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## Impact of Linear and Nonlinear Channel Equalization Algorithms on User Achievable Rates and Energy Efficiency on Uplink Large Scale MIMO Techniques for Next Generation Networks

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**Abstract** In multiuser wireless communication systems, interference and fading are two key factors that impact the energy efficiency and user data transmission capacity which can be dealt with by means of channel equalization. In this paper, the influence of linear and non-linear channel equalization on single cell MIMO system with equivalent dimensions of antennas at the receiver and transmitter is demonstrated. Specifically, the achievable rates and energy efficiency performances of four channel equalization algorithms which includes the minimum-mean-square-error (MMSE), zero forcing (ZF), Zero forcing successive interference-cancellation (ZF-SIC) and minimum-mean-square-error successive interference-cancellation (MMSE-SIC) are studied extensively through computer simulations. The results indicate that the non-linear MMSE-SIC and ZF-SIC equalization schemes outperform the linear ones by 24 to 29% in terms user achievable rates and energy efficiency. This can be justified by the fact that the linear channel equalization schemes does not fully suppress interference among users for enhanced data transmission compared to nonlinear schemes which exploit extra signal processing techniques by means of SIC to provide improved user information transmission thereby delivering higher performance gains at the receivers.

**Keywords** Large scale antenna systems, propagation environment, interference, linear channel equalization, Non-linear channel equalization, Energy throughput ratio, user achievable rates

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### 1. Introduction

The mobile wireless telecommunication technology has and is still experiencing amazing dramatic changes brought about by the introduction of apps tools and market forces [1]. The continuous growing demand for modern-day radio communication systems that can deliver high speed multimedia wireless services to multiuser subscribers also places very challenging requirements on the systems in return. The radio communication systems process signal information symbols in digital mode with the exclusive aim of transmitting the information symbols and also recovering them appropriately at the receiving end. However in entirety, this process is not an easy one especially in wireless mobile communication systems where a number of issues come into play. For instance, interference, multipath fading, noise, limitations in bandwidth, signal propagation loss and time variance, among other factors, impact the wireless channel negatively over which data transmission passes through.

Wireless multimedia communications systems based on the multiple input multiple (MIMO) technique are exploited in radio network applications that require high bits data rates. When the data is communicated at high bit rates through mobile radio channels like that of MIMO systems, the channel impulse response can outspread over sundry symbol periods, which results to inter-symbol interference (ISI) [2]. Thus, ISI is known hazard that severely upsets data transmission speed and error rate performance over wireless media MIMO systems, especially at receivers' end as shown in figure 1.



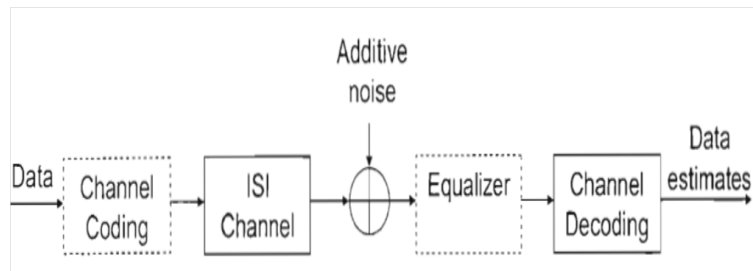


Figure 1: Baseband receiver model

One key efficient means to cater for the ISI multipath propagation environment is by employing a filtering method termed equalization [3][4]. In this chapter, two linear and nonlinear channel equalization algorithms which includes the minimum-mean-square-error (MMSE), zero forcing (ZF), minimum-mean-square-error successive interference cancellation (MMSE-SIC) and Zero forcing- successive interference cancellation (ZF-SIC), are studied extensively through computer simulations to examine their impact on energy efficiency and data transmission capacity of large scale MIMO systems. In our previous chapters, the focus were solely on massive MIMO communication the downlink single cell scenarios. Even though the large-scale MIMO antenna systems proven to be very advantages in the previous chapters, it may be challenging to equip large-scale MIMO antenna into mobile communication devices such as smart phones, palmtop computers and tablets. The employment of the large antenna number into the small devices may result to high cost, high hardware complexity and low battery life. In this chapter, the focus is in the uplink, wherein both the single base transmitter and mobile terminals are equipped with multiple antennas.

