



Optimum Signal Denoising based on wavelet shrinkage thresholding techniques: White Gaussian Noise and White Uniform Noise case study

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Abstract Wavelets are distinctive mathematical functions for analyzing data to scale or resolution. In this work, we present a comparative investigation and analysis of signal processing and denoising technique employing four distinctive wavelet shrinkage thresholding algorithms. The four wavelet thresholding techniques are Rigrsure, Minimax, heursure and Sqtwolog. Simulation based tests were conducted to explore the fitness of the four distinctive wavelet shrinkage thresholding techniques on simulated noisy sine and noisy block signal data, respectively. The results reveals that the denoising capability of each wavelet shrinkage thresholding technique depends largely on the type of noise present in the signal data. Specifically, for white Gaussian noise removal, the result reveals that the Rigrsure thresholding technique attained the best performance in terms SNR, SRER, PAD and COE values. This is followed by Minimax, then heursure and sqtwolog thresholding. But for white uniform noise removal, Minimax performed best compared to others.

Keywords Noisy signal, Wavelets, Wavelets denoising, level decomposition, shrinkage thresholding techniques

Introduction

Signal coverage processing often suffers serious setback due to various distortions engendered by noise [1, 2]. Generally, a noise can be defined as an undesirable and detrimental signal that depreciates the actual characteristics of original signal. There exist different kinds of noises present in radio signal propagation environment, among the key ones are White noise, burst noise, colored noise, etc. These noise signals may interfere with either the whole frequency band or some specific parts of frequency band. When noise interfere with the entire parts of the frequency band, at that point it becomes very challenging and demanding to eliminate or suppress the noise without losing the actual signal information. As a result, noise removal or suppression without losing the original signal characteristics is a demanding task and has become a dynamic area of research among scientists and engineers in recent time.

Some good research work has been carried out and reported in literature, using the conventional signal and image denoising techniques, as revealed in [3-10]. Among the key conventional denoising techniques are the Kalman filter [5], High-pass filter [4], low-pass filter [4], the median filter [6] and neural networks based adaptive filter [7]. However, all these conventional approaches have some key inherent limitations. For instance, spatial Low-pass filters can smoothen away noise but also will also distort signals power [8]. Also the high-pass filters possess the ability to and improve spatial signal, on the other hand, will also strengthen the background noisy. Another popular conventional noise filtering technique is the Fast Fourier Transform (FFT) [8], [9]. FFT is principally a low pass filtering method which can cater noisy signal with short time behavior, but has bad convergence property and poor time resolution of noisy signal on longer time scales, owing to its short time transform window [10].



Over the above highlighted conventional ones, wavelet transform turn out to be the first choice in recent years, as regard to effective signal denoising, owing to its multi-resolution signal processing approach and capability to cater for localized nature of signals both in time and space [11].

The prime objective of this work is to exploit various essential wavelet families based on wavelet transform for optimum processing of simulated noisy signal dataset. The target is to selectively determine the most suitable wavelet family for the optimum removal of noise from the signal dataset, using White Gaussian Noise (WGN) and White Uniform Noise (WUN) as case studies.

Wavelet Function

Wavelets were specially developed in relevant fields of Physics, electrical engineering and seismic geology. Transactions among these fields during the past few years have in turn resulted to numerous new wavelet applications like signal data processing, denoising or filtering, image compression, turbulence, earthquake, human vision, and radar prediction.

Wavelet function is a mathematical function employed to transform a given function into different components, via scaling and translation. Generally, a frequency range can be assign to each scale component in such a way that its resolution matches the scale. The scaled and translated (also termed "daughter wavelets") copies of the wavelets are a fast-decomposing or finite-length oscillating waveform (also referred to as the "mother wavelet"). Wavelet transforms possess some key advantages over accustomed Fourier transforms when representing and analyzing functions with incoherent and sharp peaks. It is also more beneficial for precisely decomposing and restructuring finite, non-stationary and/or non-periodic signals.

