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A Mathematical Model on Kidnapping

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Abstract We construct a simple mathematical model on kidnapping. Its features mimic the dynamics of an infectious disease. It accounts for the interaction between kidnappers and susceptible humans leading to their abduction for the main purpose of ransom payment. The model consisting of a system of ordinary differential equations describes the evolution and propagation of kidnapping as a crime in human society. In what seems to be a new approach to mathematical criminology, the model divides the human population into susceptible humans, kidnappers and kidnapped victims. The analysis includes establishment of a 'crime propagation number', C_{p_n}, in which a $C_{p_n} < 1$ guarantees a kidnap free state that is locally and globally asymptotically stable. The analysis shows that reducing the recruitment rate of kidnappers and increasing the rescue rate without any causality on the part of kidnapped victims will lead to eradication of kidnapping in society.

Keywords Kidnapers, Kidnappees, Modelling, Crime propagation Number.

1. Introduction

Kidnapping is a violent crime and it occurs when a person without lawful authority physically abducts another person without that person's consent and with the intent to use the abduction for financial ransom, political objective or other nefarious objectives. Many people think of kidnapping as something that is very rare and not something that could possibly happen to them. The reality is that it is a huge risk, which can happen to anyone including people of any class range [3]. However some people may be more vulnerable than others. Kidnapping for ransom has been on the increase probably due to terrorism, drug addiction, cultism or gangsterism and the high level of corruption and insecurity in some countries. Kidnapping for ransom involves series of negotiations between the kidnapper(s) and the family of the victim [10]. Limiting the family's ability to pay reduces the frequency of the offence but opens the possibility of unintended consequences in terms of fatalities and duration of abduction [2]. To the best of our knowledge, none of the papers considers kidnapping as a transmission process depicting the dynamics of a compartmental model. Here we propose a simple mathematical model incorporating these features. Section 1 includes a brief introduction whereas the formulation of the model is given in section 2. In section 3, we present the model analysis followed by numerical simulations in section 4. The paper was rounded up in section 5 with a brief discussion and conclusion. We divide the human population into compartments of susceptible, kidnappers and kidnapees. State variables in the model are given in Table 1 and the movement between compartments is summarised in Figure 1, the individual pathways to be discussed below.

Table 1: The state variables in the model

State variable	Description
N	Total human population
K_s	Number of humans susceptible to kidnapping
K_{p_a}	Kidnappers population
K_{p_e}	Number of kidnapped victims (kidnappees)

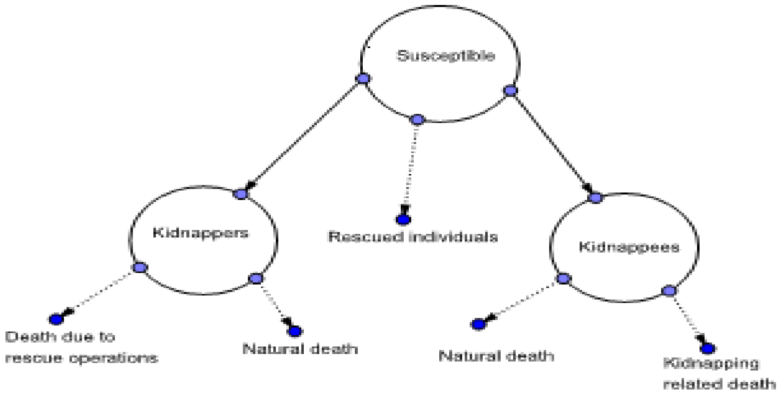


Figure 1: Schematic representation of kidnappers and susceptible individuals interaction model

