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On the Coefficient Bounds of Gamma and Beta Starlike Functions

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Abstract In this paper, we introduce and investigate a new subclass of the analytic functions in the open unit disk in the complex plane. Here, we give sharp estimates for the first three coefficients for the functions belonging to this class. Also, we give sharp estimates for some initial coefficients for the inverse function and for the function ln (*f*(*z*)/*z*).

Keywords Analytic function, coefficient bound estimate, gamma and beta starlike functions, logarithmic coefficient

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1. Introduction and preliminaries

In this section, we give the necessary information and preliminaries which shall need in our investigation.

Let A be the class of analytic functions $f(z)$ in the open unit disk $U=\{z\in\mathbb{C}:\ |z|<1\}$, normalized by $f(0)=0=f'(0)-1$ with expansion series

$$f(z)=z+a_2z^2+a_3z^3+\cdots+a_nz^n+\cdots=z+\sum_{n=2}^{\infty}a_nz^n,\ a_n\in\mathbb{C}. \tag{1.1}$$

It is well-known that a function $f:\mathbb{C}\to\mathbb{C}$ is said to be univalent if the following condition is satisfied: $z_1=z_2$ if $f(z_1)=f(z_2)$ or $f(z_1)\neq f(z_2)$ if $z_1\neq z_2$. We denote by S the subclass of A consisting of functions which are also univalent in U.

Some of the important and well-investigated subclasses of the univalent function class S are the classes S^* and C, respectively, starlike and convex in the open unit disk U.

By definition, we have (see for details, [7, 10], also [16])

$$S^*=\left\{f\in S:\ \operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right)>0,\ z\in U\right\},$$

and

$$C=\left\{f\in S:\ \operatorname{Re}\left(1+\frac{zf''(z)}{f'(z)}\right)>0,\ z\in U\right\}.$$

For some recent investigations of various subclasses of the univalent functions class S, see the works by Altintaş *et al.* [5], Gao *et al.* [9] and Owa *et al.* [13]

Interesting generalization of the function classes S^* and C, are classes S^*_β and C_β for $\beta \in [0,1)$, which defined by

$$S^*_\beta = \left\{ f \in S : \operatorname{Re}\left(\frac{zf'(z)}{\beta zf'(z) + (1-\beta) f(z)} \right) > 0,\ z \in U \right\}$$

and

$$C_\beta = \left\{ f \in S : \operatorname{Re}\left(\frac{f'(z) + zf''(z)}{f'(z) + \beta zf''(z)} \right) > 0,\ z \in U \right\},$$

respectively. Note that $S^*_0 = S^*$ and $C_0 = C$.

The class S^*_β with negative coefficient was extensively studied by Altintaş and Owa [4] and certain conditions for hypergeometric functions and generalized Bessel functions for these classes were studied Moustafa [11] and Porwal and Dixit [14]. Also, the class S^*_β recently was studied by Porwal [15].

Inspired by the studies mentioned above, we introduce a generalization of the function classes S^*_β and C_β defined as follows.

Definition 1.1. *A function $f \in S$ given by (1.1) is said to be in the class $\aleph(\beta,\gamma) = S^*_\beta C_\beta(\gamma)$, $\beta \in [0,1)$, $\gamma \geq 0$ gamma and beta – starlike function if the following condition is satisfied*

$$\operatorname{Re}\left\{ \frac{zf'(z) + \gamma z^2 f''(z)}{\gamma z\left[f'(z) + \beta zf''(z) \right] + (1-\gamma)\left[\beta zf'(z) + (1-\beta) f(z) \right]} \right\} > 0, z \in U. \qquad (1.2)$$

Remark 1.1. *Taking $\gamma = 0$ in Definition 1.1 and considering the above note, we have $\aleph(\beta,0) = S^*_\beta C_\beta(0) = S^*_\beta$, $\beta \in [0,1)$; that is,*

$$f \in \aleph(\beta,0) \Leftrightarrow \operatorname{Re}\left[\frac{zf'(z)}{\beta zf'(z) + (1-\beta) f(z)} \right] > 0, z \in U.$$

Remark 1.2. *Taking $\gamma = 1$ in Definition 1.1, we have $\aleph(\beta,1) = S^*_\beta C_\beta(1) = C_\beta$, $\beta \in [0,1)$; that is,*

$$f \in \aleph(\beta,1) \Leftrightarrow \operatorname{Re}\left[\frac{f'(z) + zf''(z)}{f'(z) + \beta zf''(z)} \right] > 0, z \in U.$$

Note 1.1. In the special case we write $\aleph(0,0) = S^*$, $\aleph(0,1) = C$.