In general, wavelets are doggedly crafted to possess explicit characteristics that make them very suitable for signal processing. As special mathematical tools, they can be utilised to extract valuable information from multifarious datasets. The wavelet-based signal processing works with the implementation of different archetype function. The wavelet transform of a predictable signal $u(t)$, with respect to wavelet basis function $\psi(t)$, can be expressed by [11-13]:

$$U(q, a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \psi' \left(\frac{t-q}{a} \right) u(t) dt \quad (1)$$

where,

$\psi(q, a) * (t)$ indicates the wavelet function, a represents the scale, q is any given point in time, and, and $u(t)$ represent input signal.

Generally, the wavelet-based signal data processing method can be described using the following three steps:

- (a) Decomposition- which is the transformation of the input signal into different approximation levels and coefficients,
- (b) Thresholding-deals with the extraction of the coefficients that contains the original signal, and then discarding others, i.e., the utilization of different denoising algorithms to signal to minimise the noise effect, or the extraction of the original signal employing various denoising algorithms,
- (c) Reconstruction- back transformation of decomposed signal using inverse wave transform.

Thresholding and Estimation Algorithms

The thresholding algorithm employs statistical regression of the noisy data coefficients to provide a nonparametric estimate of the reconstructed signal devoid of noise. A principal challenge in thresholding is determining the best threshold value (λ), which assist in the estimation noise level from a noising signal or data. If the chosen threshold value is too big, then some essential components signal features may be filtered out. On the other hand, if the chosen threshold value is too small then a substantial quantity noise will still present in the data. Described briefly below are the four thresholding algorithms employed exploited in this work. They are:

(a) **Rigrsure thresholding** - it is a threshold algorithm or soft threshold assessor based on quadratic loss function with respect to Stein's Unbiased Estimate of Risk. Decreasing the risks in (λ) provides a selection in in correspondence to desired threshold value. The selection threshold is given by



$$\lambda = \sigma \sqrt{\omega_b} \quad (2)$$

where ω_b is the b^{th} coefficient at minimum risk.

(b) Sqrtwolog thresholding- It utilises a fixed-form threshold and the threshold value λ is calculated employing square root log (i.e. universal threshold method):

$$\lambda = \sigma_j \sqrt{2 \log(N_j)} \quad (3)$$

$$\sigma_j = \frac{MAD_j}{0.6745} \quad (4)$$

Where, N_j and MAD_j are the respective noisy signal length, and Median Absolute Deviation (MAD) at j^{th} scale.

(c) Heursure thresholding-It is a mixture of the Rigrsure thresholding and Sqrtwolog thresholding. Accordingly, if one denote the Sqrtwolog method and Rigrsure method by λ_s and λ_r respectively, then Heursure thresholding method can be expressed as

$$\lambda = \sigma_j \sqrt{2 \log(N_j)} \quad (5)$$

(d) Minimax thresholding- it also utilises a fixed threshold selection rule like the Sqrtwolog thresholding to provide minimax performance, but with a different threshold value.

Simulation-based Experimental Setting

A simulation-based experiment is employed in this work to generate signal data and examine the performance of different wavelet thresholding techniques. Specifically, two benchmark signal types, namely: Heavy sine signal and Block signal, were performed with MATLAB 2015a software platform. All the generated signals contain 3000 data point samples. For comparison purpose, two different noise test cases with variable amplitudes were generated and added to the four benchmark signal types. They are additive White Gaussian Noise (WGN) and additive White Uniform Noise (WUN). The noisy WGN and WUN signals are transformed into wavelet coefficients by the sym5 wavelet family. A 5 wavelet decomposition level is selected to denoise the WGN and WUN signals test method.

Results and Analysis

For the purpose of detail comparison, four different quantitative performance evaluation parameters were employed to investigate the denoising capability of the four aforementioned wavelet thresholding algorithms. In this case, the performance evaluation parameters (i.e. metrics) are Noise Ratio (SNR), Signal to reconstruction ratio (SRER), Pulse amplitude distortion (PAD) and coefficient of efficiency. Generally, a good denoising technique would result in a larger SNR, SRER, PAD and a lower PAD values.

The first assessment is performed to identify the precise decomposition level for a robust denoising using heavy sine signal with sym8 wavelet family as a case study and the metrics are shown in Table 1. The displayed result in Table 1 indicates that level 1 is the most precise decomposition level compared to others. Thus, level 1 being the best in terms of SNR, SRER and PAD, it is chosen for denoising performance evaluation throughout this work. Although wavelets on itself can denoise a noisy signal data to some extent as revealed in table 1, their performance can be further enriched by applying the right shrinkage thresholding algorithms. Hence in the second assessment, Rigrsure, Minimax, Heursure and sqrtwolog, have been experimented and their performance is displayed in figures 1 to 9. Table 2.