2. The formulation of the model

The human population satisfies the equation

$$N = K_s + K_{p_a} + K_{p_e}.$$

Following some contact between kidnappers and susceptible humans people are either kidnapped or introduced into kidnapping. We assume that the contact is necessitated by the willingness of kidnappers to gain some benefits through ransom or susceptible humans being enticed by their kidnapper friends from the proceeds of kidnapping. Susceptible individuals progress into the K_{p_a} compartment at a rate $\gamma K_{p_a}\frac{K_s}{N}$, where $\frac{K_s}{N}$ is the probability that the contact is with a susceptible human and γ is the rate constant. We also assume a kidnapping rate constant β such that the transition rate, $K_{p_a}\frac{K_s}{N}$, of susceptible humans into the K_{p_e} class is proportional to the contact between kidnappers and susceptible humans. Security agents go for rescue operations in response to kidnap cases. This is done through the gathering of relevant information about the incidence including possible suspects and tracking of the negotiator. Since a rescued victim becomes susceptible again, we assume that recruitment into the susceptible class is through natural birth, $\lambda N$and rescued kidnapped victims, wK_{p_e}, where λ and w are per capita birth rate and rescue rate constant respectively. Rescue operations of kidnapped victims are carried out when kidnapping has taken place and the kidnapper is in contact with the kidnappee. Thus we assume that the death rate of kidnappers due to rescue operations, $a_1K_{p_a}K_{p_e}$and the kidnapping related death rate of kidnap victims,$a_2K_{p_a}K_{p_e}$are both proportional to the contact between kidnappers and kidnappees. Using the above assumptions, the system of equations are

$$\frac{dK_s}{dt} = \lambda N + wK_{p_e} - \gamma K_{p_a}\frac{K_s}{N} - \mu K_s - \beta K_{p_a}\frac{K_s}{N} \quad (2.1)$$

$$\frac{dK_{p_a}}{dt} = \gamma K_{p_a}\frac{K_s}{N} - \mu K_{p_a} - a_1K_{p_a}K_{p_e} \quad (2.2)$$

$$\frac{dK_{p_e}}{dt} = \beta K_{p_a}\frac{K_s}{N} - \mu K_{p_e} - a_2K_{p_a}K_{p_e} - wK_{p_e} \quad (2.3)$$

$$\frac{dN}{dt} = (\lambda - \mu)N - (a_1 + a_2)K_{p_a}K_{p_e} \quad (2.4)$$

where (2.4) is derived from adding $(2.1) - (2.3)$. We impose

$$t = 0, N = N_0$$

as initial human population.

2.1. Parameter values

All the model parameters are listed in Table 2 together with values taken from various sources. We note that the values for these parameters which have some regional variation may display some global semblance. A detailed work done in [6] on ransom kidnapping and its duration reveals that kidnapping, while ranging from 1 to 325 days average just less than 2 months in duration. We assume that a kidnappee remains in the kinappers` den for 60 days. In a research work on kidnapping and national security, [8] aver that Nigeria, despite ranking$11th$

position in the 12 top kidnapping countries 690 cases were recorded in 2009 and 2,0184 cases in 4 years. We therefore assume a kidnapping rate of 1.52 per day. Other parameter values are also obtained in this form. Especially [9] in [3] maintain that 64 percent of kidnapped victims are released with ransom payment, 18 percent without payment, 6 percent are rescued and 2 percent escape.

Table 2: Model parameters and their dimensions. Values marked with $(*)$ are assumed values and the rest are obtained from data

Parameters	**Description**	**Value**	**Unit**	**Source**
w	Rescue rate of kidnapped victims	0.098	day^{-1}	[6], [9]
γ	recruitment rate of kidnappers	$0.305 *$	day^{-1}	assumed
β	kidnapping rate	1.52	day^{-1}	[8]
μ	per capita death rate	0.0000356	day^{-1}	[11]
a_1	death rate of kidnappers due to operations by security agents	$2.35 \times 10^{-9} *$	$human^{-1}day^{-1}$	assumed
λ	per capita birth rate	0.000104	day^{-1}	[11]
a_2	kidnapping induced death rate of kidnapped victims	$1.18 \times 10^{-9} *$	$human^{-1}day^{-1}$	assumed

Thus using the results of [6] and [9], we assume an average rescue rate of 0.098. However, we have made some assumptions on parameters that do not seem to have well defined values. Data on recruitment of kidnappers are not readily available but it could take few people with arms to kidnap a reasonable number of victims for ransom. Thus we assume that kidnappers recruitment rate constant γ is proportional to kidnapping rate constant β with a constant of proportionality, 0.02.