Remark 1.3. Numerous subclasses of the classes given by the Definition 1.1 can be obtained by specializing the various parameters involved. Many of these classes were studied by earlier researches (cf., e.g., [2 - 6, 15]).

The object of the present paper is to give a series of sharp inequalities involving the initial coefficients of the functions in the class $\aleph(\beta,\gamma), \beta \in [0,1)$, $\gamma \geq 0$ and its special cases. Also, in this paper we give sharp estimates for some initial coefficients for the inverse function and for the logarithmic function $\ln\left(f(z)/z \right)$. To prove our main results, we need require the following well known lemmas.

Lemma 1.1 ([17]). *If $p \in \mathrm{P}$, then the estimates $|p_n| \leq 2, n = 1,2,3,...$ are sharp, where P is the family of all functions p, analytic in U for which $p(0) = 1$ and $\operatorname{Re}\left(p(z) \right) > 0,\ z \in U$, and*

$$p(z)=1+p_1z+p_2z^2+\cdots,\ z\in U. \tag{1.3}$$

Lemma 1.2 ([1, 12]). *Let* $p\in \mathrm{P}$. *Then,*

$$\left|p_2-\frac{\mu}{2}p_1^2\right|\le \max\{2,2|\mu-1|\}=\begin{cases}2, & \mu\in[0,2],\\ 2|\mu-1|, & elsewhere,\end{cases}$$

where P *is the family of all functions* p *in the form (1.3), analytic in* U *for which* $p(0)=1$ *and* $\mathrm{Re}(p(z))>0,\ z\in U$.

2. Upper bound estimates for the coefficients of the function class $\aleph(\beta,\gamma)$

In this section, we give the following theorem on the sharp estimates for the initial three coefficients of the function class $\aleph(\beta,\gamma)$.

Theorem 2.1. *Let the function* $f(z)$ *given by (1.1) be in the class* $\aleph(\beta,\gamma)$, $\beta\in[0,1)$, $\gamma\ge 0$. *Then,*

$$|a_2|\le\frac{2}{(1-\beta)(1+\gamma)}\ \ and\ \ |a_3|\le\frac{3+\beta}{(1-\beta)^2(1+2\gamma)}.$$

Also,

$$|a_4|\le\frac{2(\beta^2+5\beta+6)}{3(1-\beta)^3(1+3\gamma)}.$$

All the inequalities obtained here are sharp.

Proof. Let $f\in\aleph(\beta,\gamma),\beta\in[0,1),\gamma\ge 0$; that is,

$$\frac{zf'(z)+\gamma z^2f''(z)}{\gamma z[f'(z)+\beta zf''(z)]+(1-\gamma)[\beta zf'(z)+(1-\beta)f(z)]}=p(z),z\in U, \tag{2.1}$$

where the function $p(z)=1+\sum_{n=1}^{\infty}p_nz^n$ is in the class P.

If we take into account the expansion series (1.1) of the function $f(z)$, from (2.1), we can write

$$\sum_{n=2}^{+\infty}(n-1)(1-\beta)[1+(n-1)\gamma]a_nz^n=\left\{z+\sum_{n=2}^{+\infty}[1+(n-1)\gamma][1+(n-1)\beta]a_nz^n\right\}\sum_{n=1}^{+\infty}p_nz^n.$$

It follows that

$$\begin{aligned}&(1-\beta)(1+\gamma)a_2z^2+2(1-\beta)(1+2\gamma)a_3z^3+3(1-\beta)(1+3\gamma)a_4z^4+\ldots=p_1z^2+\\&[(1+\beta)(1+\gamma)a_2p_1+p_2]z^3+[(1+\beta)(1+\gamma)a_2p_2+(1+2\beta)(1+2\gamma)a_3p_1+p_3]z^4+\ldots.\end{aligned} \tag{2.2}$$

Comparing the coefficients of the like power of z in both sides of (2.2), we have

$$(1-\beta)(1+\gamma)a_2=p_1, \tag{2.3}$$

$$2(1-\beta)(1+2\gamma)a_3=p_2+(1+\beta)(1+\gamma)a_2p_1, \tag{2.4}$$

$$3(1-\beta)(1+3\gamma)a_4=p_3+(1+\beta)(1+\gamma)a_2p_2+(1+2\beta)(1+2\gamma)a_3p_1. \tag{2.5}$$

