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Research Article

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Mathematical Modelling of Kidnapping with Rehabilitation

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Abstract We construct a simple mathematical model on kidnapping by integrating the concept of de-radicalising and rehabilitation of kidnappers in a system of ordinary differential equations describing the evolution and propagation of kidnapping as a crime in human society It accounts for the interaction between kidnappers and vulnerable humans leading to their abduction for the main purpose of ransom payment. We establish the crime propagation number, C_{p_n} in which a $C_{p_n} < 1$ guarantees a kidnap free state that is locally and globally asymptotically stable. Simulations are carried out on the combination of different levels kidnappers recruitment and rehabilitation. The analysis shows that increasing the rehabilitation rate of kidnappers is a better and more effective way of ensuring a kidnapping free society.

Keywords Kidnapers, Kidnappees, Rehabilitation, Vulnerable, Crime propagation Number

1. Introduction

Kidnapping is a violent crime and it occurs when a person without lawful authority physically abducts another person without that person's consent and with the intent to use the abduction for financial ransom, political objective or other nefarious objectives. Kidnap victims that purportedly have a 'kidnap value' are usually identified by 'catchers' who work for a kidnap syndicate. Many people think of kidnapping as something that is very rare and not something that could possibly happen to them because they do not have a kidnap value. The reality is that it is a huge risk, which can happen to anyone including people of any class range [3]. However, some people may be more vulnerable than others. Kidnapping for ransoms has been on the increase probably due to terrorism, drug addiction, cultism or gangsterism and the high level of corruption and insecurity in some countries. Kidnapping for ransom involves series of negotiations between the kidnapper(s) and the family of the victim [15]. Limiting the family's ability to pay reduces the frequency of the offence but opens the possibility of unintended consequences in terms of fatalities and duration of abduction [2]. In our previous work [11], which appears to be the first of its kind we presented a mathematical model on kidnapping in which we introduced a new concept, the ''Crime propagation number", $C_{p_n,}$ Our analysis revealed that in order to eliminate kidnapping we need to increase the rescue rate of kidnapped victims and reduce the recruitment rate of kidnappers. Rehabilitation of offenders can be defined and understood in a number of ways. For instance, [16] defined rehabilitation as a program designed to assist offenders in addressing needs which are related to the offending behaviour and in achieving a more productive and satisfying lifestyle. Rehabilitation can be referred to as "re-enabling" or "making fit again" for the purpose of encouraging and assisting offenders or people with challenging behaviour lead a good and useful life. It provides the basic education to tackle the problem by equipping the life with work skills. A successful rehabilitation program will recognise the needs of the offenders and proceed to address them as best as they can [9]. Hence, it is very important that rehabilitation planners must be clear about the objectives of the varied interventions. In response to various suggestions and contributions to curb crime in society, [14] considers a number of justifications to support some approaches to rehabilitation.

Rehabilitation of criminals is also reviewed in [16] with the purpose of directing researchers and clinicians in the field of correctional psychology. Some of these methods of intervention of rehabilitation include education which addresses the educational deficits of the person involved, vocational training which provides transferable skills which has the tendency of increasing the prospects of employment of the person. Behavioural and cognitive skills program is also a method of intervention of rehabilitation which provides treatment for misuse of substance. Drug abuse offenders programs are the most common form of rehabilitation offered [8]. The process of rehabilitation will help a person to readapt to a society or restoring someone to a former position. The educational, vocational and psychological based programs and other specialised services programs can be used as a means of reformation of offenders. Rehabilitation will also help to develop the important work of reducing re-offending by ex-offenders.

Here we propose a simple mathematical model incorporating a rehabilitation parameter that is capable of reducing the recruitment rate of kidnappers. Section 1 includes a brief introduction whereas the formulation of the model is given in section 2. In section 3, we present the model analysis followed by numerical simulations and discussion in section 4. The paper was rounded up in section 5 with a brief conclusion. We divide the human population into compartments of susceptible, kidnappers and kidnapees. State variables in the model are given in Table 1 and the movement between compartments is summarised in Figure 1, the individual pathways to be discussed below.

Table 1: The state variables in the model

State Variable	Description
N	Total human population
V	Number of vulnerable humans
A	Kidnappers population
E	Number of kidnapped victims (kidnappees)