2.2. Nondimensionalisation

Since the variable N is the sum of the relevant compartment values, it is convenient to re-express the compartment values as population fractions using

$$\hat{S} = \frac{K_s}{N}, \ \hat{A} = \frac{K_{pa}}{N}, \ \hat{V} = \frac{K_{pe}}{N}$$

so that

$$\hat{S} + \hat{A} + \hat{V} = 1.$$

The time derivatives for the variables will become, using variable S as an example

$$\frac{dN\hat{S}}{dt} = N\frac{d\hat{S}}{dt} + \hat{S}\frac{dN}{dt} = N\frac{d\hat{S}}{dt} + \left(\lambda - \mu - (a_1 + a_2)\hat{A}\hat{V}N\right)N\hat{S},$$

Kidnappers are mainly encouraged through negotiations and ransom payment. Rescue of victims by security agents without payment of ransom may make kidnapping less attractive. The timescale in which a victim remains in the kidnappers' den before rescue is very important determinant of the safety and survival of the victim. Hence we scale time with the rescue parameter w, and write

$$t = \frac{\hat{t}}{w}$$

Assuming that N_0 is the initial population of humans, we write

$$N = N_0\hat{N}$$

and by defining the following dimensionless parameters:

$$b = \frac{\lambda}{w}, \ c = \frac{\gamma}{w}, \ e = \frac{\beta}{w}, f = \frac{a_1 N_0}{w}, g = \frac{a_2 N_0}{w}, h = \frac{\mu}{w},$$

and by substituting these new parameters into $(2.1) - (2.4)$ and dropping the hats for clarity we get

$$\frac{dS}{dt} = b(1 - S) + V - (c + e)SA + (f + g)AVN, \quad (2.6)$$

$$\frac{dA}{dt} = cSA - bA - fAVN + (f + g)A^2VN, \quad (2.7)$$

$$\frac{dV}{dt} = eSA - (1 + b)V - gAVN + (f + g)AV^2N, \quad (2.8)$$

$$\frac{dN}{dt} = (b - h)N - (f - g)AVN^2. \quad (2.9)$$

The dimensionless parameters and their values are given in Table 3.

Table 3: List of dimensionless parameters and their definitions in terms of the dimensional parameter values

Dimensional form	Nondimensional parameter	Value
$\frac{\lambda}{w}$	b	0.0163
$\frac{\gamma}{w}$	c	0.31
$\frac{\beta}{w}$	e	15.5
$\frac{a_1 N_0}{w}$	f	0.024
$\frac{a_2 N_0}{w}$	g	0.012
$\frac{\mu}{w}$	h	0.00036

3. Model Analysis

3.1. Establishing the Crime Propagation Number, C_{p_n}

We define a new parameter, crime propagation number, denoted by C_{p_n} as the expected number of secondary kidnap cases that would arise from the introduction of a single kidnapper into a fully susceptible or kidnap-free population. The method of next generation matrix used in [4], [5] in determining the basic reproduction number of an infectious disease may be used in deriving C_{p_n}. By considering a small perturbation of the kidnap-free state ($S = 1,\ A = 0,\ V = 0$), we investigate the linearised system expressed in the form

$$R' = FR - VR, \tag{3.1}$$

Where

$$R' = \frac{dR}{dt}, F = \begin{bmatrix} c & 0 \\ e & 0 \end{bmatrix}, V = \begin{bmatrix} b & 0 \\ 0 & 1+b \end{bmatrix}, R = \begin{bmatrix} V \\ A \end{bmatrix}.$$

Here, FR represents the matrix of new kidnappers and kidnap cases, VR is the transition of these cases between compartments and R the "reservoir of kidnapping''.

