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Research Article

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The Generalization from Four 4's to Four n's

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Abstract The four 4's problem is an interesting mathematics puzzle which attracts wide attention from famous mathematicians, college professors, to high school teachers. In this paper, we generalize the four 4's problem to the general four n's, which covers all the existing results in the literature. This research may have applications in data management field.

Keywords Four 4's, mathematics puzzle, four n's, data transmission

Introduction

The "Four 4's" is a famous mathematics puzzle and has been discussed widely in mathematics classes. The goal of four 4's is to find the simplest mathematical expression for every whole number from 1 to some big number, using only common mathematical symbols and the digit 4 (no other digit is allowed). Most versions of the four 4's require that each expression have exactly four 4's.

The primary difference for different variations of four 4's is which mathematical symbols are allowed to be used in their formulas. Essentially all variations allow, at least, addition, subtraction, multiplication, division, and parentheses, as well as concatenation (e.g., "44" is allowed as forty-four). Most also allow the factorial ("!"), exponentiation (e.g. "44^4"), the decimal point (".") and the square root ("$\sqrt{\ }$") operation. Other operations allowed by some variations include overline (an infinitely repeated digit), and the greatest integer ([ ]). Thus, $.\overline{4} = .4444... = \frac{4}{9}$, and $[4.444] = 4$.

Many results for four 4's have been reported in the literature (see [1-4, 6-10). Some similar problems for four 5's, and four 6's are also developed, see [5], for example. But so far, there is no result for four n's puzzle where $n = 1, 2, 3, ..., 9$. In this article, we generalize the four 4's problem to the four n's and provide a set of formulas for four n's, when *n* is between 1 and 9.

We first introduce one famous examples of four 4's discussed by Martin Gardner [2],

$$1 = \frac{44}{44}, \qquad 2 = \frac{4}{4} + \frac{4}{4}$$

$$3 = \frac{4+4+4}{4}, \qquad 4 = 4 \times (4-4) + 4$$

$$5 = \frac{(4 \times 4) + 4}{4}, \qquad 6 = 4 + \frac{4+4}{4}$$

$$7 = \frac{44}{4} - 4, \qquad 8 = 4 + 4 + 4 - 4$$

$$9 = 4 + 4 + \frac{4}{4}, \qquad 10 = \frac{44-4}{4}$$

$$11 = \frac{44}{\sqrt{4} + \sqrt{4}}, \qquad 12 = \frac{44+4}{4},$$

and, some results about four 5's and four 7's (see, for example, [6]):

$$1=\frac{5}{5}+5-5, \qquad 2=\frac{5}{5}+\frac{5}{5}$$

$$3=\frac{5+5+5}{5}, \qquad 4=\frac{5\times 5-5}{5}$$

$$5=5+5\times(5-5), \qquad 6=\frac{5\times 5+5}{5}$$

$$7=5+\frac{5+5}{5}, \qquad 8=\frac{5!}{5+5+5}$$

$$9=5+5-\frac{5}{5}, \qquad 10=\frac{55-5}{5}$$

$$11=5+5+\frac{5}{5}, \qquad 12=\frac{55+5}{5},$$

and

$$1=\frac{77}{77}, \qquad 2=\frac{7+7}{\sqrt{7}\sqrt{7}}$$

$$3=\frac{7+7+7}{7}, \qquad 4=\frac{77}{7}-7$$

$$5=7-\frac{7+7}{7}, \qquad 6=\frac{7\times 7-7}{7}$$

$$7=7+(7-7)\times 7, \qquad 8=\frac{7\times 7+7}{7}$$

$$9=7+\frac{7+7}{7}, \qquad 10=\frac{77-7}{7}$$

$$11=\frac{77}{\sqrt{7}\sqrt{7}}, \qquad 12=\frac{77+7}{7}.$$

We now generalize the results to the four n's, for $n=1,\ 2,\ \ldots,\ 9$,

$$1=\frac{nn}{nn}; \qquad \frac{n}{n}+n-n$$

$$2=\frac{n}{n}+\frac{n}{n}$$

$$3=\frac{n+n+n}{n}; \qquad \sqrt{\frac{n}{.\bar{n}}}+n-n$$

$$4=\sqrt{\frac{n}{.\bar{n}}}+\frac{n}{n}; \qquad \frac{n-.\bar{n}}{.\bar{n}+.\bar{n}}$$