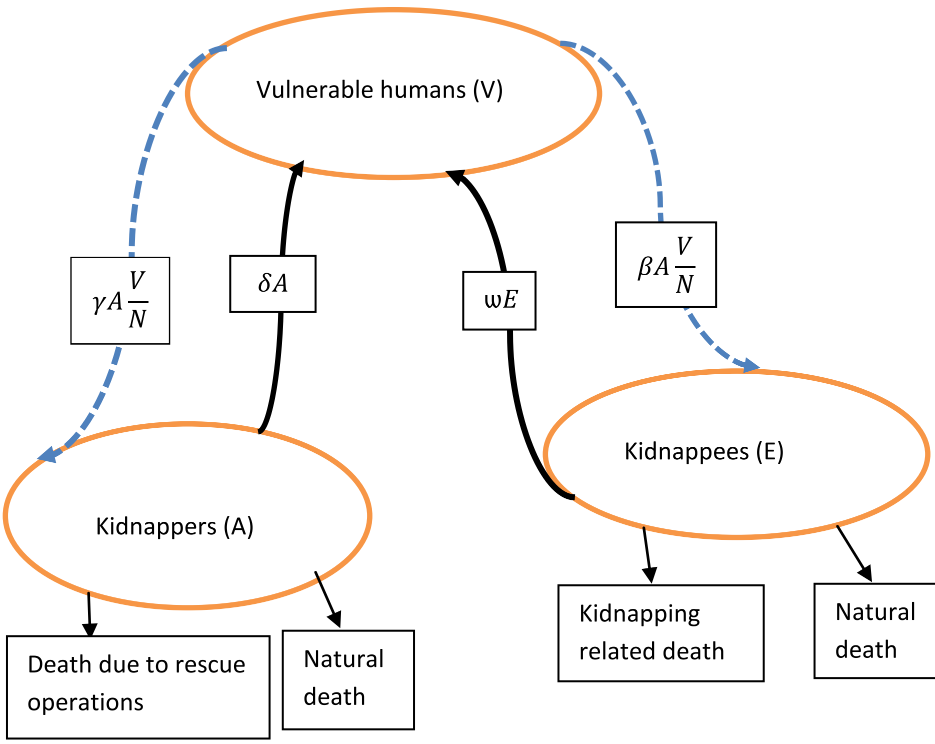


Figure 1: Schematic representation of kidnappers and vulnerable individuals interaction model

2. The formulation of the model

The entire human population is described by the equation

$$N = V + A + E.$$

Interaction or contact between kidnappers and vulnerable humans may lead to either people being kidnapped or introduced into kidnapping. We assume that the contact is necessitated by the willingness of kidnappers to gain some benefits through ransom or vulnerable humans being enticed by their kidnapper friends from the proceeds of kidnapping. Vulnerable individuals progress into the A compartment at a rate $\gamma A\frac{V}{N}$, where $\frac{V}{N}$ is the probability that the contact is with a vulnerable human and γ is the rate constant. We also assume a kidnapping rate constant β such that the transition rate, $\beta A\frac{V}{N}$, of susceptible humans into the E class is proportional to the contact between kidnappers and vulnerable humans.

Kidnap cases are usually reported to security agents who eventually swing into action to rescue the victims. The process involves among other things, the gathering of relevant information about the incidence including possible suspects and tracking of the negotiator. We assume that rehabilitated kidnappers are vulnerable and could be kidnapped. Since a rescued victim becomes vulnerable again, we assume that recruitment into the vulnerable compartment is through natural birth, λN rescued kidnapped victims, wE and rehabilitated kidnappers $\delta A,$ where λ, w and δ are per capita birth rate, rescue rate and rehabilitation rate constants respectively. Rescue operations of kidnapped victims are carried out when kidnapping has taken place and the kidnapper is in contact with the kidnappee. This may lead to casualties on both sides, hence we assume that the death rate of kidnappers due to rescue operations, $a_1 AE$ and the kidnapping related death rate of kidnap victims, $a_2 AE$ are both proportional to the contact between kidnappers and kidnappees. The above assumptions lead to the following system of equations.

$$\frac{dV}{dt} = \lambda N + wE + \delta A - \gamma A\frac{V}{N} - \mu V - \beta A\frac{V}{N} \tag{2.1}$$

$$\frac{dA}{dt} = \gamma A\frac{V}{N} - \mu A - a_1 AE - \delta A \tag{2.2}$$

$$\frac{dE}{dt} = \beta A\frac{V}{N} - \mu E - a_2 AE - wE \tag{2.3}$$

$$\frac{dN}{dt} = (\lambda - \mu)N - (a_1 + a_2)AE \tag{2.4}$$

where (2.4) is derived from adding $(2.1) - (2.3)$. We impose

$$t = 0, N = N_0$$

as initial human population. We note that by setting $\delta = 0$, that is in the absence of rehabilitation, we obtain a semblance of the system analysed in [11].

2.1. Parameter values

All the model parameters are listed in Table 2 together with values taken from various sources. We note that the values for these parameters which have some regional variation may display some global semblance. A detailed work done in [6] on ransom kidnapping and its duration reveals that kidnapping, while ranging from 1 to 325 days average just less than 2 months in duration. We assume that a kidnappee remains in the kinappers` den for 60 days. In a research work on kidnapping and national security, [10] aver that Nigeria, despite ranking $11th$ position in the 12 top kidnapping countries 690 cases were recorded in 2009 and 2,0184 cases in 4 years. We assume a kidnapping rate of 0.0612 per day. Other parameter values are also obtained in this form. Especially [13] in [3] maintain that 64 percent of kidnapped victims are released with ransom payment, 18 percent without payment, 6 percent are rescued and 2 percent escape.

