



Using Matlab-Simulink for Solving Differential Equations

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Abstract Purpose of this project is to solve the multivariable differential equation with any order by using Matlab-Simulink. The analogue computer can be simulated by using Matlab-Simulink for different types of differential equation such as (1st, 2nd, 3rd, etc...) we can be solved these types of differential equation by using library of Simulink like (integrator, differentiator, sum, clock, and gain) to workspace, this project utilized MATLAB and Simulink facilities to solve the differently of differential equations. Two methods were used to solve general differential equations (i.e. MATLAB and Simulink). One can change the coefficient parameters and see the results afterwards for each case graphically.

Keywords Differential equations, Matlab-Simulink

1. Introduction

In the last two decades many types of software are developed in the design and simulation of solving the differential equations, but one of the most powerful software is MATLAB. This is due to its great capabilities in numerical differential, integral equations, and simplicity in use.

The aim of this work is to build a MATLAB/SIMULINK for the design and simulation to solve the differential equation, the contents of this project are divided as follows:

First part contains the general introduction and the aim of the project. Second part is a brief introduction to differential equation while part three contains the cases studies and results, the software design steps and case studies to test the validity of the designed software unit.

The aim of this job is to design software unit to solve the differential equation using MATLAB/SIMULINK to simulate the design of different cases studies. These software unit lead to a significant in time and effort spent during the design and simulate of these case studies [1].

2. Order of the Differential Equations

Common ordinary differential conditions (ODE) of the first order can be considered as essential units in the investigation of higher conditions and their properties. Standard properties study and good properties in the differential equations called the settled coefficient equations, which frame the premise of the coefficient of the second degree factor and which are produced later and have various applications.

The need to ponder ordinary differential equations is given by introducing various regular issues that prompt diverse kinds of (DE). The primary application incorporates the choice of orthogonal ways. A regular case of orthogonal ways is created in the two-dimensional warmth circulations of the settled state, where one parcel of ways relates to the lines where the temperature is settled, while the other gathering identifies with lines along which warm streams. Diverse cases outlines considered are the radioactive spoil of a substance, the calculated condition and its association with populace development, discourage motions, the state of a suspended electrical cable, and the twisting of shafts. The section begins by characterizing an n^{th} request common differential condition, of which a first request condition is a unique case. Different essential terms are characterized, and the physical noteworthiness of introductory and limit conditions for differential conditions are presented and



clarified [2]. The geometrical translation of the subordinate dy/dx as the slant of the bend is utilized to build up the idea of the course field related with the main request condition $dy/dx = f(x, y)$. This idea is especially helpful as it prompts a geometrical view demonstrating the subjective conduct of all arrangements of the differential condition. It will be seen later that the thought fundamental a bearing field frames the premise of the straightforward Euler strategy for the numerical arrangement of an underlying worth issue [3].

An (ODE) is a condition that proportional the function $y(x)$ to some of its subsidiaries $y^f(x) = d^f/dx^f$. It is normal to know as x the autonomous variable and y the reliant variable, the most broad conventional DE compose as:

$$F(x, y, y^1, y^2, \dots, y^n) = 0 \quad (1)$$

The number n in equation (1) is known as the order of the ODE, and it is the request of the most elevated subsidiary of y that happens in the equation. One type of ODE's of specific significance in building and science due to their recurrence of event and the broad diagnostic techniques that are accessible for their answer are the linear ODE.

The most broad n^{th} order linear differential equation can be composed as:

$$a_0(x) \frac{d^n y}{dx^n} + a_1(x) \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1}(x) \frac{dy}{dx} + a_n(x) y = f(x) \quad (2)$$

$a_0(x) \neq 0$ what's more, we will view it as characterized over some interim $a \leq x \leq b$. Functions $a_0(x), a_1(x), \dots, a_n(x)$, called equation coefficients, which are known functions, known function and $f(x)$ are known as the non-homogeneous term. The name constraining function is likewise in some cases given to $f(x)$, because in light of the fact that in applications it speaks to the impact of outside information that drives a physical framework spoke to by the DE. Equation (2) is called homogeneous when $f(x) = 0$.

It will be seen later that the arrangement of the non-homogeneous equation is connected in a key way to the arrangement of its related homogeneous condition [4].