$$5=\frac{\sqrt{n}\sqrt{n}}{.n+.n}$$

$$6=\frac{n}{.\bar{n}}-\sqrt{\frac{n}{.\bar{n}}}; \qquad \left(\sqrt{\frac{n}{.\bar{n}}}\right)!+n-n$$

$$7 = \frac{n}{.n} - \sqrt{\frac{n}{.\bar{n}}}; \quad \left(\sqrt{\frac{n}{.\bar{n}}}\right)! + \frac{n}{n}$$

$$8 = \frac{n}{.\bar{n}} - \frac{n}{n}; \quad \frac{\sqrt{n}\sqrt{n} - .\bar{n}}{.\bar{n}}$$

$$9 = \frac{n}{.\bar{n}} + n - n; \quad \frac{n}{.n} - \frac{n}{n}$$

$$10 = \frac{n}{.n} + n - n; \quad \frac{nn - n}{n}$$

$$11 = \frac{nn}{\sqrt{n}\sqrt{n}}; \quad \frac{n}{.n} + \frac{n}{n}; \quad \left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right] + n - n$$

$$12 = \frac{nn + n}{n}; \quad \frac{n}{.\bar{n}} + \sqrt{\frac{n}{.\bar{n}}}; \quad \frac{n + .n + .n}{.n}; \quad \left(\sqrt{\frac{n}{.\bar{n}}}\right)! + \left(\sqrt{\frac{n}{.\bar{n}}}\right)!$$

$$13 = \frac{n}{.n} + \sqrt{\frac{n}{.\bar{n}}}$$

$$14 = \left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right] + \sqrt{\frac{n}{.\bar{n}}}$$

$$15 = \frac{n}{.\bar{n}} + \left(\sqrt{\frac{n}{.\bar{n}}}\right)!$$

$$16 = \frac{n}{.n} + \left(\sqrt{\frac{n}{.\bar{n}}}\right)!$$

$$17 = \frac{n + n - .\bar{n}}{.\bar{n}}$$

$$18 = \frac{n}{.\bar{n}} + \frac{n}{.\bar{n}}; \quad \sqrt{\frac{n}{.\bar{n}}} \times \left(\sqrt{\frac{n}{.\bar{n}}}\right)!$$

$$19 = \frac{n}{.n} + \frac{n}{.\bar{n}}; \quad \frac{n + n - .n}{.n}$$

$$20 = \frac{n}{.n} + \frac{n}{.n}$$

$$21 = \frac{n + n + .n}{.n}$$

$$22 = \left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right] + \left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right]$$

$$23 = \left[\sqrt{\left(\sqrt{\frac{n}{.\bar{n}}}\right)!}\right]\sqrt{\frac{n}{.\bar{n}}}$$

$$24 = \left[\sqrt{\left(\sqrt{\frac{n}{.\bar{n}}}\right)!}\right]\left[\sqrt{\sqrt{\sqrt{\left(\frac{n}{.\bar{n}}\right)!}}}\right]; \quad \sqrt{\frac{n}{.\bar{n}}} \times \left[\sqrt{\sqrt{\sqrt{\left(\left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right]\right)!}}}\right].$$