Table 2: Model parameters and their dimensions. Values marked with $(*)$ are assumed values and the rest are obtained from data

Parameters	Description	Value	Unit	Source
w	Rescue rate of kidnapped victims	0.0136	day^{-1}	[6]
γ	recruitment rate of kidnappers	3.8×10^{-4} *	day^{-1}	assumed
β	kidnapping rate	0.0612	day^{-1}	[10]

μ	per capita death rate	0.0000356	day^{-1}	[12]
a_1	death rate of kidnappers due to operations by security agents	2.35×10^{-9} *	$human^{-1}day^{-1}$	[11]
λ	per capita birth rate	0.000104	day^{-1}	[12]
a_2	kidnapping induced death rate of kidnapped victims	1.18×10^{-9} *	$human^{-1}day^{-1}$	[11]
Δ	Rehabilitation rate	0 – 0.000816	day^{-1}	assumed

Thus using the results of [6], we assume an average rescue rate of 0.0136. However, we have made some assumptions on parameters that do not seem to have well defined values. Data on recruitment of kidnappers are not readily available but it could take few people with arms to kidnap a reasonable number of victims for ransom. Thus we assume that kidnappers recruitment rate constant γ is proportional to kidnapping rate constant β with a constant of proportionality, 0.02.

2.2. Nondimensionalisation

Since the variable N is the sum of the relevant compartment values, it is convenient to re-express the compartment values as population fractions using

$\hat{V} = \frac{V}{N},\ \hat{A} = \frac{A}{N},\ \hat{E} = \frac{E}{N}$

so that

$\hat{V} + \hat{A} + \hat{E} = 1.$

The time derivatives for the variables will become, using variable S as an example

$\frac{dN\hat{V}}{dt} = N\frac{d\hat{V}}{dt} + \hat{V}\frac{dN}{dt} = N\frac{d\hat{V}}{dt} + \left(\lambda - \mu - (a_1 + a_2)\hat{A}\hat{E}N\right)N\hat{V},$

Kidnappers are mainly motivated or encouraged through negotiations and ransom payment. Rescue of victims by security agents without payment of ransom may make kidnapping less attractive. The timescale in which a victim remains in the kidnappers' den before rescue is very a important determinant of the safety and survival of the victim. Hence we scale time with the rescue parameter w, and write

$$t = \frac{\hat{t}}{w}$$

Assuming that N_0 is the initial population of humans, we write

$$N = N_0\hat{N}.$$

By defining the following dimensionless parameters:

$b = \frac{\lambda}{w},\ c = \frac{\gamma}{w},\ e = \frac{\beta}{w},\ f = \frac{a_1 N_0}{w},\ g = \frac{a_2 N_0}{w},\ h = \frac{\mu}{w},\ r = \frac{\delta}{w},$

and by substituting these new parameters into $(2.1) - (2.4)$ and dropping the hats for clarity we get

$$\frac{dV}{dt} = b(1 - V) + E + rA - (c + e)VA + (f + g)VAEN, \quad (2.5)$$

$$\frac{dA}{dt} = cVA - (b + r)A - fAEN + (f + g)A^2EN, \quad (2.6)$$

$$\frac{dE}{dt} = eVA - (1 + b)E - gAEN + (f + g)AE^2N, \quad (2.7)$$

$$\frac{dN}{dt} = (b - h)N - (f + g)AEN^2 \quad (2.8)$$

The dimensionless parameters and their values are given in Table 3.

Table 3: List of dimensionless parameters and their definitions in terms of the dimensional parameter values

Dimensional form	Nondimensional parameter	Value
$\frac{\lambda}{w}$	b	0.0076
$\frac{\gamma}{w}$	c	0.028
$\frac{\beta}{w}$	e	4.5

$\frac{a_1 N_0}{w}$	f	0.024
$\frac{a_2 N_0}{w}$	g	0.0086
$\frac{\mu}{w}$	h	0.00036
$\frac{\delta}{w}$	r	0 – 0.06

3. Model Analysis

3.1. Establishing the Crime Propagation Number, C_{p_n}

The crime propagation number, denoted by C_{p_n} is the expected number of secondary kidnap cases that would arise from the introduction of a single kidnapper into a fully vulnerable or kidnap-free population [11]. The method of next generation matrix used in [4], [5] in determining the basic reproduction number of an infectious disease may be used in deriving C_{p_n}. By considering a small perturbation of the kidnap-free state ($V = 1,\ A = 0,\ E=0$, we investigate the linearised system expressed in the form

$$R' = FR - MR, \tag{3.1}$$

Where

$$R' = \frac{dR}{dt}, F = \begin{bmatrix} c & 0 \\ e & 0 \end{bmatrix}, M = \begin{bmatrix} b+r & 0 \\ 0 & 1+b \end{bmatrix}, R = \begin{bmatrix} A \\ E \end{bmatrix} \tag{3.2}$$