For white Gaussian noise removal, the result in tables 2 and 3 reveal that the Rigrsure thresholding technique attained the best performance in terms SNR, SRER, PAD and COE values. This is followed by Minimax, then heursure and sqrtwolog thresholding. But for white uniform noise removal, Minimax performed best compared to others. Furthermore, comparing Table 1 and 2, the results clearly reveals the importance of utilizing



thresholding techniques as compared to without thresholding as in Table 1. Noise can be better removed only to specified amount of tolerance as shown in Table 3.

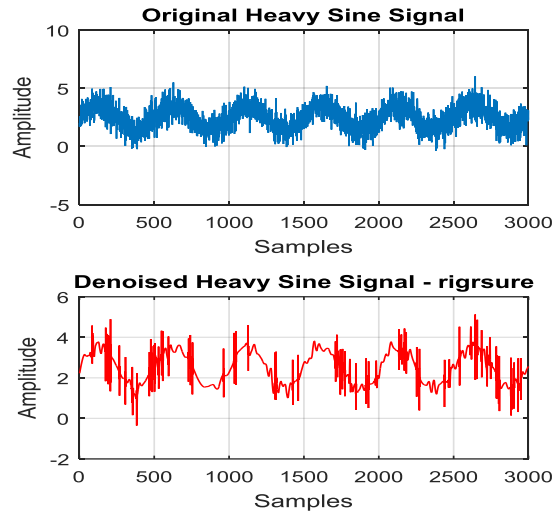


Figure 1: Denoised Sine Signal performance with Rigrsure Thresholding Technique

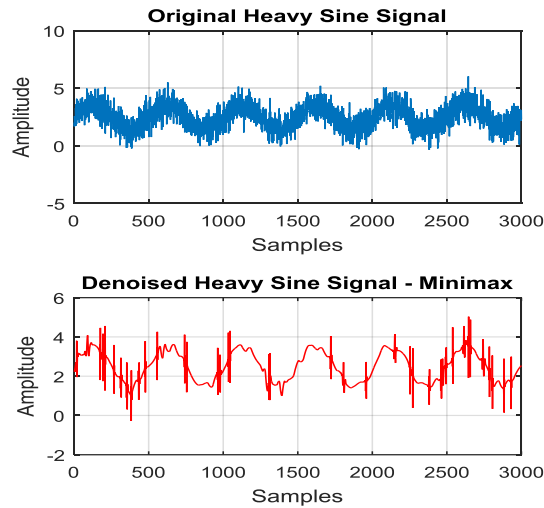


Figure 2: Denoised Sine Signal performance with Minimax Thresholding Technique

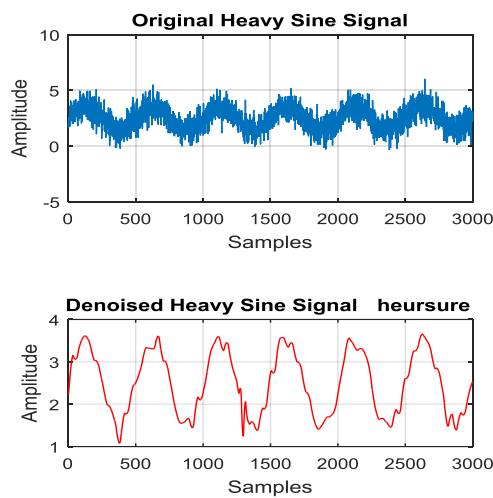


Figure 3: Denoised Sine Signal performance with Heursure Thresholding Technique

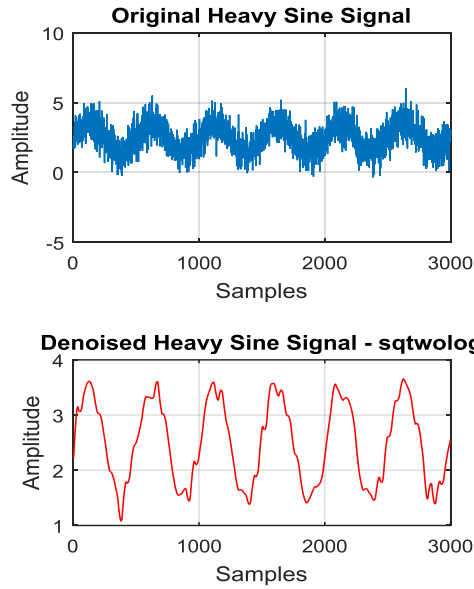


Figure 4: Denoised Sine Signal performance with Sqtrwolog Thresholding Technique

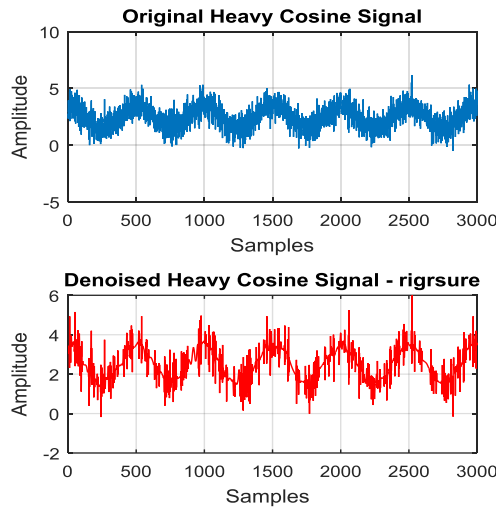


Figure 5: Denoised Cosine Signal performance with Rigrsure Thresholding Technique

