal to it. So the y axis lies in the horizontal plane. The plate is inclined at angle α from vertical. The magnetic field B0 of uniform strength is applied perpendicular to the flow. The viscous dissipation and induced magnetic field has been neglected due to its small effect. Initially it has been considered that the plate as well as the fluid is at the same temperature T_∞ and the concentration level C_∞ everywhere in the fluid is same in stationary condition. At time t>0, temperature of the plate is raised to T_w. The concentration near the plate is C_w. The suction velocity normal to the plate is $V_0 = -u'_0$. The flow model is as under:

Equation of Momentum

$$\frac{\partial u}{\partial t}+V_0\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta Cos\alpha(T-T_\infty)+g\beta^* Cos\alpha(C-C_\infty)+\frac{k'N(v-u)}{\rho}-\frac{\sigma B_0^2 u}{\rho} \quad (1)$$

Equation of Dust Particle:

$$\frac{\partial v}{\partial t}=\frac{k'(u-v)}{m'} \quad (2)$$

Equation of Concentration

$$\frac{\partial C}{\partial t}+V_0\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}+D_m\frac{K_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Equation of Energy:

$$\frac{\partial T}{\partial t}+V_0\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The following boundary conditions have been considered.

$$\left.\begin{array}{l} t\le 0: u=0,\ v=0,\ T=T_\infty,\ C=C_\infty,\ \text{for every } \bar{y}, \\ t>0: u=0, T=T_w, C=C_w,\ \text{at y=0} \\ u\to 0, v\to 0, T\to T_\infty, C\to C_\infty \ \text{ as } \ y\to\infty \end{array}\right\} \tag{5}$$

Here u is the velocity of the fluid, v - the velocity of dust particle, V_0 is section velocity, g - the acceleration due to gravity, β - volumetric coefficient of thermal expansion, t- time, m is the mass of dust partical α is the thermal conductivity, k' is the stoke's resistance coefficient, T- temperature of the fluid, β^*- volumetric coefficient of concentration expansion, C - species concentration in the fluid, ν - the kinematic viscosity, y- the coordinate axis normal to the plates, ρ - the density, C_p- the specific heat at constant pressure, k - thermal conductivity of the fluid, D - the mass diffusion coefficient , N – the number of dust particles per unit volume, m' - mass concentration of dust particles, D_m- the effective mass diffusivity rate, T_w- temperature of the plate at y= 0, C_w - species concentration at the plate y= 0, B_0- the uniform magnetic field, σ - electrically conductivity.
By use of the following dimensionless quantities:

$$\left.\begin{array}{l} \bar{y}=\dfrac{yu'_0}{\upsilon}, \bar{u}=\dfrac{u}{u'_0}, \bar{v}=\dfrac{v}{u'_0}, \theta=\dfrac{(T-T_\infty)}{(T_w-T_\infty)}, S_c=\dfrac{\upsilon}{D}, \mu=\rho\upsilon, P_r=\dfrac{\mu c_p}{k}, \\ G_r=\dfrac{g\beta\upsilon(T_w-T_\infty)}{u_0'^{\,3}}, M=\dfrac{\sigma B_0^2\upsilon}{\rho u_0'^2}, G_m=\dfrac{g\beta^*(C_w-C_\infty)}{u_0'^{\,3}}, \bar{C}=\dfrac{(C-C_\infty)}{(C_w-C_\infty)} \\ \bar{t}=\dfrac{tu_0'^2}{\upsilon}, S_r=\dfrac{D_mK_T(T_w-T_\infty)}{\upsilon T_m(C_w-C_\infty)}, \tau_p=\dfrac{mu_0'^2}{\upsilon k'}, f'=\dfrac{\upsilon k' N}{u_0'^2\rho} \end{array}\right\} \tag{6}$$

The symbols in dimensionless form are as under:
$\bar{u}$ - is the velocity of fluid, $\bar{v}$ - the velocity of dust particle, $\bar{t}$ - time, θ - the temperature, $\bar{C}$ - the concentration, G_r - thermal Grashof number, G_m - mass Grashof number, μ- the coefficient of viscosity, P_r - the Prandtl number, S_c - the Schmidt number, S_r- Soret number, K_T - Thermal diffusion ratio, T_m -The mean

fluid temperature , M - the magnetic parameter and $\overline{K}$ - is the permeability parameter, f' Dust fluid parameter, τ_p- Relaxation time parameter for dust particles parameter.