Here, FR represents the matrix of new kidnappers and kidnap cases, MR is the transition of these cases between compartments and $R$the "reservoir of kidnapping''.This method assumes that there is a non-negative matrix $G = FM^{-1}$ that guarantees a unique, positive and real eigenvalue strictly greater than all others. Computing the inverse of M yields

$$G = \frac{1}{(b+r)(1+b)} \begin{bmatrix} c(1+b) & 0 \\ e(1+b) & 0 \end{bmatrix} \tag{3.3}$$

The characteristic equation of (3.3) in terms of the eigenvalue, σ, gives the largest eigenvalue as$\frac{c}{b+r}$. Thus the crime propagation number is expressed as

$$C_{p_n} = \frac{c}{b+r} \tag{3.4}$$

3.2. Positivity, Existence and Uniqeness of Solution

The model is described in the domain

$$\Phi \in \mathcal{R}^4 = \{V, A, E, N : V \geq 0, A \geq 0, E \geq 0, N > 0, V + A + E = 1\} \tag{3.5}$$

Suppose at $t = 0$ all variables are non-negative, then $V(0) + A(0) + E(0) = 1$. If $V = 0$, and all other variables are in Φ, then $\frac{dV}{dt} \geq 0$. This is also the case for all other variables in $(2.6) - (2.8)$. We note from (2.7) that $A = 0$ implies $\frac{dV}{dt} = 0$ meaning $A = 0$ at all times and there will be no kidnapping in the absence of kidnappers. If $N = 0$, then $\frac{dN}{dt} = 0$. But if $N > 0$ and assuming $b > h$, then with appropriate initial conditions, $\frac{dN}{dt} > 0$ for all values of $t > 0$. We note that the right-hand side of $(2.6) - (2.9)$ is continuous with continuous partial derivatives, so solutions exist and are unique. The model is therefore mathematically and criminologically well posed with solutions in Φ for all $t \in [0, \infty)$.

3.3. Steady State Solution and Stability Analysis

It can easily be shown from the system that the kidnap free state is $(V, A, E) = (1, 0, 0)$. In the absence of a kidnapper, $A = 0$ and substituting this into the right hand side of (2.8) we obtain $E = 0$. Further substitution of the values of A and E into (2.6) gives $V = 1$. The kidnap free state is locally asymptotically stable when $C_{p_n} < 1$ and globally asymptotically stable when $C_{p_n} < 1$, $g = 0$, and unstable for $C_{p_n} > 1$, where C_{p_n} is as defined in (3.2). We note that $C_{p_n} = 1$ is a bifurcation surface in which the system changes its stability status. We derive sufficient conditions for local and global stability of the kidnap free state from all initial conditions in

Φ. The Jacobian matrix obtained by linearising system $(2.6)-(2.8)$ about the kidnap free equilibrium point, $(V, A, E) = (1, 0, 0)$ is

$$J_{d_f} = \begin{bmatrix} -b & r-(c+e) & 1 \\ 0 & c-(b+r) & 0 \\ 0 & e & -(1+b) \end{bmatrix} \tag{3.6}$$

Lemma 3.1. *The kidnap free equilibrium is locally asymptotically stable if* $C_{p_n} < 1$ *and unstable if* $C_{p_n} > 1$.

Proof

The characteristic polynomial equation of (3.6) with eigenvalues, Ƙ is

$$(b+ Ƙ)\{(b+r-c)+ Ƙ\}\{(1+b)+ Ƙ\} = 0$$

Two of the eigenvalues are strictly negative and the remaining eigenvalue is expressed by the linear equation

$$Ƙ + (b+r)\left(1-\frac{c}{b+r}\right) = 0 \tag{3.7}$$

From (3.4), C_{p_n} is given by

$$C_{p_n} = \frac{c}{b+r} \tag{3.8}$$

If $C_{p_n} < 1$, then $c < b$ and the coefficients of the linear polynomial of (3.7) are all positive and non zero; so by the Descartes' rule of signs there is no positive real eigenvalue. Thus Routh Hurwitz stability condition for a linear polynomial as stated in [1] and given in this case by $(b+r)(1-\frac{c}{b+r}) > 0$ is satisfied. We observe that if $C_{p_n} > 1$, $c > b+r$ and the constant, $(b+r)(1-\frac{c}{b+r})$ is negative. Therefore there is one sign change and by using Descartes' rule of sign there exists one positive real eigenvalue, we conclude that the kidnap free state is unstable if $C_{p_n} > 1$. When $C_{p_n} = 1$,
$c = b+r$ and (3.7) has zero eigenvalue, which shows that $C_{p_n} = 1$ is a bifurcation surface in (b, τ) parameter plane, where $\tau = b+r$.