We can even find more formulas,

$$25=\left[\sqrt{\left(\sqrt{\frac{n}{.\overline{n}}}\right)!}\right]\left[\sqrt{\sqrt{\left(\left(\sqrt{\frac{n}{.\overline{n}}}\right)!\right)!}}\right]$$

$$26=\left[\sqrt{\left(\sqrt{\frac{n}{.\overline{n}}}\right)!}\right]\left(\sqrt{\frac{n}{.\overline{n}}}\right)!$$

$$27=\frac{n+n+n}{.\overline{n}};\qquad \frac{n}{.\overline{n}}\times\sqrt{\frac{n}{.\overline{n}}}$$

$$28=\left[\sqrt{\left(\sqrt{\frac{n}{.\overline{n}}}\right)!}\right]\left[\sqrt{\sqrt{\sqrt{\left(\left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right]\right)!}}}\right]$$

$$29=\left[\sqrt{\left(\sqrt{\frac{n}{.\overline{n}}}\right)!}\right]\frac{n}{.\overline{n}}$$

$$30=\frac{n+n+n}{.n};\qquad \left(\sqrt{\frac{n}{.\overline{n}}}\right)\times\frac{n}{.n}$$

$$31=\sqrt{\frac{n}{.\overline{n}}}\,\frac{n}{n}$$

$$32=\sqrt{\frac{n}{.\overline{n}}}\left[\sqrt{\left(\frac{n}{.\overline{n}}\right)!}\right]$$

$$33=\left(\left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right]\right)\times\sqrt{\frac{n}{.\overline{n}}}$$

$$34=\sqrt{\frac{n}{.\overline{n}}}\left[\sqrt{\sqrt{\sqrt{\left(\frac{n}{.\overline{n}}\right)!}}}\right]$$

$$35=\sqrt{\frac{n}{.\overline{n}}}\left[\sqrt{\sqrt{\left(\left(\sqrt{\frac{n}{.\overline{n}}}\right)!\right)!}}\right]$$

$$36=\sqrt{\frac{n}{.\overline{n}}}\left(\sqrt{\frac{n}{.\overline{n}}}\right)!$$

$$37=\sqrt{\frac{n}{.\overline{n}}}\left(-\left[-\sqrt{\sqrt{\sqrt{\left(\frac{n}{.n}\right)!}}}\right]\right)$$

$$38=\sqrt{\frac{n}{.\overline{n}}}\left(\left[\sqrt{\sqrt{\sqrt{\left(\left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right]\right)!}}}\right]\right)$$

$$39=\sqrt{\frac{n}{.\overline{n}}}\,\frac{n}{.\overline{n}}$$

$$40=\left[\sqrt{\sqrt{\sqrt{\left(\frac{n}{.\overline{n}}\right)!}}}\right]\times\frac{n}{.n}\ ,......$$

In fact, for $n=1,\ 2,\ ...,\ 9$, we have the following identities:

$$\left[\sqrt{\sqrt{n}}\right]=1,\qquad \left[\sqrt{\left(\sqrt{\frac{n}{.\bar{n}}}\right)!}\right]=2$$

$$\sqrt{\frac{n}{.\bar{n}}}=3,\qquad \left[\sqrt{\sqrt{\sqrt{\left(\frac{n}{.\bar{n}}\right)!}}}\right]=4$$

$$\left[\sqrt{\sqrt{\left(\left(\sqrt{\frac{n}{.\bar{n}}}\right)!\right)!}}\right]=5,\qquad \left(\sqrt{\frac{n}{.\bar{n}}}\right)!=6$$

$$-\left[-\sqrt{\sqrt{\sqrt{\left(\frac{n}{.n}\right)!}}}\right]=7$$

$$\left[\sqrt{\sqrt{\sqrt{\left(\left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right]\right)!}}}\right]=8$$

$$\frac{n}{.\bar{n}}=9,\qquad \frac{n}{.n}=10$$

$$\left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right]=11$$

…… …… ……

Using these identities, it is not difficult to create more formulas for the natural number such as 37, 53, 82, 110, 122, or 911, for any $n=1,2,3,\ldots\ldots,9$