**MIMO channel Model**

Before looking at the various equalization methods examined in this works, let us consider a typical multiuser large-scale MIMO system that possess  $N_t$  transmit antennas and  $N_r$  receive-antennas as illustrated in figure 2. The received signal in the  $j$ th antenna can be expressed as

$$y_i^k = \sum_{j=1}^{N_t} H_{ij} x_j^k + n_i^k \tag{1}$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{2}$$

The subscript  $i$  and  $j$  indicate the  $i$ -th receiver and  $j$ -th transmitter respectively. The superscript  $k$  designates the  $k$ th transmit-antennas, where

$\mathbf{H} = N_t \times N_r$  channel matrix,

$\mathbf{y} = N_r \times I$  received signal,

$\mathbf{x} = N_t \times I$  transmitted signal, and

$\mathbf{n}$  = additive white Gaussian noise corresponding to the receive-antenna.

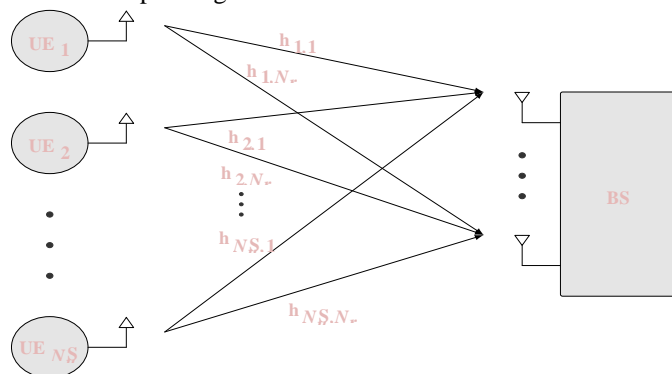


Figure 2: Uplink multiuser MIMO receiver model structure

To estimate and cater for the transmitted signal symbols that has been impacted by Inter symbol Interference (ISI) and noise in the MIMO systems communication link, an equalization method is employed. A brief general overview of the four channel equalization techniques considered at the receiver in this work provided below.

The receiver is assumed to possess a perfect knowledge of the channel state. Also, the weights of the equalizing filters are calculated dynamically.

### 2.1 Zero-forcing (ZF)

ZF equalizer was first proposed by Robert Lucky and it is a type of linear equalization algorithm employed to invert the frequency response of the channel in communication systems. That is, the ZF equalizer exploits the inverse of the radio channel to remove signal distortion if there were no noise and bring down the inter symbol before the channel. When perfect CSI is available at the BS, the ZF equalization employs a Pseudo-inverse method of the  $N_t \times N_r$  channel matrix, and it can be expressed as:

$$\mathbf{G}_{ZF} = \text{Pinv}(\mathbf{H}) = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \quad (3)$$

where  $\mathbf{G}$  is the channel equalization matrix,  $\text{Pinv}(H)$  denotes the Pseudo-inverse, the superscript  $H$  stands for Hermitian conjugation of a matrix and the superscript  $-1$  stands for the inverse of matrix.

Let

$$\mathbf{G} = [\mathbf{g}_1, \dots, \mathbf{g}_i, \dots, \mathbf{g}_{N_t}]^T \quad (4)$$

and

$$\mathbf{h} = [\mathbf{h}_1, \dots, \mathbf{h}_i, \dots, \mathbf{h}_{N_t}] \quad (5)$$

The estimated transmitted symbol,  $\hat{x}$  is therefore given by:

$$\hat{x} = \mathbf{G} \cdot \mathbf{y} \quad (6)$$

Thus, with ZF equalization, the signal quality,  $SINR_k$  and the achievable data rate,  $R_k$  user  $k$  can be expressed as:

$$SINR_{k,ZF} = \frac{P_k |\mathbf{G}_k \mathbf{h}_k|^2}{N_0} \quad (7)$$

$$R_{k,ZF} = B \cdot \text{Log}_2 \left( 1 + \frac{P_k |\mathbf{G}_k \mathbf{h}_k|^2}{N_0} \right) \quad (8)$$

$P_k$  denotes the transmit power constraint.

Similar to references [5, 11], the achievable sum rate,  $R_{sum}$  of  $k$ -th user in the uplink is given by:

$$R_{sum} = \sum_{k=1}^{N_t} \zeta^{ul} \left( 1 - \frac{\tau^{ul} K}{U \tau^{ul}} \right) R_{k,ZF} \quad (9)$$

Where the term  $1 - \frac{\tau^{ul} K}{U \tau^{ul}}$  describes the reduction of effective transmission rate which occurs as a result of the overheard of training.

### 2.2 Minimum mean square error (MMSE)

The ZF equalizer is employed to get rid of all linear distortion, nonetheless noise is increased significantly in the process, especially when the channel response possesses a small amplitude. A better equalizer to cater for the total power of the noise channel and also partially remove ISI distortion is the MMSE equalizer. Aptly, it is a method that attempts to find an equalization matrix  $G$  minimizing the criterion:

$$E \{ (\mathbf{G} \cdot \mathbf{y} - x)^H (\mathbf{G} \cdot \mathbf{y} - x) \} \quad (10)$$

where,

$E = \{ \}$  denotes the statistical ensemble.