The flow model becomes

$$\frac{\partial \overline{u}}{\partial \overline{t}} - \frac{\partial \overline{u}}{\partial \overline{y}} = \frac{\partial^2 \overline{u}}{\partial \overline{y}^2} + G_r \theta Cos\alpha + G_m \overline{C} Cos\alpha + f'(\overline{v} - \overline{u}) - M\overline{u} \quad (7)$$

$$\frac{\partial \overline{v}}{\partial \overline{t}} = \frac{1}{\tau_p}(\overline{u} - \overline{v}) \quad (8)$$

$$\frac{\partial \overline{C}}{\partial \overline{t}} - \frac{\partial \overline{C}}{\partial \overline{y}} = \frac{1}{S_c}\frac{\partial^2 \overline{C}}{\partial \overline{y}^2} + S_r \frac{\partial^2 \theta}{\partial \overline{y}^2} \quad (9)$$

$$\frac{\partial \theta}{\partial \overline{t}} - \frac{\partial \theta}{\partial \overline{y}} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial \overline{y}^2} \quad (10)$$

The boundary conditions (5) reduced to:

$$\left.\begin{array}{l} \overline{t} \le 0 : \overline{u} = 0,\ \overline{v} = 0,\ \theta = 0,\ \overline{C} = 0, \text{ for every } \overline{y}, \\ \overline{t} > 0 : \overline{u} = 0, \theta = 1, \overline{C} = 1 \text{ at } \overline{y} = 0, \\ \overline{u} \to 0, \overline{v} \to 0,\ \theta \to 0, \overline{C} \to 0 \text{ as } \overline{y} \to \infty \end{array}\right\} \quad (11)$$

Dropping bars in the above equations, we get

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_r \theta Cos\alpha + G_m C Cos\alpha + f'(v - u) - Mu \quad (12)$$

$$\frac{\partial v}{\partial t} = \frac{1}{\tau_p}(u - v) \quad (13)$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} + S_r \frac{\partial^2 \theta}{\partial y^2} \quad (14)$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} \quad (15)$$

Finally, the boundary conditions become:

$$\left.\begin{array}{l} t \le 0 : u = 0,\ v = 0,\ \theta = 0,\ C = 0, \text{ for every } \overline{y}, \\ t > 0 : u = 0, \theta = 1, C = 1 \text{ at } y = 0, \\ u \to 0, \theta \to 0, C \to 0 \text{ as } y \to \infty \end{array}\right\} \quad (16)$$

Method of solution

To find the solutions of dimensionless governing equations (12) to (15), subject to the boundary conditions (16), we assume the solutions as

$$u(y,t) = U_0(y)e^{-\lambda t} \tag{17}$$

$$v(y,t) = v_0(y)e^{-\lambda t} \tag{18}$$

$$C(y,t) = C_0(y)e^{-\lambda t} \tag{19}$$

$$\theta(y,t) = \theta_0(y)e^{-\lambda t} \tag{20}$$

With boundary condition

$$\left.\begin{array}{l} t \le 0 : U_0 = 0, v_0 = 0,\ \theta_0 = 0, C_0 = 0, \text{ for every } \bar{y}, \\ t > 0 : U_0 = 0, \theta_0 = 1, C_0 = 1 \text{ at } y = 0, \\ U_0 \to 0, v_0 \to 0, \theta_0 \to 0, C_0 \to 0 \text{ as } y \to \infty \end{array}\right\} \tag{21}$$

Solution of the equation (15) by using (21), we have

$$\theta(y,t) = e^{-m_1 y - \lambda t}$$

Solution of the equation (14) by using (19), we have

$$C(y,t) = \{A_2 e^{-m_2 y} - A_1 e^{-m_1 y}\}e^{-\lambda t}$$

Solution of the equation (12) by using (17) , we have

$$u(y,t) = \frac{Cos\alpha(G_r - A_1 G_m)(e^{-m_3 y} - e^{-m_1 y})}{m_1^2 - m_1 + A} e^{-\lambda t} + \frac{Cos\alpha G_m A_2 e^{-\lambda t}(e^{-m_3 y} - e^{-m_2 y})}{m_2^2 - m_2 + A}$$