From these

$$a_2 = \frac{p_1}{(1-\beta)(1+\gamma)}, \tag{2.6}$$

$$a_3 = \frac{p_2}{2(1-\beta)(1+2\gamma)} + \frac{(1+\beta)\, p_1^2}{2(1-\beta)^2 (1+2\gamma)}, \tag{2.7}$$

$$a_4 = \frac{p_3}{3(1-\beta)(1+3\gamma)} + \frac{(3+4\beta)\, p_1 p_2}{6(1-\beta)^2 (1+3\gamma)} + \frac{(1+\beta)(1+2\beta)\, p_1^3}{6(1-\beta)^3 (1+3\gamma)}. \tag{2.8}$$

Since $|p_1| \le 2$, from (2.6), we have

$$|a_2| \le \frac{2}{(1-\beta)(1+\gamma)}. \tag{2.9}$$

From (2.7) since the coefficients p_2 and p_1^2 are positive for $\beta \in [0,1)$ and $\gamma \ge 0$, using triangle inequality and applying the inequalities $|p_n| \le 2, n = 1,2$, we obtain

$$|a_3| \le \frac{3+\beta}{(1-\beta)^2 (1+2\gamma)}. \tag{2.10}$$

Similarly, from (2.8) for $|a_4|$ we have

$$|a_4| \le \frac{2(\beta^2 + 5\beta + 6)}{3(1-\beta)^3 (1+3\gamma)}. \tag{2.11}$$

Thus, the proof of the inequalities in the theorem is completed.

To see that the inequalities obtained in the theorem are sharp, we note that equality is attained in the inequalities when $p_1 = p_2 = p_3 = 2$.

Moreover, we can easily show that all inequalities obtained in the theorem are sharp for the particular solution of the following linear homogeneous differential equation

$$\left[(1+\beta)\gamma z + (1-\beta)\gamma\right] z^2 y'' + \left\{\left[1+\beta+(1-\beta)\gamma\right] z + (1-\beta)(1-\gamma)\right\} z y' -$$
$$(1-\beta)(1-\gamma)(1-z)\, y = 0.$$

Thus, the proof of Theorem 2.1 is completed.

Setting $\gamma = 0$ in Theorem 2.1, we can readily deduce the following result.

Corollary 2.1. *Let the function $f(z)$ given by (1.1) be in the class S_β^*, $\beta \in [0,1)$. Then,*

$$|a_2| \le \frac{2}{1-\beta} \quad \textit{and} \quad |a_3| \le \frac{3+\beta}{(1-\beta)^2}.$$

Also,

$$|a_4| \le \frac{2(\beta^2 + 5\beta + 6)}{3(1-\beta)^3}.$$

All the inequalities are sharp.

Setting $\beta = 0$ in Corollary 2.1, we have the following result.

Corollary 2.2. *Let the function* $f(z)$ *given by (1.1) be in the class* S^*. *Then,*

$$|a_n| \le n, n = 2,3,4.$$

All the inequalities are sharp.

Note 2.1. As you can see, Corollary 2.2 confirmed that the Bieberbach's Conjecture (see for example [7]) $|a_n| \le n$ on the coefficient estimates for the starlike function class has been provided for $n = 2,3,4$.

Setting $\gamma = 1$ in Theorem 2.1, we can readily deduce the following result.

Corollary 2.3. *Let the function* $f(z)$ *given by (1.1) be in the class* C_β, $\beta \in [0,1)$. *Then,*

$$|a_2| \le \frac{1}{1-\beta} \quad \text{and} \quad |a_3| \le \frac{3+\beta}{3(1-\beta)^2}.$$

Also,

$$|a_4| \le \frac{\beta^2 + 5\beta + 6}{6(1-\beta)^3}.$$

All the inequalities are sharp.

Setting $\beta = 0$ in Corollary 2.3, we have the following result.

Corollary 2.4. *Let the function* $f(z)$ *given by (1.1) be in the class* C. *Then,*

$$|a_n| \le 1, n = 2,3,4.$$

All the inequalities are sharp.

3. Coefficient estimates for the inverse function

In this section, we give the following theorem on the sharp estimates for some initial coefficients of the inverse function of the function $f \in \aleph(\beta,\gamma)$.