Minimising equation (10) yields:



$$\mathbf{G}_{MMSE} = \left( N_0 \mathbf{I}_{N_r} + \sum_{i \neq k}^{N_t} P_i \mathbf{h}_i \mathbf{h}_i^* \right)^{-1} \mathbf{h}_k \quad (11)$$

The achieving output  $SINR_k$ ,  $R_k$  and  $R_{sum}$  can be expressed as:

$$SINR_{k, MMSE} = P_k \mathbf{h}_k^* \left( N_0 \mathbf{I}_{N_r} + \sum_{i \neq k}^{N_t} P_i \mathbf{h}_i \mathbf{h}_i^* \right)^{-1} \mathbf{h}_k \quad (12)$$

$$R_{k, MMSE} = B \cdot \text{Log}_2 \left( 1 + \frac{P_k \mathbf{h}_k^* \mathbf{h}_k}{\left( N_0 \mathbf{I}_{N_r} + \sum_{i \neq k}^{N_t} P_i \mathbf{h}_i \mathbf{h}_i^* \right)} \right) \quad (13)$$

$$R_{sum, MMSE} = \sum_{k=1}^{N_t} R_{k, MMSE} \quad (14)$$

where  $B$  indicates the bandwidth.

### 2.3 Zero forcing- successive interference cancellation (ZF-SIC)

With the ZF-SIC equalizer, the user 1 to  $k - 1$ 's signals are presumed to be ideally decoded and canceled. Thus, the equalizer for user  $k$  is given by:

$$\mathbf{G}_{ZF-SIC} = \left[ \left( \mathbf{H}_k^H \mathbf{H}_k \right)^{-1} \mathbf{H}_k \right]_{:,1}, \quad (15)$$

where  $[ \cdot ]_{:,1}$  indicates taking the first column of a matrix.

With the assumption that ideal interference cancellation is realised at the receiver, the  $k$ -th users' signals and the achievable rates can be expressed as:

$$SINR_{k, ZF-SIC} = \frac{P_k}{N_0 \|\mathbf{G}_k\|^2} \quad (16)$$

$$R_{k, ZF-SIC} = B \cdot \text{Log}_2 \left( 1 + \frac{P_k}{N_0 \|\mathbf{G}_k\|^2} \right) \quad (17)$$

where  $B$  indicates the bandwidth.

### 2.4 Minimum mean square error- successive interference cancellation (MMSE-SIC)

With the MMSE-SIC equalizer, the user 1 to  $k - 1$ 's signals are assumed to be ideally decoded and treated as noise. Thus, the equalizer for user  $k$  is given by:

$$\mathbf{G}_{MMSE-SIC} = p_k \left( N_0 \mathbf{I}_{N_r} + \sum_{j=k}^N P_j \mathbf{h}_j \mathbf{h}_j^* \right)^{-1} \mathbf{h}_k \quad (18)$$

Assuming ideal interference cancellation at the receiver, the  $k$ -th users' signals and the achievable rates can be expressed as:

$$SINR_{k, MMSE-SIC} = P_k \mathbf{h}_k^H \left( N_0 \mathbf{I}_{N_r} + \sum_{j=k+1}^K P_j \mathbf{h}_j \mathbf{h}_j^H \right)^{-1} \mathbf{h}_k \quad (19)$$

$$R_{k, MMSE-SIC} = B \cdot \log \left( 1 + P_k \mathbf{h}_k^H \left( N_0 \mathbf{I}_{N_r} + \sum_{j=k+1}^K P_j \mathbf{h}_j \mathbf{h}_j^H \right)^{-1} \mathbf{h}_k \right) \quad (20)$$

where  $B$  indicates the bandwidth.



## B. Power Consumption Model

A total power consumption model of [6-8], which takes both transmit and receive antennas into account is adopted in this chapter. The power model is evaluated as a function of two main components, namely:

- (i) Circuit blocks power consumption,  $P_C$
- (ii) Amplifier power consumption,  $P_{PA}$

The Circuit blocks power consumption,  $P_C$  is given by:

$$P_C = N_t P_t + 2P_{sym} + N_r P_r \quad (21)$$

Such that,

$$P_t = (P_{DAC} + P_{MOD} + P_{filt} + P_{mix}), \text{ and}$$

$$P_r = (P_{mix} + P_{LNA} + P_{filr} + P_{IFA} + P_{MeMOD} + P_{ADC})$$

where:

$N_t$ : number of transmit antennas

$N_r$ : number of receive antennas

$P_{DAC}$ : digital-to-analog converter power consumption.