When at least one of the coefficients of equation (2) rely upon x , it is known as a variable coefficient equation. Easier than variable coefficient direct equations, yet at the same time of impressive significance, are the straight equations in which the coefficients are the constants a_0, a_1, \dots, a_n , so that equations (2) becomes:

$$a_0(x) \frac{d^n y}{dx^n} + a_1(x) \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1}(x) \frac{dy}{dx} + a_n(x) y = f(x) \quad \text{for } a \leq x \leq b \quad (3)$$

Equations of this write are called constant coefficient linear equations. In the event that the interim $a \leq x \leq b$ on which equations (2) and (3) are characterized isn't indicated.

An ordinary differential equation that isn't direct is said to be nonlinear. Non- nonlinear equation and degree linearity emerges in ODE as a result of the event of a nonlinear function of the needy parameter y that occasionally happens as a power or a radical. The terms homogeneous and non-homogeneous have no significance for nonlinear equations [5].

A term that is additionally being used, chiefly as a sign of the intricacy normal of an answer, is the level of an equation. The degree is the best vitality to which the most astounding request subordinate in the DE is raised after the radicals have.

3. The types of differential equation:

a) **The 1st order D.E:** $\dot{y} + 2xy = \sin x$

This equation is a **linear variable** coefficient **non-homogeneous** first order equation.

(b) **The 2nd order D.E:** $(1 - x^2) - 2xy\dot{y} + 6y = 0$ with $-1 < x < 1$

This equation is a **linear variable** coefficient **homogeneous** second order equation.

(c) **The 2nd order D.E:** $\ddot{y} + ay + by = \sin \omega t$ with $\omega = \text{constant}$

This equation is a **linear constant** coefficient **non-homogeneous** second order equation.

(d) **The 2nd order D.E:** $\ddot{\theta} + k \sin \theta = 0$ with $k = \text{constant}$

This is a **nonlinear** second order equation because **θ happens nonlinearly** in the function $\sin \theta$.

(e) **The 2nd order D.E:** $k\ddot{y} = f(x)[1 + (\dot{y})^2]^{2/3}$ with $k > 0$ a constant

This is a **nonlinear** second order equation of **degree 2** involving a power and a radical.



A solution of an ordinary differential equation is a function $y = \Phi(x)$ that when substituted into the equation, makes it indistinguishably zero over the interim on which the equation is characterized. An answer of an n^{th} order equation that contains n subjective constants is known as the general arrangement of the equation. In the event that the discretionary general and particular solutions and integral curves constants in the general arrangement are appointed particular esteems, the outcome is known as a particular solution of the equation. For clear reasons the arrangement solution of an ordinary differential equation is also called an integral curve. An answer that can't be gotten from the general arrangement for any selection of its self-assertive constants is known as a singular solution. For the situation singular solution of linear equations every single conceivable arrangement of the equation can be gotten from the general arrangement, so linear equations have no particular arrangements. Nonlinear equations have a more convoluted structure that regularly permits the presence of at least one solitary arrangement [6].

(a) The general arrangement of the linear constant coefficient non-homogeneous equation:

$$\ddot{y} - 4y = x$$

Is $y = Ae^{2x} + Be^{-2x} - x/4$, where A and B are discretionary constants. This is effectively checked, in light of the fact that substituting for y in the equation prompts the character x . (b).

The nonlinear equation:

$$\dot{y}^2 + y^2 = 1$$

Has the general arrangement $y = \sin(x+A)$. Be that as it may, $y = \pm 1$ are likewise observed to be solutions, in spite of the fact that as these can't be gotten from the general answer for any decision of A , they are particular arrangements.

The linear equation (2) is frequently composed in the more smaller frame:

$$L[y] = f(x) \tag{4}$$

$$L[.] \equiv a_0(x) \frac{d^n}{dx^n} + a_1(x) \frac{d^{n-1}}{dx^{n-1}} + \dots + a_{n-1}(x) \frac{d}{dx} + a_n(x) \tag{5}$$

With coefficients that might be elements of x . Just when L follows up on an n times differentiable function does it deliver a function. Equation (2) is called direct on the grounds that if y_1 and y_2 are any two arrangements of the homogeneous type of the equation $L[y] = 0$, the straight mix.