Theorem 3.1. *Let the function* $f(z)$ *given by (1.1) be in the class* $\aleph(\beta,\gamma)$, $\beta \in [0,1)$, $\gamma \ge 0$ *and* $f^{-1}(w)$ *is the inverse function of* $f(z)$. *Then, for the initial two coefficients of the inverse function* f^{-1}, *we have*

$$|A_2| \le \frac{2}{(1-\beta)(1+\gamma)},$$

$$|A_3| \le \begin{cases} \dfrac{1}{(1-\beta)(1+2\gamma)} \left\{ \dfrac{2\left[4(1+2\gamma) - (1+\beta)(1+\gamma)^2\right]}{(1-\beta)(1+\gamma)^2} - 1 \right\}, & \gamma \in \left[0, 1+\sqrt{2}\right], \\ \dfrac{1}{(1-\beta)(1+2\gamma)}, & \gamma \in \left[1+\sqrt{2}, +\infty\right). \end{cases}$$

All the inequalities obtained here are sharp.

Proof. Let $f \in \aleph(\beta,\gamma), \beta \in [0,1), \gamma \ge 0$. Then, since $\aleph(\beta,\gamma) \subset S$, inverse function $f^{-1}(w)$ exist and defined in the open disk $D = \{w : |w| < r_0(f)\}$, $r_0(f) \ge 1/4$ in the series expansion (see, for example [8])

$$f^{-1}(w) = w + A_2 w^2 + A_3 w^3 + A_4 w^4 \cdots, \; w \in D,$$

where

$$A_2 = -a_2, \quad A_3 = 2a_2^2 - a_3, \quad A_4 = -5a_2^3 + 5a_2 a_3 - a_4. \tag{3.1}$$

From Theorem 2.1 and the first equality of (3.1) the inequality for $|A_2|$ is obvious.

From the second equality of (3.1), using (2.6) and (2.7), we can write expression for A_3 as

$$A_3 = \frac{-1}{2(1-\beta)(1+2\gamma)}\left[p_2 - \frac{4(1+2\gamma)-(1+\beta)(1+\gamma)^2}{(1-\beta)(1+\gamma)^2}p_1^2\right].$$

From this, we write

$$|A_3| \le \frac{1}{2(1-\beta)(1+2\gamma)}\left|p_2 - \frac{\mu}{2}p_1^2\right|,$$

with $\mu = 2\left[4(1+2\gamma)-(1+\beta)(1+\gamma)^2\right]/(1-\beta)(1+\gamma)^2$.

First note that $\mu \ge 2$, when $\gamma \in \left[0, 1+\sqrt{2}\right]$ and $\mu \in [0,2]$, when $\gamma \in \left[1+\sqrt{2}, +\infty\right)$. We now use Lemma 1.2 with $\mu = 2\left[4(1+2\gamma)-(1+\beta)(1+\gamma)^2\right]/(1-\beta)(1+\gamma)^2$, and obtain

$$|A_3| \le \begin{cases} \dfrac{1}{(1-\beta)(1+2\gamma)}\left\{\dfrac{2\left[4(1+2\gamma)-(1+\beta)(1+\gamma)^2\right]}{(1-\beta)(1+\gamma)^2}-1\right\}, & \gamma \in \left[0, 1+\sqrt{2}\right], \\ \dfrac{1}{(1-\beta)(1+2\gamma)}, & \gamma \in \left[1+\sqrt{2}, +\infty\right). \end{cases}$$

Thus, the proof of the inequalities in the theorem is completed.

To see that the inequality for $|A_2|$ is sharp, we note that equality is attained in the inequality when $p_1 = 2$, and the first inequality for $|A_3|$ is sharp when $p_1 = p_2 = 2$. The second inequality for $|A_3|$ is sharp when $p_1 = 0 = p_2 - 2$.

Thus, the proof of Theorem 3.1 is completed.

From Theorem 2.1 we arrive at the following results.

Corollary 3.1. *Let the function* $f(z)$ *given by (1.1) be in the class* S_β^*, $\beta \in [0,1)$ *and* $f^{-1}(w)$ *is the inverse function of* $f(z)$ *. Then, for the initial two coefficients of the inverse function* f^{-1}, *we have*

$$|A_2| \le \frac{2}{1-\beta}, \ |A_3| \le \frac{5-\beta}{(1-\beta)^2}.$$

All the inequalities obtained here are sharp.

Corollary 3.2. *Let the function* $f(z)$ *given by (1.1) be in the class* C_β, $\beta \in [0,1)$ *and* $f^{-1}(w)$ *is the inverse function of* $f(z)$ *. Then, for the initial two coefficients of the inverse function* f^{-1}, *we have*

$$|A_2| \le \frac{1}{1-\beta}, \ |A_3| \le \frac{3-\beta}{3(1-\beta)^2}.$$

All the inequalities obtained here are sharp.