$P_{MOD}$ : modulator power consumption.

$P_{filt}$ : active filter power consumption at the transmitter.

$P_{mix}$ : mixer power consumption.

$P_{sym}$ : frequency synthesizer power consumption.

$P_{LNA}$ : low-noise amplifier power consumption.

$P_{filr}$ : active filter power consumption at the receiver.

$P_{IFA}$ : intermediate frequency amplifier power consumption.

$P_{DeMOD}$ : demodulator power consumption.

$P_{ADC}$ : analog-to-digital converter power consumption.

The amplifier power consumption,  $P_{PA}$  is given by:

$$P_{PA} = \frac{\xi P_{out}}{\eta} \quad (22)$$

Such that,

$$\xi = 3 \cdot \frac{(M - 2\sqrt{M} + 1)}{M - 1}, \text{ and}$$

$$P_{out} = \left( E_b \cdot R_b \frac{(4\pi)^2}{g_t g_r \lambda^2} \cdot d^n \cdot M_t \cdot N_f \right)$$

where:

$\eta$ : drain efficiency.

$M$ : constellation size.

$R_b$ : Bit rate.

$E_b$ : transmission energy per bit.

$g_t$ : transmit antenna gain

$g_r$ : receive antenna gain

$\lambda$ : carrier wavelength

$M_t$ : link gain

$N_f$ : receiver noise figure

$n$ : propagation exponent

$d$ : transmit-receiver communication distance



For the energy efficiency model, the Energy Throughput Ratio (ETR) of [9-11] is adopted. It is given by:

$$ETR = \frac{P_{tr}}{R_k^2} \quad (23)$$

where  $T_p$  is the throughput which represent the user achievable rates,  $R_k$  and  $P_{tr} = (P_{PA} + P_C)$ , represents total transmit-receive power consumption.

### Simulation Example, Results and Analysis

This section present simulations parameters as revealed in table 1 and results to verify the various models obtained in Section V for ZF, MMSE, ZF-SIC and MMSE-SIC processing schemes. We provide simulation results obtained using Matlab 2015a software considering perfect CSI, in single-cell scenarios.

**Table 1:** Simulation parameter and Value

| Parameter  | Value         |
|--|---------------|
| Number of transmit antennas, $N_t$                             | [10, 20, 30]  |
| Number of receive antennas, $N_r$                              | [10, 20, 30]  |
| Digital-to-analog converter power consumption, $P_{DAC}$ :     | 40mW          |
| Modulator power consumption, $P_{MOD}$                         | 30mW          |
| Active filter power consumption at the transmitter, $P_{firt}$ | 2.5mW         |
| Mixer power consumption, $P_{mix}$                             | 30.3mW        |
| Frequency synthesizer power consumption, $P_{sym}$             | 50mW          |
| Low-noise amplifier power consumption, $P_{LNA}$               |               |
| Active filter power consumption at the receiver, $P_{fir}$     | 2.5mW         |
| Intermediate frequency amplifier power consumption, $P_{IFA}$  | 20mW          |
| Demodulator power consumption, $P_{DeMOD}$                     | 30mW          |
| Analog-to-digital converter power consumption, $P_{ADC}$       | 40mW          |
| Drain efficiency, $\eta$                                       | 0.35          |
| Constellation size, $M$  | 2             |
| Bit rate, $R_b$  | 10kbits       |
| Transmit antenna gain, $g_t$                                   | 17.5          |
| Receive antenna gain, $g_r$                                    | 1             |
| Carrier frequency, $f$   | 2600MHz       |
| Link gain, $M_l$   | 40dB          |
| Receiver noise figure, $N_f$ :                                 | 10dB          |
| Propagation exponent, $n$                                      | [2, 2.7, 4.2] |
| Transmit-receiver communication distance, $d$                  | [1,30m]       |

Figures 3 to 5 presents the best-case sum achievable user capacity and energy efficiency versus SNR transmission requirement at the UE terminals. The plots reveals that achievable data transmission capacity by users improves with increasing SNR transmission condition at the receivers. That is, all the graphs show that best-case sum achievable user capacity gradually improves for all the schemes upon increment in the minimum SNR requirement at the UE terminals. Particularly, it can be noticed from the graphs that the nonlinear MMSE-SIC and ZF-SIC schemes consistently outperforms the linear ones (that is MMSE and ZF) schemes in terms of sum achievable user capacity. This can be justified by the fact that the linear channel equalization schemes does not fully suppress interference among users for enhanced data transmission compared to nonlinear schemes which exploit extra signal processing techniques by means of SIC to provide improved user information transmission thereby delivering higher performance gains at the receivers. For instance, at SNR = 10 dB in Figure 5 , the sum achievable user capacity attained by the MMSE-SIC and ZF-MMSE schemes stood at 153 and 150 compared to MMSE and ZF which stood at 100 and 50 respectively.