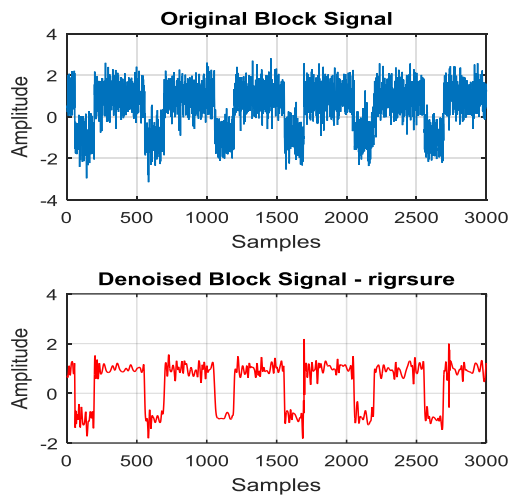


Figure 7: Denoised Block Signal performance with Rigrsure Thresholding Technique

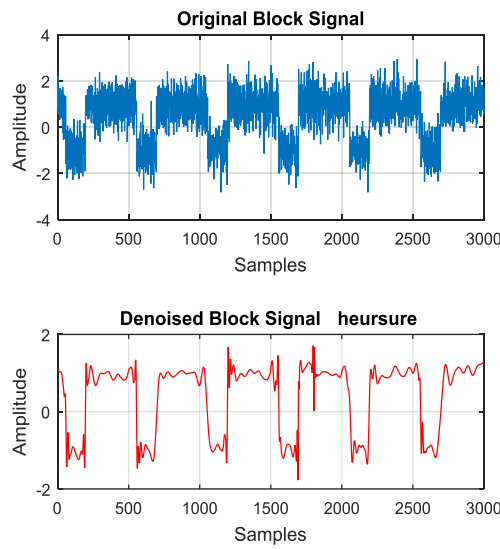


Figure 7: Denoised Block Signal performance with Heursure Thresholding Technique

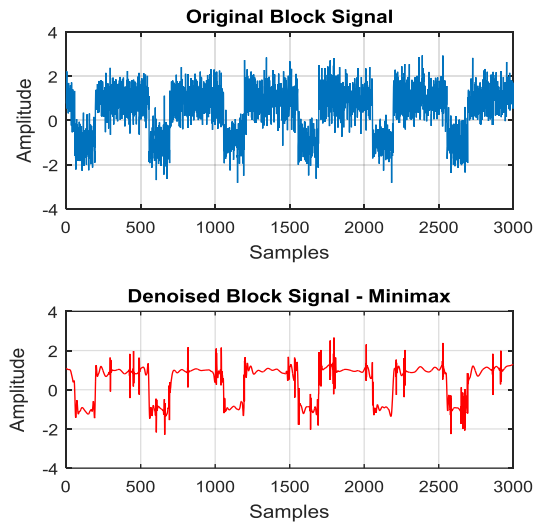


Figure 8: Denoised Block Signal performance with Minimax Thresholding Technique

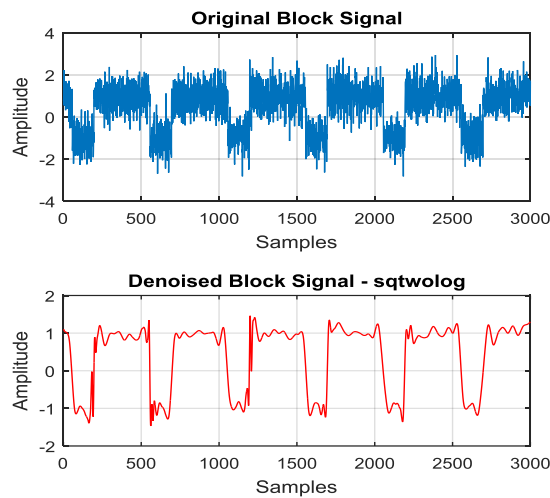


Figure 9: Denoised Block Signal performance with Rigrsure Thresholding Technique

Table 1: Denoised Sine Signal performance at Different Decomposition Level

Metric	Decomposition Level	White Gaussian Noise	White Uniform Noise
SNR	1	14.72	26.23
	2	12.95	24.42
	3	12.15	23.73
	4	11.81	23.59
PSNR	1	21.08	29.31
	2	19.53	27.51
	3	19.41	26.84
	4	18.94	26.68
PAD	1	1.73	0.39
	2	2.05	0.47
	3	2.38	0.48
	4	2.47	0.51
COE	1	86.86	98.07
	2	80.48	97.07
	3	74.97	96.61
	4	73.24	96.45

Table 2: Denoised Sine Signal performance with Different Thresholding techniques

Metric	Threshold Type	White Gaussian Noise	White Uniform Noise
SNR	Rigsure	15.87	26.31
	Heursure	14.58	26.30
	Minimax	15.51	26.33
	sqtwolog	14.58	26.31
PSNR	Rigsure	22.66	29.40
	Heursure	21.42	29.40
	Minimax	21.93	29.42
	sqtwolog	21.41	29.40
PAD	Rigsure	1.42	0.35
	Heursure	1.75	0.40
	Minimax	1.90	0.40
	sqtwolog	1.76	0.40
COE	Rigsure	90.35	98.10
	Heursure	86.89	98.10
	Minimax	88.46	98.11
	sqtwolog	86.89	98.10