Lemma 3.2. *The kidnap free equilibrium is globally asymptotically stable in* Φ *if*

$$C_{p_n} < 1, \quad g = 0 \tag{3.9}$$

Proof

Consider the function $\Psi : \{(V, A, E) \in \Phi : V > 0\} \to \mathcal{R}$, where

$$\Psi = A. \tag{4.0}$$

We note that $\Psi \geq 0$ and is continuously differentiable on the interior of Ψ. We shall show that the kidnap free equilibrium is a global minimum of Ψ on Φ if (3.9) holds. The derivative of Ψ computed along solutions of the system is

$$-c(A+E)A-(b+r)(1- C_{p_n})A-(f+g)(V+E)AEN+gAEN \tag{4.1}$$

We can see clearly that $\frac{d\Psi}{dt} < 0$ whenever $C_{p_n} < 1$ and $g = 0$. In fact, for $(A, E) = (0,0)$, $\frac{d\Psi}{dt} \leq 0$ and (A, E) is the largest positively invariance subset in the interior of Φ and by LaSalle's invariant principle [7], $(A, E) \to (0,0)$ as $t \to \infty$ while $V \to 1$ on the boundary of Φ. Thus, the kidnap free state is globally stable if (3.9) are true, noting $c < b+r$.

4. Numerical Simulations

The numerical solution is obtained by using MATLAB's ode45, a variable order Runge-Kutta method with a relative and absolute tolerance of 10^{-9}. The parameters used for the simulations as defined in Table 3 are $b = 0.0076, c = 0.028, e = 4.5, f = 0.004, g = 0.0086, h = 0.00036, r = 0$. At time $t = 0$ we have the following initial conditions in the proportions: $V = 0.99$, $A = 0.01, E = 0, N = 1$. This is a situation where the entire vulnerable human population is exposed to a small fraction of kidnappers. The program was run in MATLAB andMAPLE's ode15s with different sets of initial conditions.

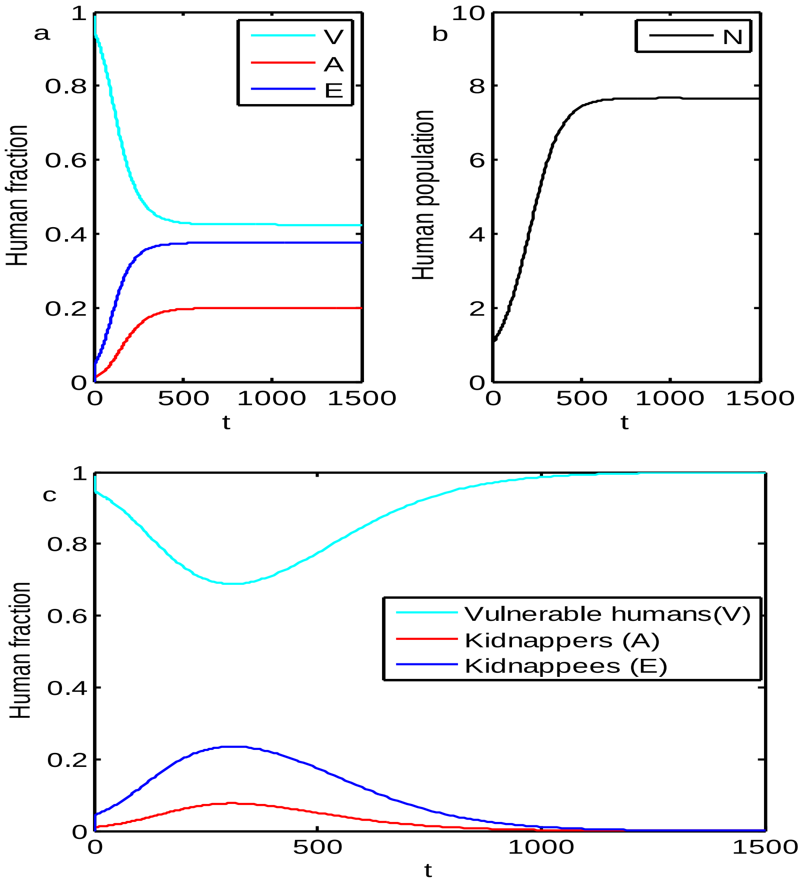
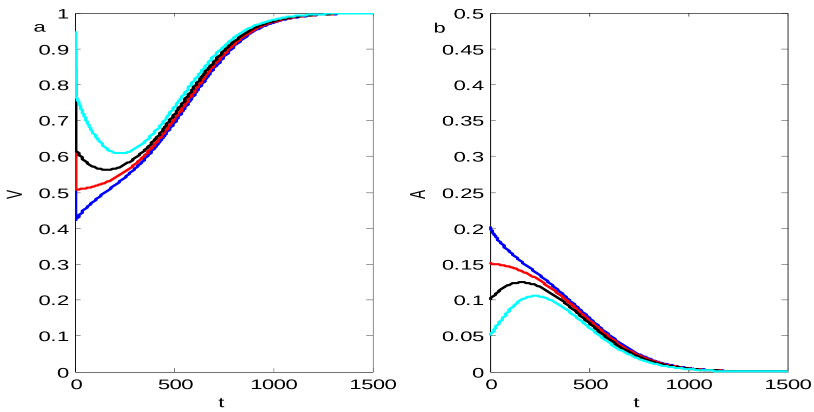