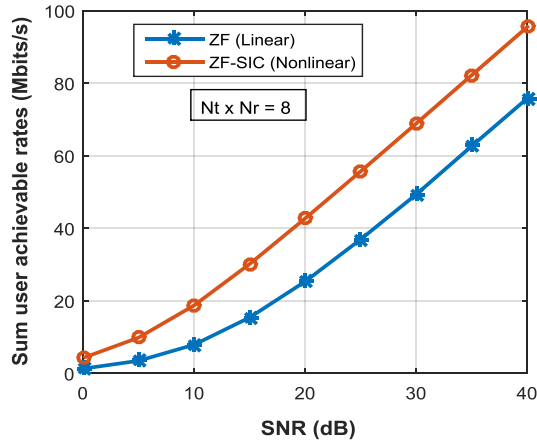


Figure 3a: User achievable rates versus SNR for ZF and ZF-SIC with  $N_t \times N_r = 10$

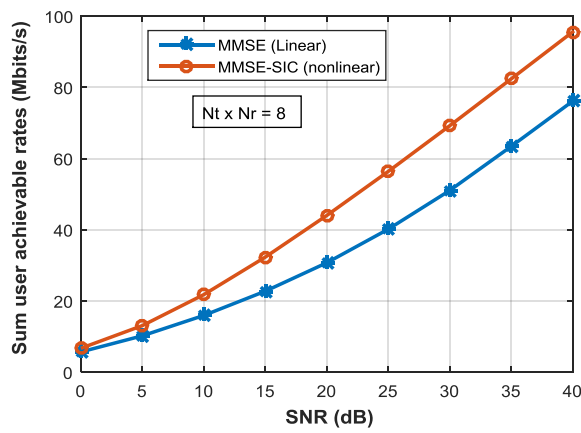


Figure 3b: User achievable rates versus SNR for ZF and ZF-SIC with  $N_t \times N_r = 10$

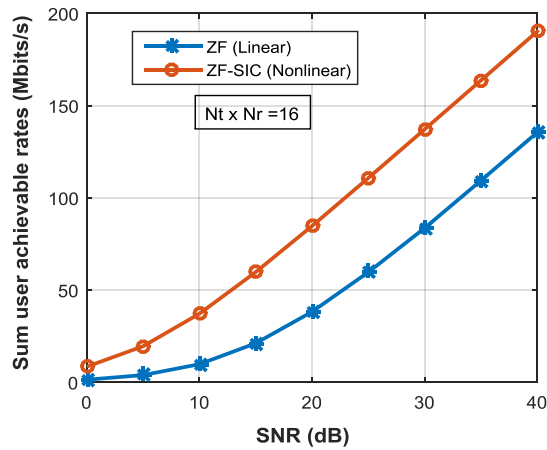


Figure 4a: User achievable rates versus SNR for MMSE and MMSE-SIC with  $N_t \times N_r = 10$

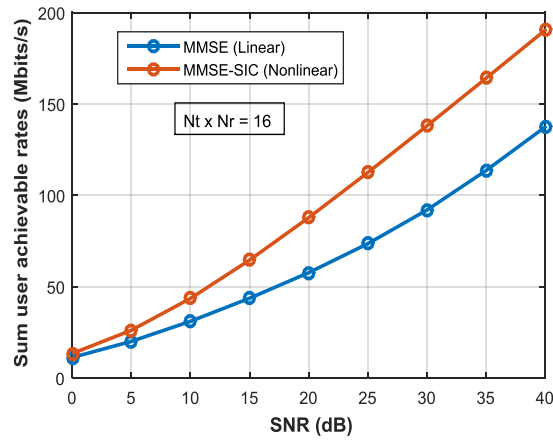


Figure 4b: User achievable rates versus SNR for MMSE and MMSE-SIC with  $N_t \times N_r = 20$