Solution of the equation (12) by using (17) , we have

$$v(y,t) = \frac{e^{-\lambda t} Cos\alpha}{1 - \lambda\tau_p}\{\frac{(G_r - A_1 G_m)(e^{-m_3 y} - e^{-m_1 y})}{m_1^2 - m_1 + A} + \frac{G_m A_2 (e^{-m_3 y} - e^{-m_2 y})}{m_2^2 - m_2 + A}\}$$

The symbols are used in above solutions are as under

$$m_1 = \frac{1}{2}(P_r + \sqrt{P_r^2 - 4nP_r}),\ m_2 = \frac{1}{2}(S_c + \sqrt{S_c^2 - 4nS_c}),\ m_3 = \frac{1}{2}(1 + \sqrt{1 - 4A}),$$

$$A = \frac{n\tau_p f'}{1 - n\tau_p} + n - M,\ A_1 = \frac{S_c S_r m_1^2}{m_1^2 - m_1 S_c + nS_c},\ A_2 = 1 - \frac{S_c S_r m_1^2}{m_1^2 - m_1 S_c + nS_c},$$

3. Results and Discussion

In this paper we have studied the effects of Soret with dust particle on the flow which is past through inclined porous plate. The velocity profiles of fluid and dust particle for different parameters are shown in figures 1 to 10. Due to gravity component gCosα, the fluid flows with higher velocity when plate is vertical as compared to flow when plate is horizontal. It is observed from figures 1 and 6 that the velocity of fluid and dust particle decreases when the angle of inclination (α) is increased. In figures 2 and 7, it is observed that the velocities of fluid and dust particle increase when the dust fluid parameter is increased throughout the boundary layer region. Figures 3 and 8 depict the velocity profile for the case of large time for different values of magnetic parameter M. We infer from these figures that the fluid velocity decrease with increase in magnetic parameter. This is due to the fact that, the application of transverse magnetic field plays an important role of a resistive type of force similar to drag force, that acts in the opposite direction of the fluid motion, thereby fluid and dust phase velocity reduces. It is in agreement since the magnetic field establishes a force which acts against the main flow resulting in slowing down the velocity of fluid. Further, it is observed that velocities increases when Soret number is increased (figures 4 and 9). From figures 5 and 10, it is observed that the velocities increase with relaxation time parameter for dust particles parameter.

