



Dynamic Response and Pd Control of a Single Joint System using Pole Placement Technique

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Abstract This paper focuses on the modelling and simulation of the dynamic response of a single joint system, consisting of a DC motor as the actuator and a coupled body that forms a single joint system with a DC motor acting as the actuator to cause rotation of the arm. A Proportional Derivative controller is designed and implemented on the system using pole- placement technique. The results are analysed both in the time and frequency domain. In the time domain, a set-point of 1rad was tracked at the output with a time constant of 0.18s and peak overshoot of 7%, while an infinite gain margin was realised and a phase margin of 101.9850° were both gotten from Bode and Nyquist plots respectively, The performance of the system was greatly improved by the implementation of the Proportional Derivative controller when compared to the open loop system whose step response was unstable with a gain margin of 0.1700 and a phase margin of 65.9822° respectively.

Keywords Pole placement, PD Controller, DC Motor, Single Joint System, Robot arm

1. Introduction

Single joint systems are mostly found in Robots whose joints are actuated by DC motors that have good torque and wide speed control and therefore their dynamic response can be studied [1]. In order to study this response and control, a mathematical model is required [2]. The parameters of the model need to be provided since the mathematical model cannot provide the actual results without the required parameters [3]. A PID controller can be applied using genetic algorithms, which involves the mechanics of the motor used in the joint taking into consideration kinematics and dynamics of the arms [3]. This will be the paradigm involved before testing the dynamic response and subsequently the controller implementation in both the time and the frequency domain.[4] Also worked on DC motors as actuators but emphasis was placed on the control of the current and the torque of the DC Motor while [5] designed a PD and PID controller only for speed control while [6,7] worked also on speed control of a DC motor using fuzzy logic and sliding mode control respectively. In this design we are not just modelling the DC motor as the actuator but also modelling of the arm coupled to a single joint by a DC motor is considered and the dynamics is used to design a PD controller whose performance will be analysed in both the time and frequency domain.

2. Modelling and Steady State Response

The single joint comprises of an arm which is coupled to a horizontal object by a DC motor acting as actuator at the Joint. The arm is free to rotate at an angle (θ) to the horizontal plane. Fig (1) shows the diagram of the single joint system.