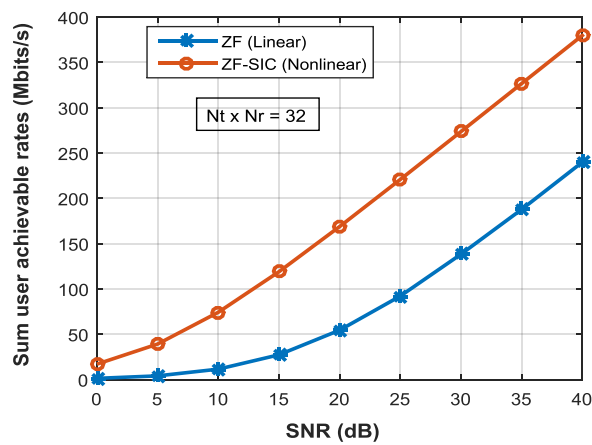


Figure 3c: User achievable rates versus SNR for ZF and ZF-SIC with  $N_t \times N_r = 30$

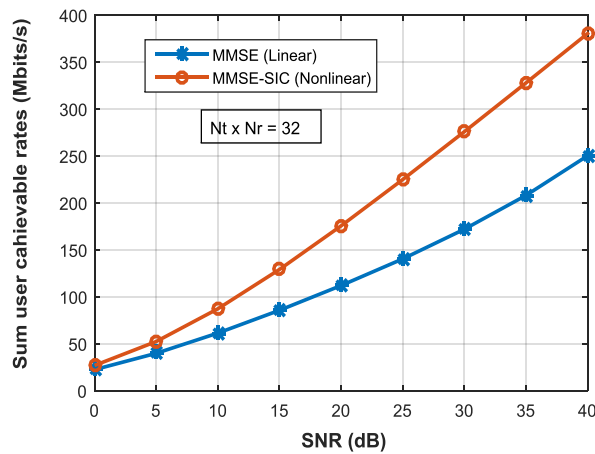


Figure 4c: User achievable rates versus SNR for MMSE and MMSE-SIC with  $N_t \times N_r = 3$



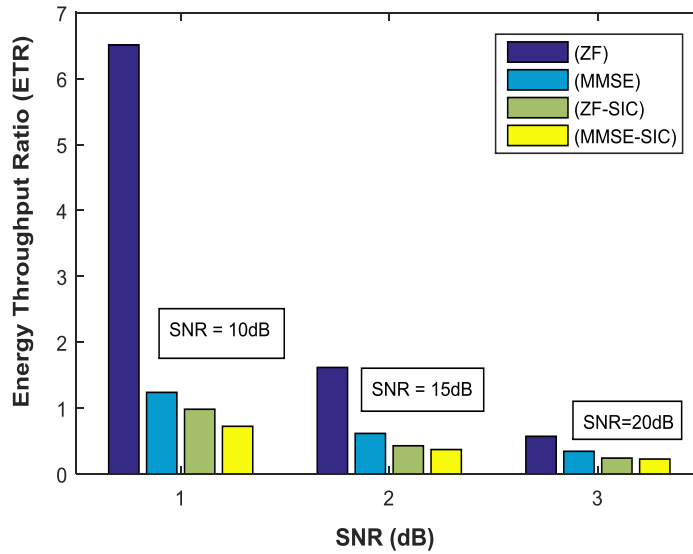


Figure 5: Energy throughput Ratio performance of ZF, MMSE, ZF-SIC and MMSE-SIC for SNR fixed at 10dB, 15dB and 20dB.

The total ETR performance for various received antenna number, i.e.  $N_r = 10$ ,  $N_r = 20$ , and  $N_r = 30$  is presented in Figure 6. As expected, the presented results show that higher received antenna number produces lower ETR values, thus saving total energy consumption at the user terminals.

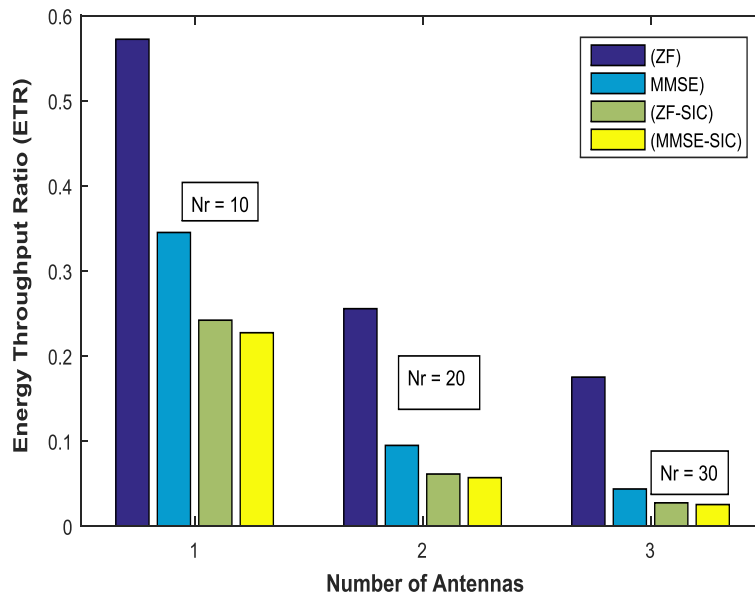


Figure 6: Energy throughput Ratio performance of ZF, MMSE, ZF-SIC and MMSE-SIC for SNR fixed at 10dB, 15dB and 20dB.