4. The coefficient of $\ln\left(f(z)/z\right)$

The logarithmic coefficients δ_n of a function $f \in S$ are defined from the following equation

$$\ln\left(f(z)/z\right)=2\sum_{n=1}^{\infty}\delta_n z^n . \tag{4.1}$$

On differentiating both sides of (4.1), it is a trivial consequence of the inequality $|p_n|\le 2$, that for $n\ge 1$, $|\delta_n|\le 1/n$ when $f\in S^*$, and $|\delta_n|\le 1/2n$ when $f\in C$. However when $f\in\aleph(\beta,\gamma)$, the same procedure does not give a convenient expression in terms of $1+zf''(z)/f'(z)$ or $zf'(z)/f(z)$ unless $\beta=0=\gamma-1$ or $\beta=0=\gamma$. We show next that it is however possible to obtain sharp estimates for the modulus of the initial coefficients of $\ln\left(f(z)/z\right)$ when $f\in\aleph(\beta,\gamma)$.

We give the following theorem on the sharp estimates for initial two coefficients of $\ln\left(f(z)/z\right)$ for the function $f\in\aleph(\beta,\gamma)$.

Theorem 4.1. *Let the function $f(z)$ given by (1.1) be in the class $\aleph(\beta,\gamma)$, $\beta\in[0,1)$, $\gamma\ge 0$. Then,*

$$|\delta_1|\le\frac{1}{(1-\beta)(1+\gamma)},\quad |\delta_2|\le\frac{1}{(1-\beta)(1+2\gamma)}\left[\frac{1}{2}+\frac{(1+\beta)(1+\gamma)^2-(1+2\gamma)}{(1-\beta)(1+\gamma)^2}\right].$$

All the inequalities obtained here are sharp.

Proof. Let $f\in\aleph(\beta,\gamma)$, $\beta\in[0,1)$, $\gamma\ge 0$. Differentiating both sides of (4.1) and comparing the coefficients of the like power of z in the both sides, we obtain

$$2\delta_1=a_2, \tag{4.2}$$

$$4\delta_2+2a_2\delta_1=2a_3. \tag{4.3}$$

From (4.2), easily write

$$\delta_1=\frac{a_2}{2}. \tag{4.4}$$

From (4.4), using Theorem 2.1, the inequality for $|\delta_1|$ is obvious.

From (4.3), (4.4), (2.6) and (2.7), we can write the expression for δ_2 as

$$\delta_2=\frac{1}{4(1-\beta)(1+2\gamma)}p_2+\frac{(1+\beta)(1+\gamma)^2-(1+2\gamma)}{4(1-\beta)^2(1+\gamma)^2(1+2\gamma)}p_1^2. \tag{4.5}$$

Since the coefficients of p_2 and p_1^2 in the above equation (4.5) are positive for $\beta\in[0,1)$ and $\gamma\ge 0$, the required inequalities for $|\delta_2|$ is valid on using the inequalities from Lemma 1.2 $|p_n|\le 2$ for $n=1,2$.

To see that the inequalities obtained in the theorem are sharp, we note that equality is attained in the inequalities when $p_1=p_2=2$.

Thus, the proof of Theorem 4.1 is completed.

From Theorem 4.1 we arrive at the following results.

Corollary 4.1. *Let the function $f(z)$ given by (1.1) be in the class S_β^*, $\beta\in[0,1)$. Then,*

$$|\delta_1|\le\frac{1}{1-\beta},\quad |\delta_2|\le\frac{1+\beta}{2(1-\beta)^2}.$$

The inequalities are sharp.

Corollary 4.2. *Let the function* $f(z)$ *given by (1.1) be in the class* C_β, $\beta \in [0,1)$. *Then,*

$$|\delta_1| \le \frac{1}{2(1-\beta)}, \quad |\delta_2| \le \frac{3+2\beta}{12(1-\beta)^2}.$$

The inequalities are sharp.

Corollary 4.3. *Let the function* $f(z)$ *given by (1.1) be in the class* S^*. *Then,*

$$|\delta_n| \le \frac{1}{n}, \ n = 1, 2.$$

The inequalities are sharp.

Corollary 4.4. *Let the function* $f(z)$ *given by (1.1) be in the class* C. *Then,*

$$|\delta_n| \le \frac{1}{2n}, \ n = 1, 2.$$

The inequalities are sharp.

Remark 4.1. Using this work, we can be examined the Fekete - Szegö problem for the coefficients of the function class $\aleph(\beta,\gamma)$. Moreover, using this work we can be find $H_2(2) = a_2a_4 - a_3^2$ second Hankel determinant for the functions belonging in this class. Hence, we find upper bound estimate for the $|a_2a_4 - a_3^2|$.

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