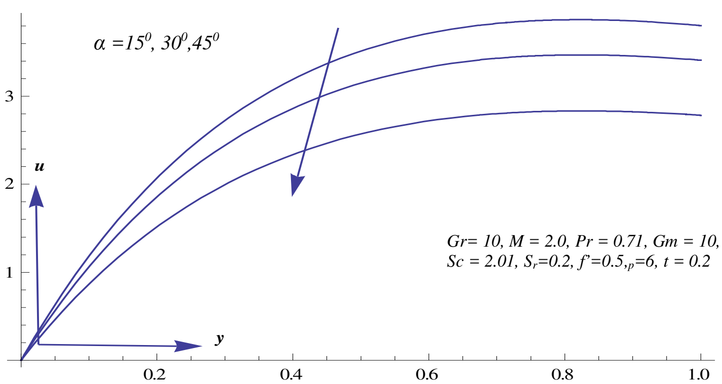

Figure1: Effect of α on fluid velocity

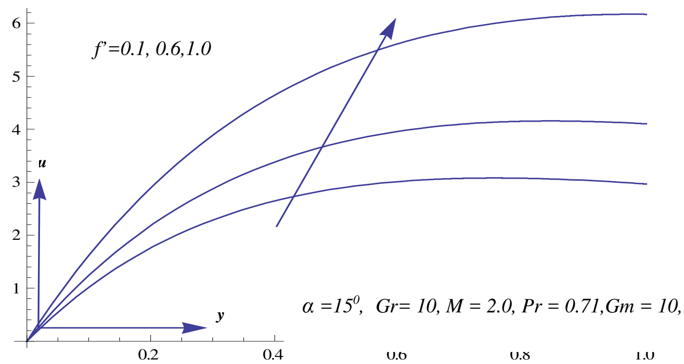

Figure2: Effect of f' on fluid velocity

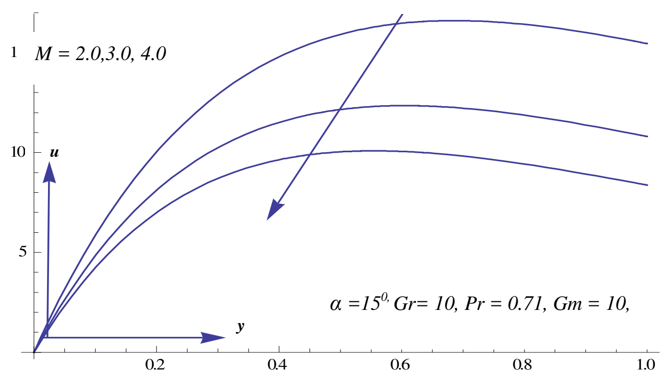

Figure 3: Effect of *M* on fluid velocity

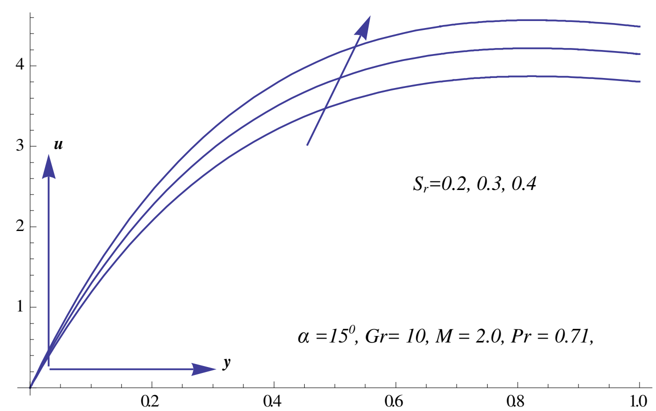

Figure 4: Effect of *Sr* on fluid velocity

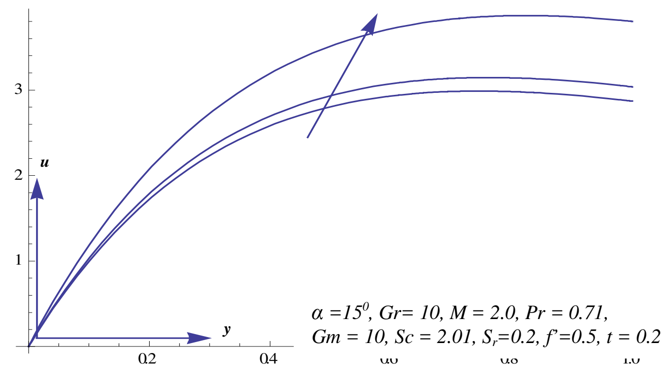

Figure 5: Effect of τ_p on fluid velocity

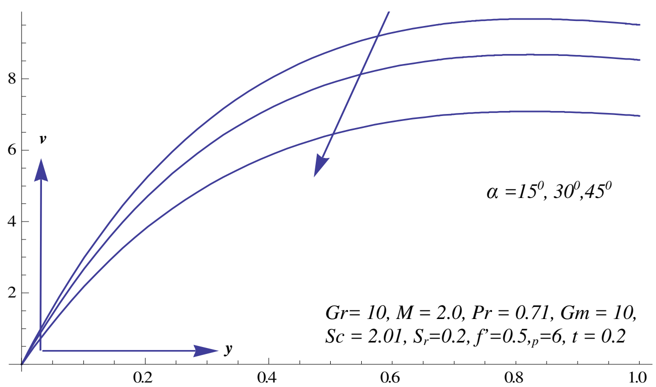

Figure6: Effect of α on dust particle velocity

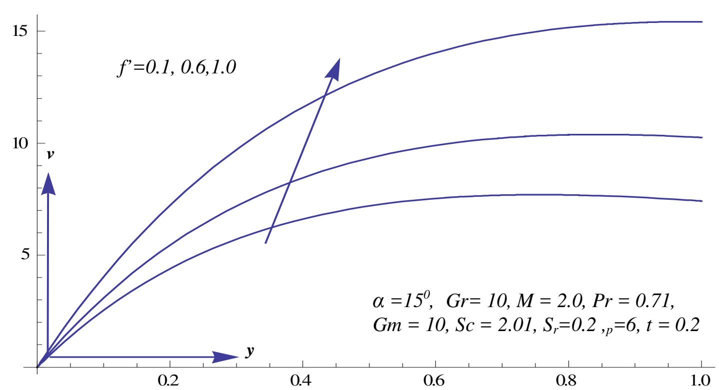

Figure 7: Effect of f' on dust particle velocity

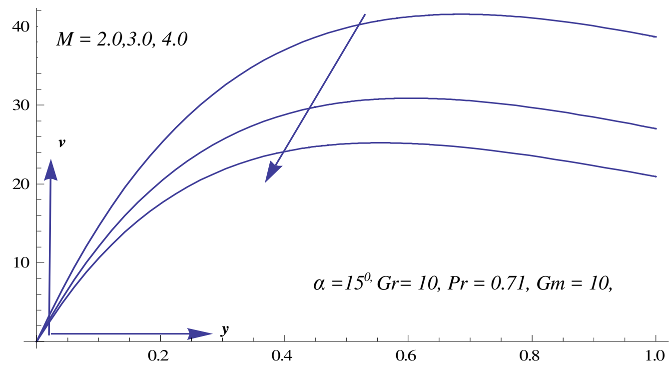

Figure 8: Effect of *M* on dust particle velocity

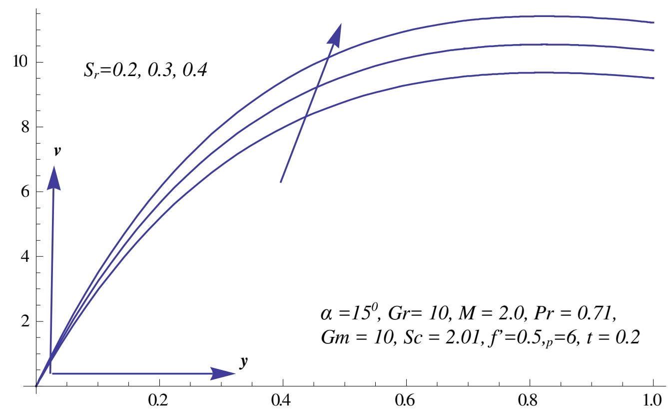

Figure 9: Effect of *Sr* on dust particle velocity

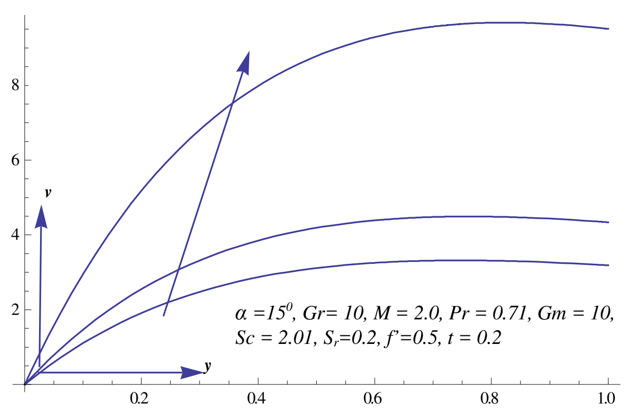

Figure 10: Effect of τ_p on dust particle velocity

4. Conclusion

In this present paper a theoretical analysis has been done to study the influence of Soret and dusty fluid on unsteady MHD Flow Past over porous inclined plate with constant plate temperature and mass diffusion. The results obtained are in agreement with the usual flow. It has been found that the velocity of the fluid and dust particle in the boundary layer region decreases with increasing the values of magnetic parameter, angle of inclination of plate and increases with increase in Soret number, Dust fluid parameter and relaxation time parameter for dust particles parameter. This study is expected to be useful in understanding the influence of Soret effect and dusty fluid on flow. Fluid flow embedded with dust particles is encountered in a wide variety of engineering problems concerned with atmospheric fallout, dust collection, nuclear reactor cooling, filtration and purification process of crude oil in chemical engineering, the investigation of underground water resources in agriculture engineering, the study of movement of natural gas, oil and water through the oil reservoirs in petroleum engineering, performance of solid fuel rock nozzles and guided missiles etc.

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