The total ETR performance for various user number, i.e.  $k = 10$ ,  $k = 20$ , and  $k = 30$  is presented in Figure 7. Contrary results in figure 7, the displaced graph show that higher user number produces higher ETR values, thus leading to higher total energy consumption at the user terminals. This results simply implies that higher user number, the more interference and higher energy consumption.

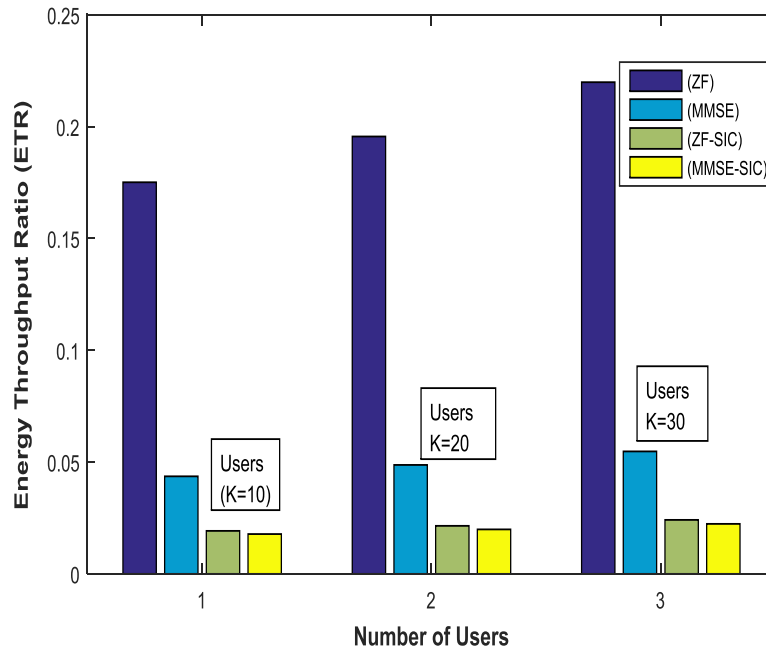


Figure 7: Energy throughput Ratio performance of ZF, MMSE, ZF-SIC and MMSE-SIC for  $N_r \times N_r$  fixed at 10, 20 and 30

The chart in figure 8 is displayed to show the impact of different propagation environment on total ETR performance. In the plot,  $n = 2$ ,  $n = 2.7$  and  $n = 4.2$  stand for free space, suburban and urban environments. Similar to figure 7, the presented result reveals that urban environments with  $n = 4.2$  consume more energy, followed by suburban environment, with  $n = 2.7$  and then lower with  $n = 2$  in free space. More importantly, it can be noticed from the graphs of Figures 5 to 7 that the nonlinear MMSE-SIC and ZF-SIC schemes consistently outperforms the linear MMSE and ZF schemes in terms of energy efficiency. The results also clearly reveal that the SIC-based receivers are better than that of the linear receivers, which can be attributed to their higher interference suppression power. Centered on this, it is judicious to consider non-linear SIC based receivers, which can enhance power efficiency at user equipment terminals.

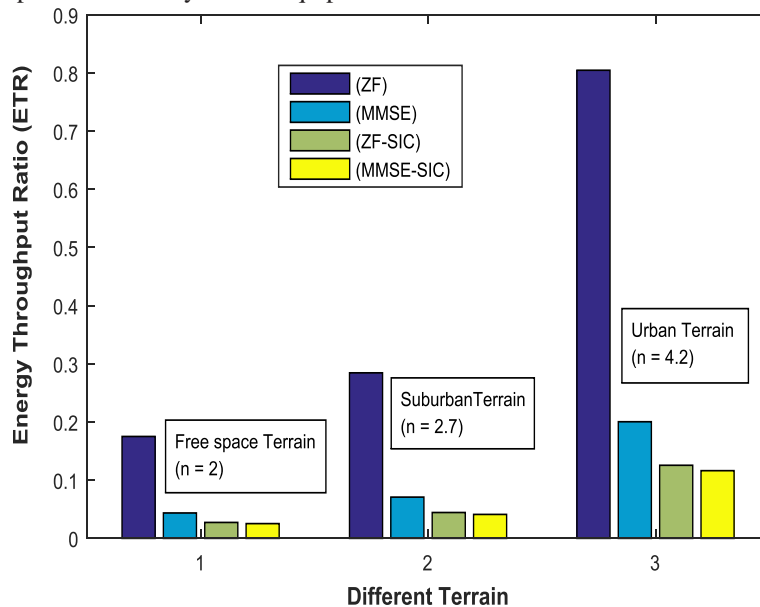


Figure 9: Energy throughput Ratio performance of ZF, MMSE, ZF-SIC and MMSE-SIC for  $n$  fixed at 2, 2.7 and 4.2