Table 3: Denoised Block Signal performance with different Thresholding techniques

Metric	Decomposition Level	White Gaussian Noise	White Uniform Noise
SNR	Rigsure	9.31	11.38
	Heursure	8.07	11.04
	Minimax	8.60	11.21
	sqtwolog	8.08	11.02
PSNR	Rigsure	18.83	18.03
	Heursure	17.44	17.71
	Minimax	17.90	17.86
	sqtwolog	17.44	17.70
PAD	Rigsure	1.08	0.33
	Heursure	1.76	0.51
	Minimax	1.21	0.33
	sqtwolog	1.76	0.60
COE	Rigsure	93.69	92.23
	Heursure	91.77	91.91
	Minimax	92.63	91.91
	sqtwolog	91.77	91.58



Conclusion

In this work, we presented a comparative investigation and analysis of noise removal signal method employing four distinctive wavelet shrinkage thresholding techniques. Simulation based Tests were conducted to investigate the fitness of employing four distinctive wavelet shrinkage thresholding techniques, which are Rigrsure, Minimax, heursure and Sqtwolog. The results reveals that the denoising capability of each wavelet with geodetic navigation examples shrinkage thresholding technique depends largely on the type of noise in any given signal data. Specifically, for white Gaussian noise removal, the result reveals that the Rigrsure thresholding technique attained the best performance in terms SNR, SRER, PAD and COE values. This is followed by Minimax, then heursure and sqtwolog thresholding. But for white uniform noise, Minimax performed best compared to others.

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