$$y = C_1y_1 + C_2y_2$$

Where C_1 and C_2 are constants is additionally an answer. Regarding the differential administrator L this property moves toward becoming:

$$L[C_1y_1 + C_2y_2] = C_1L[y_1] + C_2L[y_2]$$

What's more, it takes after straightforwardly from the linearity of the separation activity, on the grounds that:

$$\frac{d^m}{dx^m}(y_1 + y_2) = \frac{d^m y_1}{dx^m} + \frac{d^m y_2}{dx^m}$$

For $m = 0, 1, \dots, n$, with $d^0y/dx^0 = y$. On the off chance that $y_1(x), y_2(x), \dots, y_m(x)$ are arrangements of the n^{th} request homogeneous equation $L[y] = 0$, with $m \leq n$ and C_1, C_2, \dots, C_m arbitrary constants, the linear combination.

$$y(x) = C_1y_1(x) + C_2y_2(x) + \dots + C_my_m(x)$$

Is known as a direct superposition of the m arrangements and it is additionally an answer of the homogeneous equation. Later we will characterize the straight autonomy of an arrangement of functions over an interim and demonstrate that the homogeneous type of Eq. (2) has absolutely n directly free arrangements $y_1(x), y_2(x), \dots, y_n(x)$, and that its general arrangement is:

$$y_c(x) = C_1y_1(x) + C_2y_2(x) + \dots + C_ny_n(x) \tag{6}$$

where C_1, C_2, \dots, C_n are self-assertive constants. This general arrangement of the homogeneous form of equation (2) is called the complementary function or the complementary solution of equation (2). A function $y_p(x)$ that is a solution of the non-homogeneous complementary solution, particular integral and finish arrangement condition (2) yet contains no subjective constants is known as a particular integral of (2). The total arrangement $y(x)$ of equation (2) is:

$$y(x) = y_c(x) + y_p(x) \quad (7)$$

In utilizations of customary differential equations the estimations of the self-assertive constants in particular issues are acquired by picking them, so the arrangement fulfills helper conditions that distinguish a specific issue. Helper conditions determined at a solitary point $x = a$, are called beginning conditions, since x frequently speaks to the time with the goal that states of this compose depict how the arrangement begins. An underlying quality issue (IVP) includes finding an answer of a differential equation that fulfills endorsed introductory conditions. An alternate sort of issue rises when the helper conditions are determined at two unique focuses $x = a$ and $x = b$.

States of this write are called boundary conditions, since in such issues x for the most part speaks to a space variable, and boundary and initial conditions the arrangement is required to be resolved between two limits situated at $x = a$ and $x = b$ where limit conditions are recommended. A limit esteem issue (BVP) involves finding an answer of a differential equation that fulfills endorsed limit conditions [7, 8].

4. Software Development & Case Studies

This area portrays how to speak to common differential equations as frameworks for the MATLAB standard differential equation (ODE) solvers. The MATLAB ODE solvers are intended to deal with customary differential equations. These are differential equations containing at least one subordinates of a needy variable y as for a solitary autonomous variable t , for the most part alluded to as time. The subordinate of y as for t is indicated as y' , the second subsidiary as y'' , et cetera. Frequently $y(t)$ is a vector, having components y_1, y_2, \dots, y_n . Tributes regularly include various ward factors, and in addition subsidiaries of request higher than one. To utilize the MATLAB ODE solvers, you should revise such equations as a comparable arrangement of first-order differential equations as far as a vector y and its first subordinate.

$$y' = f(t, y)$$

When you speak to the equation along these lines, you can code it as an ODE M-record that a MATLAB ODE solver can utilize.

For the most part there are numerous functions $y(t)$ that fulfill a given ODE, and extra data is important to determine the arrangement of intrigue. In an underlying worth issue, the arrangement of intrigue has a particular starting condition, that is, y is equivalent to y_0 at a given beginning time t_0 . An underlying quality issue for an ODE is at that point.

$$y' = f(t, y)$$

$$y(t_0) = y_0$$

On the off chance that the function $f(t, y)$ is satisfactorily smooth, this issue has one and just a single arrangement. For the most part there is no expository articulation for the arrangement, so it is important to rough $y(t)$ by numerical means, for example, one of the solvers of the MATLAB ODE suite [9, 10, 11].