Figure 2: Results showing the effect of small amount of kidnappers on evolution of kidnapping in a kidnap free society, where $t = 1$*, represents approximately* 10 *days in real time. The initial conditions used are* $S = 0.99, A = 0.01, V = 0, N = 1$ *and the parameter values are given in Table* 3 *except that we have used* $r = 0$ *in* $2a, b$ *and* $r = 0.006$ *in* $2c$

The simulations in all cases show that the qualitative form of the steady state solutions were the same, although the system gets to a steady state faster as the initial fraction of kidnappers increases. In Figure $2a$, where there is no rehabilitation of kidnappers the proportion of vulnerable human population drops due to invasion of the population by kidnappers.Some vulnerable individuals are kidnapped and others are being wooed into kidnapping resulting in the fractions of kidnappees and kidnappers growing to settle at their respective steady state levels. The entire population is gradually increasing in Figure 2b.



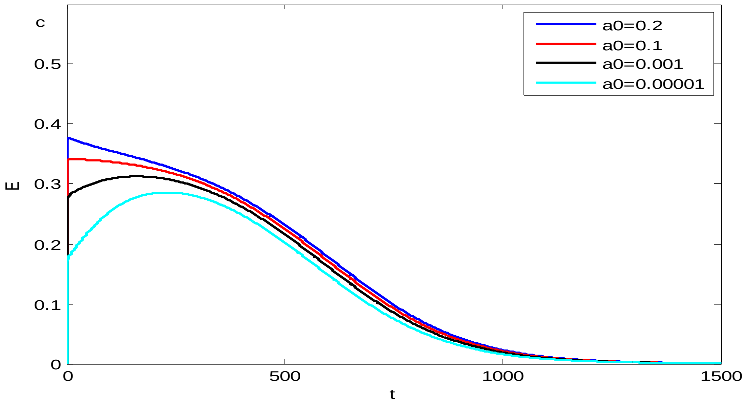


Figure 3: Results showing the effect of introducing different amount of kidnappers on the different compartments (Figure 3a, b, c). The parameter values used for the simulations are the same as those in Figure 2 except that the initial conditions, we have used $S = 1 - a_0$, $A = a_0$, $V = 0$, $N = 1$ with different values of a_0, as shown in the graphs.

In Figure $2c$, where rehabilitation is applied, the proportion of vulnerable humans drops and later grows to a steady state whereas the fractions of kinappers and kidnappees pick as a result of the initial effect of kidnappers but latter drop to their respective steady states due to effective rehabilitation strategies. Figure $3a, b, c$ show the effect of different values of a_0 on the various fractions of human population. We investigate each of the human sub-populations alongside the entire population as a_0 varies from 0.00001 to 0.2 and the results show that there is a unique steady state for each human compartment irrespective of the value of a_0 whereas the human population is gradually increasing.