## Conclusion

The demand for multimedia services in mobile broadband communication systems is expected to increase a hundred or thousand fold by 2020 for future wireless networks such as the 5G, owing to the requirement of super high speed data services, massive connectivity, low latency, high spectral and efficiency. A number of prominent techniques such as massive or large-scale multiple-input multiple-output (MIMO), millimeter (mm) wave, spatial modulation (SM) and non-orthogonal multiple access (NOMA) have been look into in recent years to meet the aforementioned requirements. In this chapter, four distinctive channel equalization algorithms which includes the MMSE, ZF, MMSE-SIC and ZF-SIC, are studied extensively through computer simulations to examine their impact on energy efficiency and data transmission capacity of large scale MIMO system. Our results indicates that the nonlinear MMSE-SIC and ZF-SIC equalization schemes outperform the linear ones by 20 to 25% in terms user achievable rates and energy efficiency. This can be justified by the fact that the linear channel equalization schemes does not fully suppress interference among users for enhanced data transmission compared to nonlinear schemes which exploit extra signal processing techniques by means of SIC to provide improved user information transmission thereby delivering higher performance gains at the receivers.

## References

- [1]. J. Isabona, Harnessing Telecommunications Revolution in Nigeria: A Case Study.” *Wireless and Mobile Technologies* Vol.1, no. 1 (2013): 20-24. doi: 10.12691/wmt-1-1-4.
- [2]. A. S. Mindaudu<sup>1</sup> and A. M. Miyim, BER Performance of MPSK and MQAM in 2x2 Alamouti MIMO Systems, *international Journal of Information Sciences and Techniques (IJIST)*, pp. 1-10, Vol.2, No.5, September 2012.
- [3]. Liu, Shoumin, Soft-decision equalization techniques for frequency selective MIMO channels, Dissertation, Michigan Technological University, 2005. <http://digitalcommons.mtu.edu/etds/62>
- [4]. V. Dawar and R. Sharma, Reduction in Bit Error Rate from Various Equalization Techniques for MIMO Technology, *International Journal of Soft Computing and Engineering (IJSCE)*, Vol. 2, Issue-4, pp. 66-70, 2012.
- [5]. E. Bjornson, L. Sanguinetti, J. Hoydis, and M. Debbah, “Optimal design of energy-efficient multi-user MIMO systems: Is massive MIMO the answer?” *IEEE Trans. Wireless Commun.*, vol. 14, no. 6, pp. 3059–3075, Jun. 2015.
- [6]. Maha Ben Zid, Kosai Raouf, and Ammar Bouallegue, MIMO Spectral Efficiency over Energy Consumption Requirements: Application to WSNs, *Int. J. Communications, Network and System Sciences*, Vol.5, pp.121-129, 2012.
- [7]. S. Cui, A. J. Goldsmith and A. Bahai, “Energy-Efficiency of MIMO and Cooperative MIMO Techniques in Sensor Networks,” *IEEE Journal on Selected Areas in Communications*, Vol. 22, No. 6, 2004, pp. 1089-1098. doi:10.1109/JSAC.2004.830916.
- [8]. S. Cui, A. J. Goldsmith and A. Bahai, “Energy-Constrained Modulation Optimization,” *IEEE Transactions on Communications*, Vol. 4, No. 5, 2005, pp. 2349-2360.
- [9]. Bohli, A & Bouallegue. R, (2014) "Energy Efficiency in Heterogeneous Wireless Networks Using Cognitive Monitoring Strategy," *Modelling Symposium (EMS), 2014 European, Pisa, 2014*, pp. 387-391, 2014
- [10]. J. Isabona, and V. M. Srivastava, Energy-efficient Communication in Large Scale Antenna Systems: Impact of Variable User Capacity and Number of Transmission Antennas, *Progress in Electromagnetic Research*, vol. 58, pp.205-213, 2017.
- [11]. J. Isabona and V. M. Srivastava, “Downlink massive MIMO systems: Achievable sum rates and energy efficiency perspective for future 5G systems” *Wireless Personal Communication*, Vol. 95, pp. No. 4, pp. 3955–3968, 2017.