This method assumes that there is a non-negative matrix $G = FV^{-1}$ that guarantees a unique, positive and real eigenvalue strictly greater than all others. Computing the inverse of V yields

$$G = \frac{1}{b(1+b)} \begin{bmatrix} c(1+b) & 0 \\ e(1+b) & 0 \end{bmatrix} \tag{3.2}$$

The characteristic equation of (3.2) in terms of the eigenvalue, σ, gives the largest eigenvalue as $= \frac{c}{b}$. Thus the crime propagation number is expressed as

$$C_{p_n} = \frac{c}{b} \tag{3.3}$$

3.2. Positivity, Existence and Uniqeness of Solution

The model is described in the domain

$$\Phi \in \mathcal{R}^4 = \{S, A, V, N : S \geq 0, A \geq 0, V \geq 0, N > 0, S + A + V = 1\} \tag{3.4}$$

Suppose at $t = 0$ all variables are non-negative, then $S(0) + A(0) + V(0) = 1$. If $S = 0$, and all other variables are in Φ, then $\frac{dS}{dt} \geq 0$. This is also the case for all other variables in $(2.6) - (2.8)$. We note from (2.7) that $A = 0$ implies $\frac{dS}{dt} = 0$ meaning $A = 0$ at all times and there will be no kidnapping in the absence of kidnappers. If $N = 0$, then $\frac{dN}{dt} = 0$. But if $N > 0$ and assuming $b > h$, then with appropriate initial conditions, $\frac{dN}{dt} > 0$ for all values of $t > 0$. We note that the right-hand side of $(2.6) - (2.9)$ is continuous with continuous partial derivatives, so solutions exist and are unique. The model is therefore mathematically and criminologically well posed with solutions in Φ for all $t \in [0, \infty)$.

3.3. Steady state solution and stability analysis

It can easily be shown from the system that the kidnap free state is $(S, A, V) = (1, 0, 0)$. In the absence of a kidnapper, $A = 0$ and substituting this into the right hand side of (2.8) we obtain $V = 0$. Further substitution of the values of A and V into (2.6) gives $S = 1$. The kidnap free state is locally asymptotically stable when $C_{p_n} < 1$ and globally asymptotically stable when $C_{p_n} < 1$, $g = 0$, and unstable for $C_{p_n} > 1$, where C_{p_n} is as defined in (3.2). We note that $C_{p_n} = 1$ is a bifurcation surface in which the system changes its stability status. We derive sufficient conditions for local and global stability of the kidnap free state from all initial conditions in Φ.The Jacobian matrix obtained by linearising system $(2.6) - (2.8)$ about the kidnap free equilibrium point, $(S, A, V) = (1, 0, 0)$ is

$$J_{d_f} = \begin{bmatrix} -b & -(c+e) & 1 \\ 0 & c-b & 0 \\ 0 & e & -(1+b) \end{bmatrix}. \quad (3.5)$$

Lemma 3.1. *The kidnap free equilibrium is locally asymptotically stable if $C_{p_n} < 1$ and unstable if $C_{p_n} > 1$.*

Proof The characteristic polynomial equation of (3.5) with eigenvalues, Ꝃ is

$$(b + Ꝃ)\{(b - c) + Ꝃ\}\{(1 + b)\, Ꝃ\} = 0$$

Two of the eigenvalues are strictly negative and the remaining eigenvalue is expressed by the linear equation

$$Ꝃ + b\left(1 - \frac{c}{b}\right) = 0 \quad (3.6)$$

From (3.3), C_{p_n} is given by

$$C_{p_n} = \frac{c}{b} \quad (3.7)$$

If $C_{p_n} < 1$, then $c < b$ and the coefficients of the linear polynomial of (3.6) are all positive and non zero; so by the Descartes' rule of signs there is no positive real eigenvalue. Thus Routh Hurwitz stability condition for a linear polynomial as stated in [1] and given in this case by $b(1 - c/b) > 0$ is satisfied. We observe that if $C_{p_n} > 1$, $c > b$ and the constant, $b(1 - c/b)$is negative. Therefore there is one sign change and by using Descartes' rule of sign there exists one positive real eigenvalue, we conclude that the kidnap free state is unstable if $C_{p_n} > 1$. When $C_{p_n} = 1$, $c = b$ and (3.6) has zero eigenvalue, which shows that $C_{p_n} = 1$ is a bifurcation surface in (b, c) parameter plane.