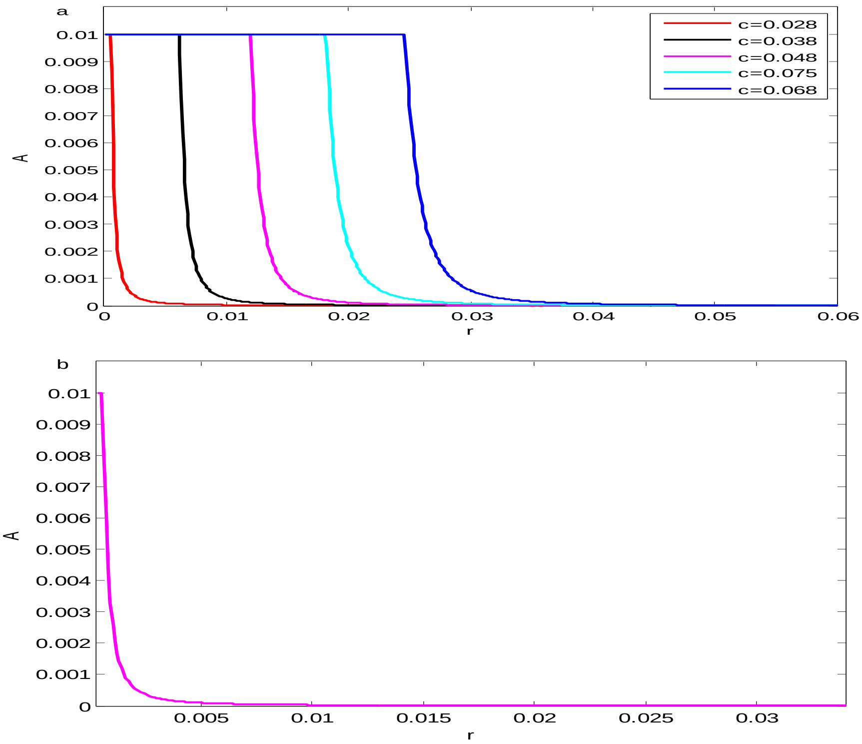


Figure 4: A plot of kidnappers' recruitment and rehabilitation effect on kidnapping control. In Figure 4b, r varies from 0 to 0.035, whilst each curve in Figure 4a represents a plot of kidnappers with a given level of recruitment rate, c against different values of the rehabilitation parameter, r. Initial conditions and other parameter values are the same as those in Figure 2

Figure 4 describes the relationship between recruitment rate and and rehabilitation of kidnappers as they impact on kidnapping control. Figure *4a*shows that with the given level of recruitment rate as obtained from data, there is the likelihood of obtaining a kidnapping free state through rehabilitation of kidnappers since a variation of r from 0 to 0.035 reduces A to 0. In Figure *4a* different values of the rehabilitation parameter (r) are plotted against different values of kidnappers recruitment rate (c) in which high and low values of r are associated high and low values of$c$ respectively.The impact of any rehabilitation programme on the kidnapping profile may be delayed depending on the recruitment rate. For instance when$c = 0.038$, r can only start having effect on A at $r = 0.0062$leading to $A = 0$ at $r = 0.0062$, whereas when $c = 0.068$, r can only start impacting on A at $r = 0.0247$ and finally reduces it to 0 at $r = 0.0741$.

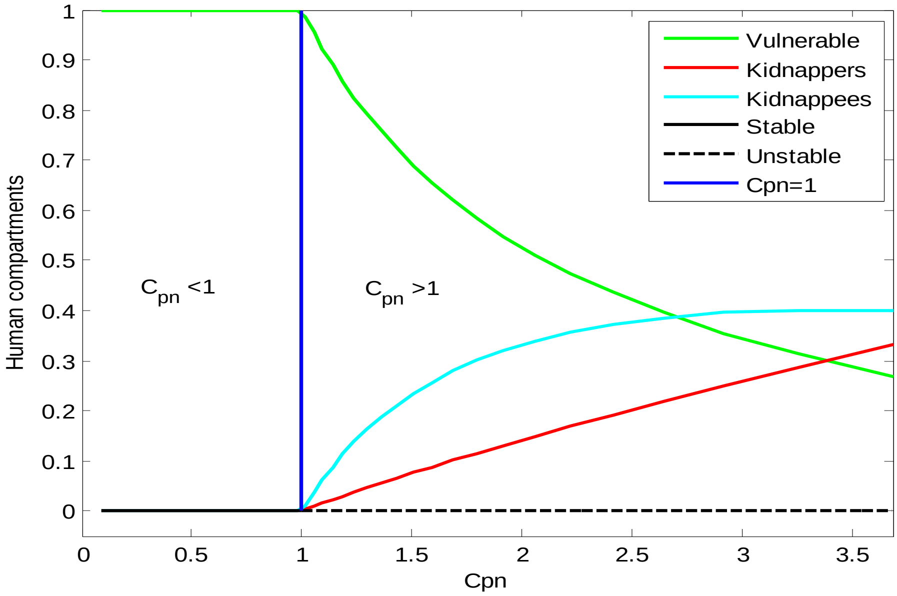


Figure 5: *Results showing the kidnap free state when* $C_{p_n} < 1$ *and the kidnap persistent state when*$C_{p_n} > 1$*for values of* $C_{p_n} \in [0.091, 3.68]$. *The parameter values used to obtain these results are given in Table* 3, *except that*$g = 0$. *We used the parameter,* r *to change* C_{p_n} *where,*$C_{p_n} = 3.68$ *corresponds to* $r = 0.0204$.