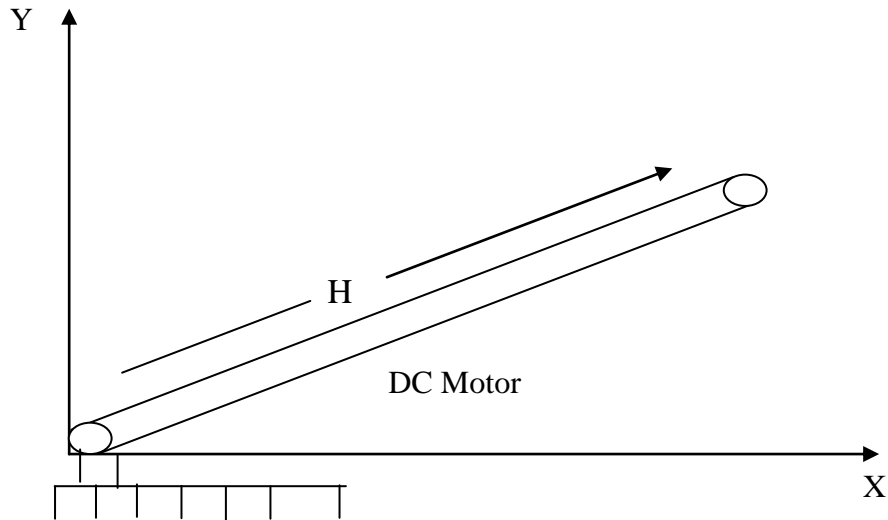


Figure 1: Single Joint System Coupled to the X-Y Axis

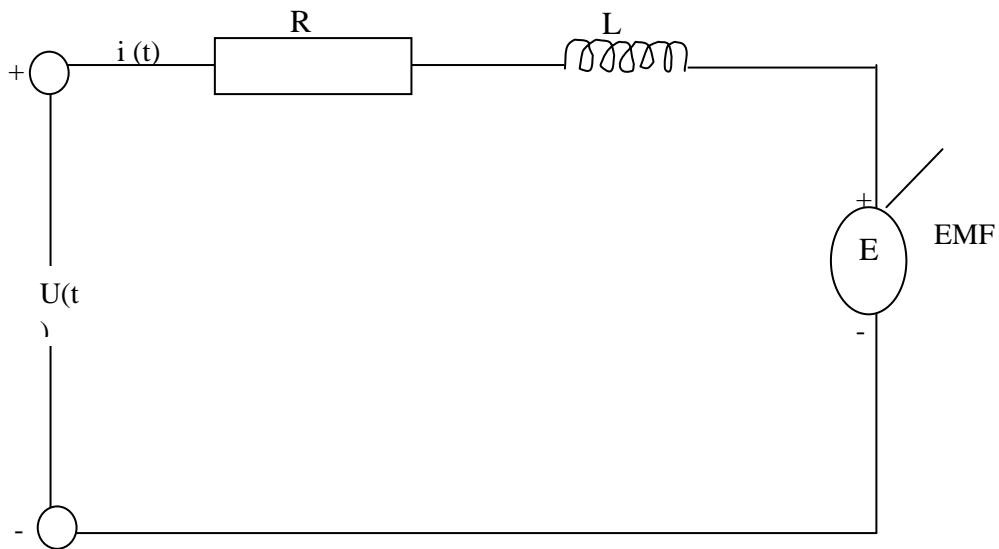


Figure 2: Electrical Part of the Single Joint System

Electrical Part Modelling

The electrical part can equivalently be shown in the circuit of fig (2). In this circuit, we denote $I(t)$ as the armature current, and using Kirchoff's voltage law, we have

$$Ri(t) + L \frac{di}{dt} + E = U(t) \tag{1}$$

- Where R = Motor resistance
- L = Inductance
- $U(t)$ = The input voltage

Assuming, the strength of the magnetic field is fixed in this case, the back EMF is proportional to the speed of the motor with a fixed field strength, as:

$$E = K_1 \frac{d\theta}{dt} \quad (2)$$

Where θ = The motor shaft angle position (Angle of Rotation of the shaft)

K_1 = The Electromotive force constant

$$\frac{d\theta}{dt} = \text{Motor speed}$$

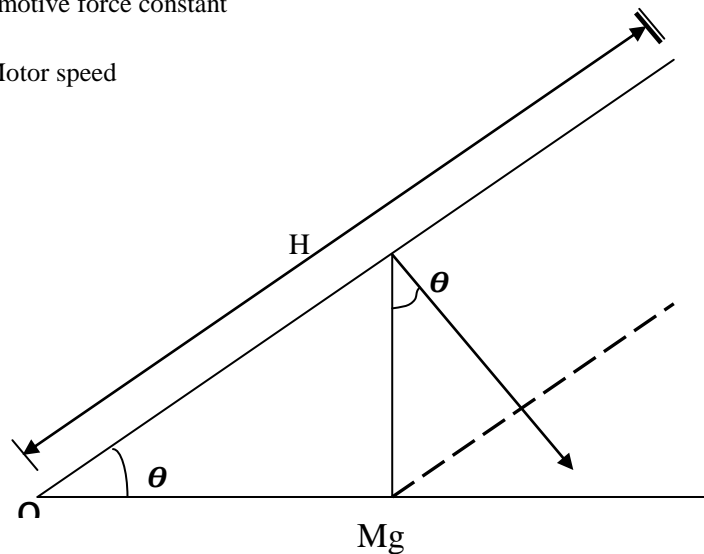


Figure 3: Mechanical Part of Single Joint System

Mechanical Part Modelling

From Newton's second law of motion, we have

$$J \frac{d^2\theta}{dt^2} = T_d - T_L^{Total} \quad (3)$$

Where:

J = Inertia Constant

T_d = Driving Torque

T_L^{Total} = Total Load Torque

T_L = Load torque applied to end effectors

When the strength of the magnetic field is fixed, then

$$T_d = K_2 i(t) \quad (4)$$

Where K_2 = constant

$$T_L^{Total} = T_L + T_M \quad (5)$$

$$T_M = \frac{H}{2} Mg \cos\theta \quad (6)$$

T_L = load Torque applied to the end effectors

T_M = load Torque caused by the weight of the arm.