$$37=\sqrt{\frac{n}{.\bar{n}}}\left(-\left[-\sqrt{\sqrt{\sqrt{\left(\frac{n}{.n}\right)!}}}\right]\right)$$

$$53=\left[\sqrt{\sqrt{\left(\left(\sqrt{\frac{n}{.\bar{n}}}\right)!\right)!}}\right]\sqrt{\frac{n}{.\bar{n}}},$$

$$82=\left[\sqrt{\sqrt{\sqrt{\left(\left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right]\right)!}}}\right]\left[\sqrt{\left(\sqrt{\frac{n}{.\bar{n}}}\right)!}\right],$$

$$110=\left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right](n-n),$$

$$122=\left(\left[\sqrt{\sqrt{\left(\left(\sqrt{\frac{n}{.\bar{n}}}\right)!\right)!}}\right]\right)!+\left[\sqrt{\left(\sqrt{\frac{n}{.\bar{n}}}\right)!}\right],$$

$$911=\frac{n}{.\bar{n}}\left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right].$$

As examples, let $n = 1, 4, 6$ we have

$$\sqrt{\frac{n}{.n}}\left(-\left[-\sqrt{\sqrt{\sqrt{\left(\frac{n}{.n}\right)!}}}\right]\right)=\sqrt{\frac{1}{.1}}\left(-\left[-\sqrt{\sqrt{\sqrt{\left(\frac{1}{.1}\right)!}}}\right]\right)=3\left(-\left[-\sqrt{\sqrt{\sqrt{10!}}}\right]\right)$$

$$=3\left(-\left[-6.6065\right]\right)=37,$$

$$\left[\sqrt{\sqrt{\left(\left(\sqrt{\frac{n}{.n}}\right)!\right)!}}\right]\sqrt{\frac{n}{.n}}=\left[\sqrt{\sqrt{\left(\left(\sqrt{\frac{4}{.4}}\right)!\right)!}}\right]\sqrt{\frac{4}{.4}}=\left[\sqrt{\sqrt{720}}\right]3=53,$$

$$\left[\sqrt{\sqrt{\sqrt{\left(\left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right]\right)!}}}\right]\left[\sqrt{\left(\sqrt{\frac{n}{.n}}\right)!}\right]=\left[\sqrt{\sqrt{\sqrt{\left(\left[\sqrt{\sqrt{6}}\right]\left[\sqrt{\sqrt{6}}\right]\right)!}}}\right]\left[\sqrt{\left(\sqrt{\frac{6}{.6}}\right)!}\right]$$

$$=\left[\sqrt{\sqrt{\sqrt{11!}}}\right]\left[\sqrt{3!}\right]=\left[\sqrt{\sqrt{6317.9744}}\right]\left[2.4495\right]=82.$$

If $n = 2, 5, 9$, we have

$$\left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right](n-n)=\left[\sqrt{\sqrt{2}}\right]\left[\sqrt{\sqrt{2}}\right](2-2)=110,$$

$$\left(\left[\sqrt{\sqrt{\left(\left(\sqrt{\frac{n}{.n}}\right)!\right)!}}\right]\right)!+\left[\sqrt{\left(\sqrt{\frac{n}{.n}}\right)!}\right]=\left(\left[\sqrt{\sqrt{\left(\left(\sqrt{\frac{5}{.5}}\right)!\right)!}}\right]\right)!+\left[\sqrt{\left(\sqrt{\frac{5}{.5}}\right)!}\right]$$

$$=\left(\left[\sqrt{\sqrt{6!}}\right]\right)!+\left[\sqrt{6}\right]=120+2=122,$$

$$\frac{n}{.n}\left[\sqrt{\sqrt{n}}\right]\left[\sqrt{\sqrt{n}}\right]=\frac{9}{.9}\left[\sqrt{\sqrt{9}}\right]\left[\sqrt{\sqrt{9}}\right]=911.$$

The above results have some potential applications in the data transmission and will be published in somewhere else.

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