Figure 5 describes the relationship between the crime propagation number and the kidnapping crime profile as it affects the population. In a previous work [11], we considered simulations with $g > 0$,where the kidnapping free state is not globally asymptotically stable and $g = 0$ is stable. This is a similar case but we only consider the case of $g = 0$ since we are interested in no casualty of kidnapped victims during rescue operations.The values of C_{p_n} were obtained by varying r and $C_{p_n} = 1$ corresponds to $r = 0.0204$. We note that$r = 0$ gives $C_{p_n} = 3.68$ from data and since$\lim_{r\to\infty} C_{p_n} = 0$, then,$C_{p_n} \in (0, 3.68]$, hence we investigate the kidnapping profile for some values of C_{p_n} in the domain $[0.091, 3.68]$. The values are not very important but the basic idea is to show that Kidnapping establishes itself for values of $C_{p_n} > 1$and dies out if $C_{p_n} < 1$. ThusFigure 5 also represents a bifurcation diagram showing a switch from a kidnap free state to a kidnap persistent state. The result is obtained by drawing the steady states of vulnerable humans, kidnap victims and kidnappees against different values of the crime propagation number, C_{p_n}. The kidnapping free state prevails when C_{p_n} is less than unity and for $C_{p_n} > 1$ kidnappers invade the population. This is a situation of transcritical bifurcation in the vicinity of $C_{p_n} = 1$, as is expected from the analysis. It is not quite certain whether or not kinappers invade at $C_{p_n} = 1$, the kidnapping free state is stable for values of $C_{p_n} < 1$, but becomes unstable when $C_{p_n} > 1$ whereas, the kidnap persistent state becomes stable as expected.

5. Discussion

Our model describes a typical situation of kidnapping as a new area of what seems to look like mathematical criminology with special reference to kidnapping by exploring the techniques of deterministic infectious disease models. Kidnapping has been on the increase when some lazy and criminally minded individuals operating of the influence of drugs and gang practices prey of hard working people by abducting them or their family members for ransom or other selfish reasons. Just as infection occurs during a contact between infectious and a susceptible individual so as a crime (kidnapping) is committed when a kidnapper interacts with vulnerable individuals in society [11]. Analysis of the crime propagation number, C_{p_n} given as *3.68*, obtained from data using (*3.7*) the numerical simulations without rehabilitation clearly show a prevalent kidnapping situation.There is a high proportion of kidnappees as shown in Figure 2,where about *39%* of the entire population will become kidnapped victims within a period of sixty years whereas *20%* of the population would be attracted to the act of kidnapping in the absence of rehabilitation. This does not only constitute a threat to global security and productivity but creates situations where victims pass through unimaginable ordeals in the form of physical, mental and emotional torture culminating in lack of trust and interpersonal relationship. This will affect the indices of national development in particular and global productivity in general as foreign investors would be dissuaded from investing in such regions irrespective of low production costs.

The crime propagation number, C_{p_n} plays a crucial role in bringing a crime under control in a population of varying size. This could be done by reducing the crime reservoirbelow a threshold value with increasing time. The results of [11] suggest viable anti-kidnapping strategies targeting recruitment of kidnappers and enhancing rescue operations by security agents could potentially be effective since this would drastically reduce the crime propagation number. Our results also lend support for these strategies while making a strong case in favour of rehabilitation of kidnappers as demonstrated in our numerical results of Figure 4. Since global stability of the kidnapping free-state hinges on the 'no casualty' syndrome during rescue operations, rehabilitation of kidnappers becomes more preferable. We note that discouraging ransom payment in order to reduce recruitment of kidnappers and promoting rescue operations without kidnappees casualty may be quite difficult to achieve in that these measures can only be carried out when kidnapping has already occurred, which may put kidnapped victims' lives at stake. Increasing rescue operations alone cannot lead to the eradication of kidnapping but could only help in the management and control of the crime [11]. Since rehabilitation seeks to avoid kidnapping, then an eclectic approach to a combination of different strategies with emphasis on rehabilitation of kidnappers could be an effective measure to eradicating kidnapping.

6. Conclusion

In this model we have made a bold attempt to use a mathematical model to proffer solution to Problem of ransom kidnapping. The key information we obtained from the analysis and simulations of the model is that there is the possibility of eradicating kidnapping by discouraging the recruitment of kidnappers through rehabilitation of kidnapped victims. Simulations are carried out on the combination of different levels of kidnappers recruitment and rehabilitation. The analysis shows that increasing the rehabilitation rate of kidnappers is a better and more effective way in eradicating kidnapping in society. However, in a situation of high kidnappers' recruitment rate more rehabilitation effort is needed to bring the crime under control than in a region of low recruitment rate. Most of the kidnapping cases are perpetrated by armed robbers under the influence of hard drugs. Therefore illegal hard drug dealers should be treated as great enemies to society. It would be difficult to persuade hard drug addict from committing crime. Our model is an extension or modification of [11] in which our results also suggest rescue operation strategies devoid of casualty on the part of kidnappees otherwise families of kidnappees would not have confidence in the rescue strategies of security agents and rather pay ransom to release their relations. If $g > 0$, $C_{p_n} < 1$ becomes globally asymptotically unstable, a situation that would likely encourage recruitment of kidnappers. Hence the need for rehabilitation and de-radicalisation of kidnappers.

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