T_L^{Total} = Total load torque

H = Length of the arm

Substituting equations (4), (5) and (6) into (3)

$$J \frac{d^2\theta}{dt^2} = K_2 i(t) - \frac{H}{2} Mg \cos\theta - T_L \quad (7)$$

Thus from (7), we have



$$i(t) = \frac{1}{K_2} \left(J \frac{d^2\theta}{dt^2} + \frac{H}{2} Mg \cos\theta + T_L \right) \quad (8)$$

Substitute (8) into (1)

$$\frac{RJ}{K_2} \frac{d^2\theta}{dt^2} + \frac{RH}{2K_2} Mg \cos\theta + \frac{RT_L}{K_2} + \frac{LJ}{K_2} \frac{d^3\theta}{dt^3} - \frac{LH}{2K_2} Mg \sin\theta \frac{d\theta}{dt} + \frac{L}{K_2} \frac{dT_L}{dt} + K_1 \frac{d\theta}{dt} = U(t) \quad (9)$$

Equation (9) represents the open loop system transfer function in the time domain linearization of the open-loop systems transfer function dynamics is denoted by;

$$\Delta U = U(t) - U_o$$

$$\Delta\theta = \theta(t) - \theta_o$$

$$\Delta i = i(t) - i_o$$

Where; ΔU = Small change in the input voltage, $\Delta\theta$ = Small change in the angle of rotation of the shaft Δi = Small change in the current.

$$\text{Therefore } \lim_{\Delta t \rightarrow 0} \frac{\Delta i}{\Delta t} = \frac{di}{dt} \Leftrightarrow \frac{d\Delta i}{dt} = \frac{di}{dt}$$

$$\text{Hence } \frac{d^2\Delta\theta}{dt^2} = \frac{d^2\theta}{dt^2}$$

Where U_o , θ_o and i_o are all constants These three constants represent the initial state of the, input voltage, angle of rotation and the current respectively.

From equation (1)

$$R\Delta i + L \frac{d\Delta i}{dt} + K_1 \frac{d\Delta\theta}{dt} = \Delta U \quad (10)$$

and from equation(2)

$$J \frac{d^2\Delta\theta}{dt^2} = K_2 \Delta i(t) - \frac{H}{2} Mg \Delta \cos\theta \quad (11)$$

Where the load torque $T_L=0$

Thus applying the Taylor series expansion linearization and assuming we start from an initial state of zero

Then equation (11) becomes

$$J \frac{d^2\Delta\theta}{dt^2} = K_2 \Delta i(t) + \frac{H}{2} Mg \sin\theta \Delta\theta \quad (12)$$

The desired transfer function is given as the ratio of the Laplace transform of the output to the Laplace transform of the input variable.

$$G(s) = \frac{\Delta\theta(s)}{\Delta U(s)}$$

Taking the Laplace transform of (10)

$$R\Delta I(s) + sL\Delta I(s) + K_1 s\Delta\theta(s) = \Delta U(s)$$

$$\Delta I(s)(R + Ls) = \Delta U(s) - K_1 s\Delta\theta(s)$$

$$\Delta I(s) = \frac{1}{Ls + R} (\Delta U(s) - K_1 s\Delta\theta(s)) \quad (13)$$

Taking the Laplace transform of equation (12), when load torque ($T_L=0$) and using the linearised model below We have



$$Js^2\Delta\theta(s) = K_2\Delta I(s) + \frac{H}{2}Mg\sin\theta_o\Delta\theta \quad (14)$$

Substituting equation (13) into equation (14)

$$Js^2\Delta\theta(s) = \frac{K_2}{Ls + R} \left(\Delta U(s) - K_1s\Delta\theta(s) + \frac{H}{2}Mg\sin\theta_o\Delta\theta \right) \quad (15)$$

Multiply left and right of equation (15) by $Ls + R$ and rearranging

$$\Delta\theta(s) \left(LJs^3 + RJs^2 + sK_1K_2 - \frac{sLH}{2}Mg\sin\theta_o - \frac{RH}{2}Mg\sin\theta_o \right) = K_2\Delta U(s)$$

$$G(s) = \frac{\Delta\theta(s)}{\Delta U(s)} = \frac{K_2}{LJs^3 + RJs^2 + \left(K_1K_2 - \frac{L}{2}HMg\sin\theta_o \right)s - \frac{R}{2}HMg\sin\theta_o} \quad (16)$$

$G(s)$ represents the open loop transfer function of the single Joint system.

From the given transfer function, we realized there is a negative coefficient in the denominator of equation (16).

Hence the system can be described as open loop unstable.

The following parameter and values were chosen as illustrated in the table below.