Matlab has function to solve both initial-value and boundary-value problems. Several functions solve an n -th order initial-value problem of the shape:

$$\frac{dy^n(t)}{dt} = f(t, y(t), \dot{y}(t), \ddot{y}(t), \ddot{\ddot{y}}(t), \dots)$$

Of the shape the starting conditions $y(t_0) = y_0, \dot{y}(t_0) = \dot{y}_0, \ddot{y}(t_0) = \ddot{y}_0, \ddot{\ddot{y}}(t_0) = \ddot{\ddot{y}}_0$, and so on. Other functions also solve both linear second-order limit esteem issues. A Linear limit esteem issue is of the shape.

$$\ddot{y} = p(x) \frac{dy}{dx} q(x)y - r(x) = 0$$

Of the shape boundary conditions $y(a) = \alpha$ and $y(b) = \beta$. We have only placing constraints on the value of y at the endpoint. Here is an example of a nonlinear boundary-value problem:

$$\ddot{y} + [\dot{y}]^2 + y - \ln(x) = 0$$

Case (1):

Let's start by solving a simple first order DE.

$$dy/dt = y + t + 1 \text{ with initial condition } y(0) = 0$$

Note that the DE is of the form

$$dy/dt = f(t, y)$$

Where $f(t, y) = y + t + 1$.



First order initial value problem example

$$\dot{y}=y+t+1$$

(8)

We can now simulate this equation by using simulink model as shown in figure 1 below [12, 13]:

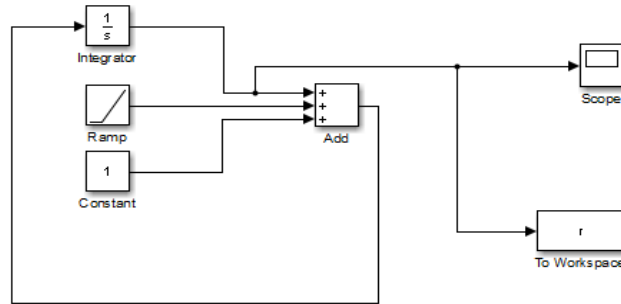


Figure 1: Matlab-Simulink for equation

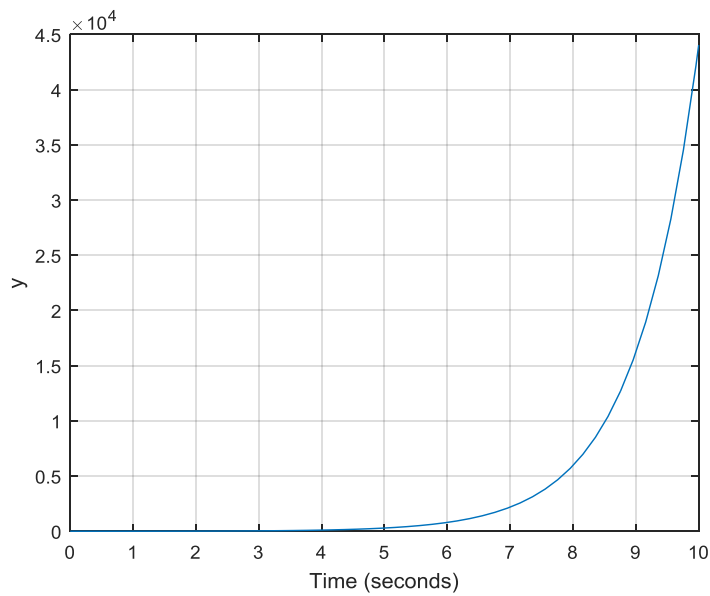


Figure 2: The result of the equation 8 model

Case (2):

The 2nd order differential equation

$$\ddot{y}=\dot{y}+y+4$$

(9)

We can now simulate the differential equation above by using Simulink model as shown in figure 3 below [12, 13]:

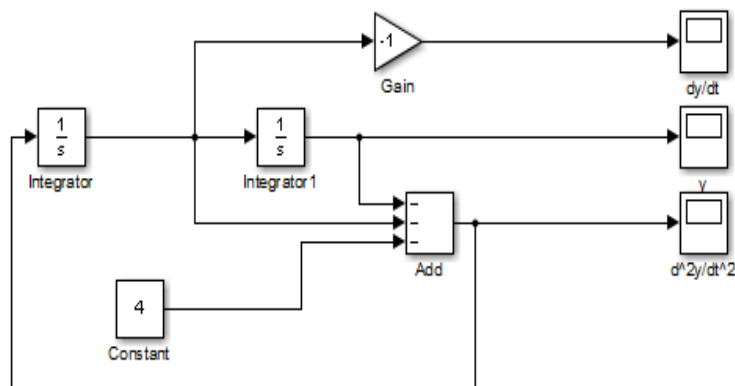


Figure 3: Matlab-Simulink module for equation 9

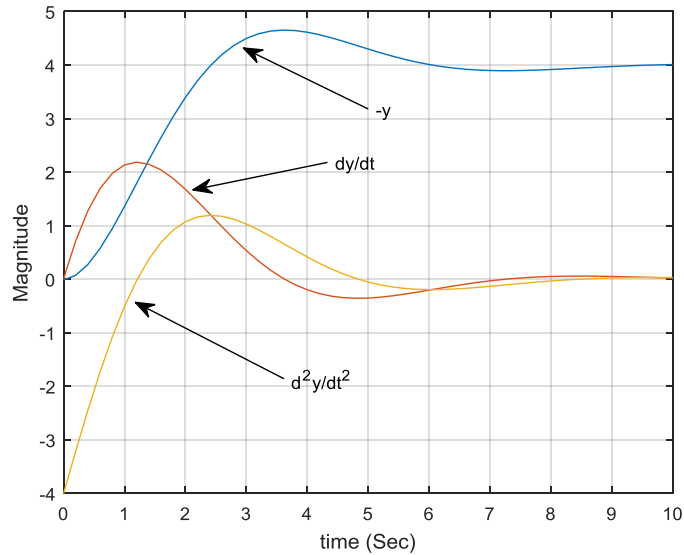


Figure 4: The result of the equation 9 model

Case (3):

The differential equations:

$$\begin{aligned} \dot{y} &= -\dot{y} - yx - 4 & (10) \\ \dot{x} &= x + 3 \end{aligned}$$

We can now simulate the differential equation above by using Matlab-Simulink model as shown below [12, 13]:

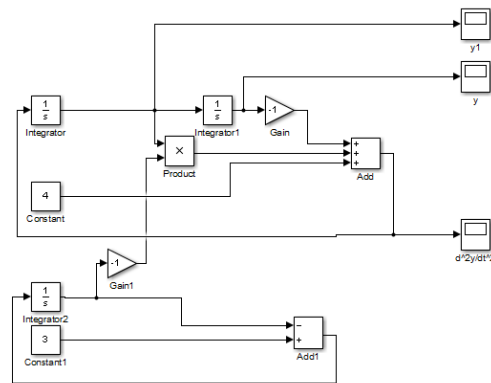


Figure 5: Matlab-Simulink module for equation 10

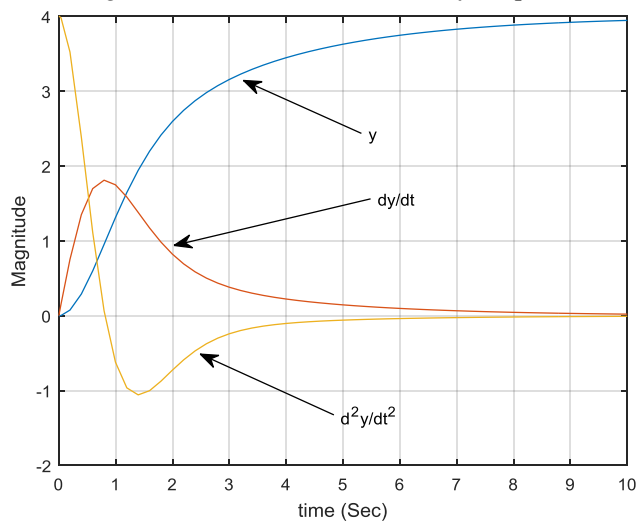


Figure 6: The result of the equation 10 model

5. Conclusions

Any system in our lives can be represented using differential equations, whether electrical or mechanical or anything else. In order to solve these equations, in other words, to solve these systems represented by the equation, we used Matlab-Semulink to represent these equations in the computer.

In this research, differential equations of different types were solved using Matlab-Semulink, as this program enables us to easily arrange the components to suit the equation and obtain the results very easily. We can easily change the components to suit any differential equation we want to solve by changing the parameters or adding or removing blocks from the system. DE of the first and second order were simulated and the results obtained are shown as curves in the research. These equations must be arranged so that we can represent them in the computer using Matlab-Semulink.

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