Table 1: Parameters and their values chosen for the design

Parameter	Value
Resistance of the resistor	0.1Ω
Inductance of the inductor	1.25mH
Motor Torque	0.1kg-m ²
K_2	0.5
K_1	0.4
Weight(Mg)	2N

When we substitute all these parameters into equation (16), we can write the new equation as;

$$G(s) = \frac{4000}{s^3 + 80s^2 + 1592s - 680} \quad (17)$$

In general, the transfer function of the closed-loop system can be written as;

$$TF = \frac{GC}{1 + GC} \quad (18)$$

Where G = Transfer function of the plant (system)

C = The controller function

Designing of the proportional derivative controller (PD)

The general formula for a proportional Derivative controller is given as

$$C = K_p + K_{ds} \quad (19)$$

Where: K_p = Proportional Controller

K_{ds} = Derivative Controller

The closed-loop dynamics can be calculated as follows:

$$G(s) = \frac{\Delta\theta(s)}{\Delta U(s)} = \frac{4000(K_p + K_{ds})}{s^3 + 80s^2 + (4000K_d + 1592)s + 4000K_p - 680} \quad (20)$$

Therefore the characteristic polynomial which determines the stability of the closed-loop system is

$$P_x = s^3 + 80s^2 + (4000K_d + 1592)s + 4000K_p - 680 \quad (21)$$



It is desired for this design to ensure that the poles of the characteristic polynomial P_x are all located in the left half side of the S-plane for stability to be acquired. Thus the following poles were chosen for the pole placement which produces coefficient comparable to the characteristics equation. This is done by considering poles, whose product after multiplication gives the coefficient of both S^3 and S^2 , thus making it possible for the characteristic polynomial and the desired polynomial to be compared.

$$P_Y = (S + 50)(S + 20)(S + 10) \tag{22}$$

$$P_Y = s^3 + 80s^2 + 1700s + 10,000 \tag{23}$$

Comparing (21) and (23)

$$4000K_{ds} + 1592 = 1700 \tag{24}$$

$$4000K_p - 680 = 10,000 \tag{25}$$

Hence $K_p = 2.67$ and $K_{ds} = 0.027$

Simulations

The simulation of the systems and control implementation are all carried out in Matlab and shown below:

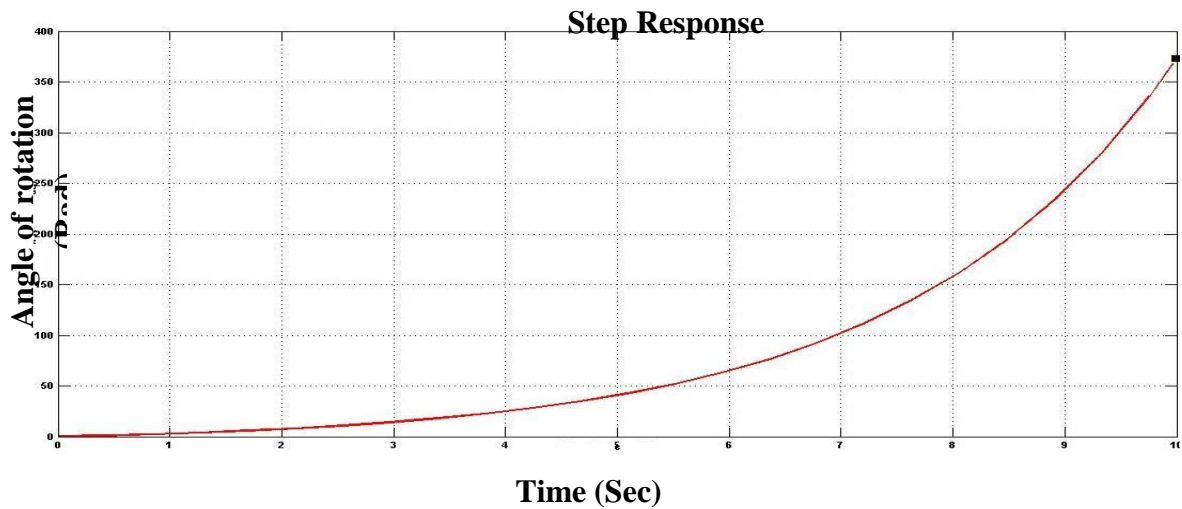


Figure 4: Step Response for Open Loop Unstable System

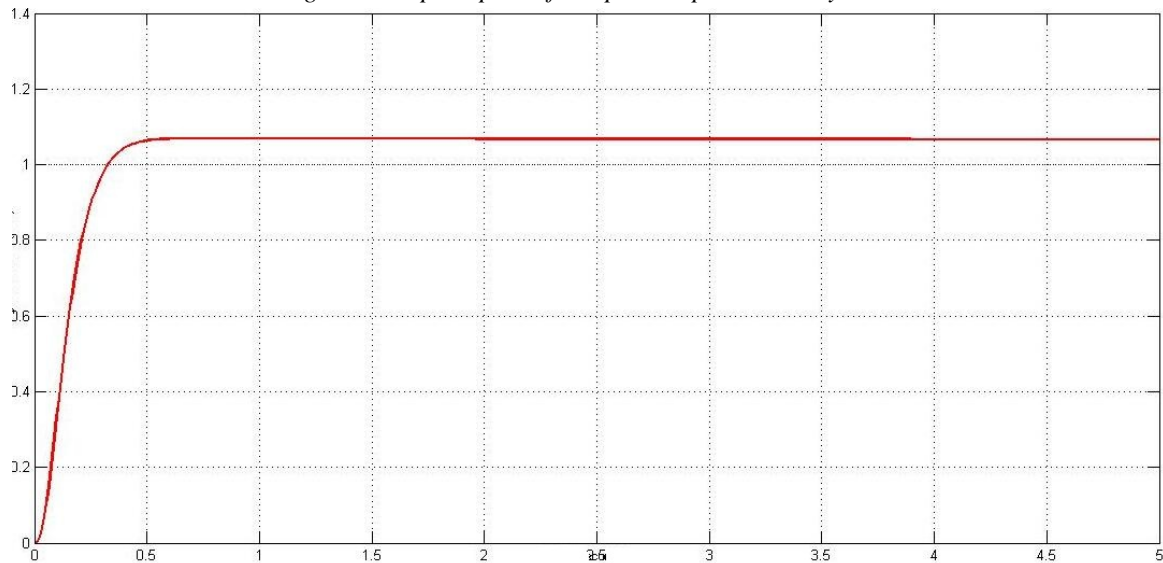


Figure 5: Step Response Using PD Controller

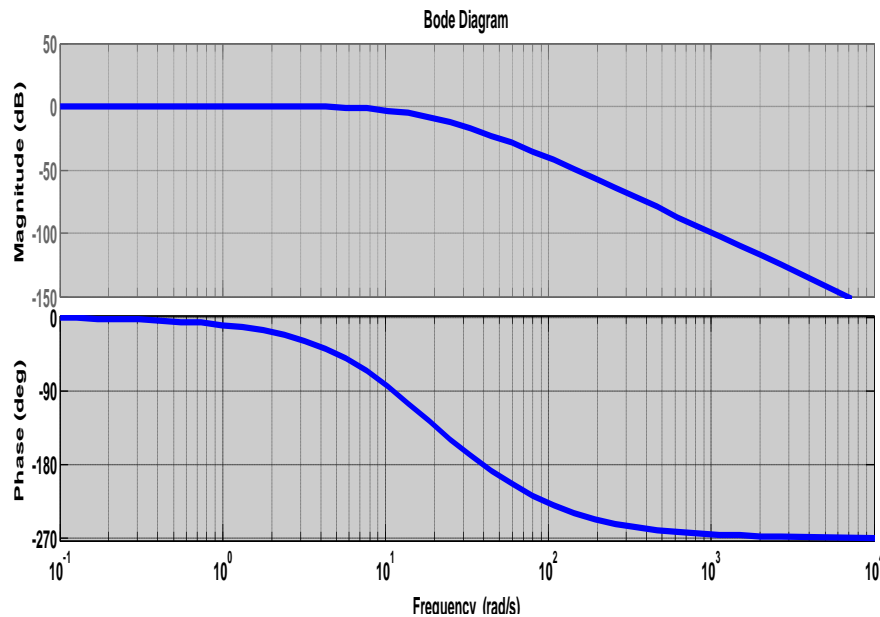


Figure 5: Bode Plot Using PD Controller

3. Results and Discussion

The graph in Fig. 3 indicate an open loop unstable system as shown by its open loop step response, while Fig. 4 shows the closed loop step response of the angle of rotation of the arm when a proportional derivative controller is applied. It is observed that such a control method has been able to force the output from an unbounded range to a bounded range which becomes stable at an angle of rotation of about 1.07 rad although, there is still a steady state offset of about 0.07 rad, which is not desirable, but then the derivative action has been able to eliminate any oscillations occurring at steady state. In the frequency domain the gain and the phase margins are infinite and 101.9850° respectively making the control method a reliable method for control and oscillation cancellations as illustrated by the bode plot.

4. Conclusion

It has been observed that the open loop system of the single joint system was unstable as shown by its step response in Fig. 3, but after implementation of the PD controller, stability was achieved both in the time domain and frequency domain although we still observe a little steady state error, the controller has been able to perform its function of stabilizing the single joint system.

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