Lemma 3.2 *The kidnap free equilibrium is globally asymptotically stable in Φ if*

$$C_{p_n} < 1, \quad g = 0. \quad (3.8)$$

Proof. Consider the function $\Psi : \{(S, V, A) \in \Phi : S > 0\} \to \mathcal{R}$, where

$$\Psi = A. \quad (3.9)$$

We note that $\Psi \geq 0$ and is continuously differentiable on the interior of Φ. We shall show that the kidnap free equilibrium is a global minimum of Ψ on Φ if (3.8) holds. The derivative of Ψ computed along solutions of the system is

$$-c(A + V)A - b(1 - C_{p_n}) - (f + g)(S + V)AVN + gAVN \quad (4.0)$$

We can see clearly that $\frac{d\Phi}{dt} < 0$ whenever $C_{p_n} < 1$ and $g = 0$. In fact, for $(A, V) = (0,0)$, $\frac{d\Phi}{dt} \leq 0$ and (A, V) is the largest positively invariance subset in the interior of Φ and by
LaSalle's invariant principle [7], $(A, V) \to (0,0)$ as $t \to \infty$ while $S \to 1$ on the boundary of Φ.
Thus, the kidnap free state is globally stable if (3.8) are true, noting $c < b$.

4. Numerical Simulations

The numerical solution is obtained by using MATLAB's ode 45, a variable order Runge-Kutta method with a relative tolerance of 10^{-8} and absolute tolerance of 10^{-9}. The parameters used for the simulations as defined in Table 3 are $b = 0.016, c = 0.31, e = 15.5, f = 0.024, g = 0.012, h = 0.00036$. At time $t = 0$ we have the following initial conditions in the proportions: $S = 0.99, A = 0.01, V = 0, N = 1$. This is a situation where the entire susceptible human population is exposed to a small fraction of kidnappers. The program was run in MATLAB with different sets of initial conditions, a check was also conducted using MAPLE's ode45 in all cases and the qualitative form of the steady state solutions were the same, although the system gets to a steady state faster as the initial fraction of kidnappers increases.

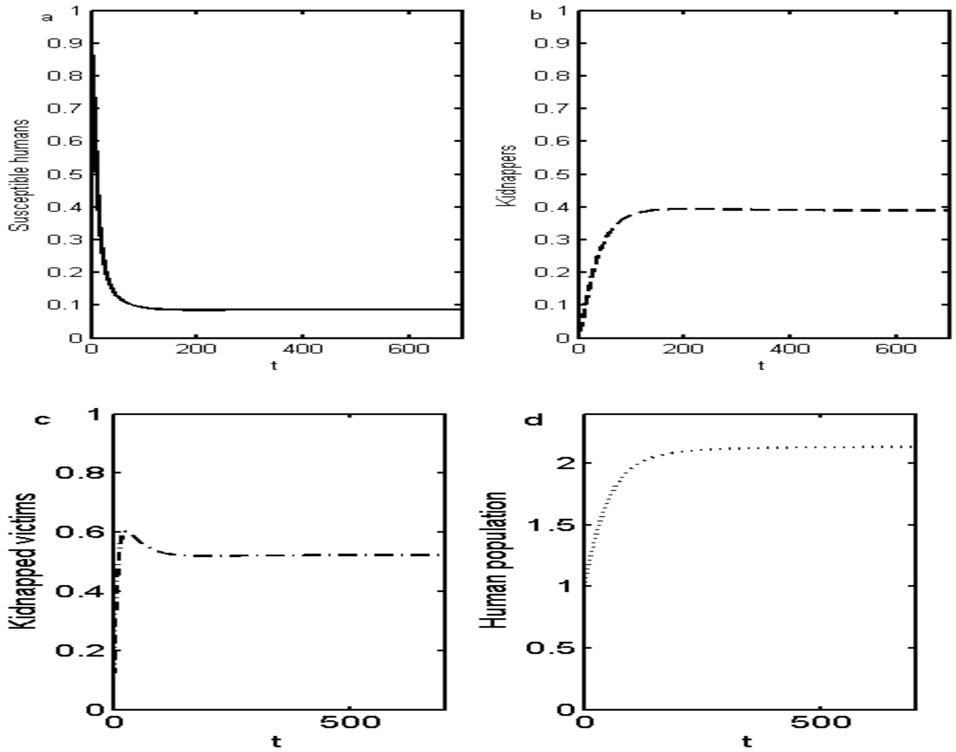
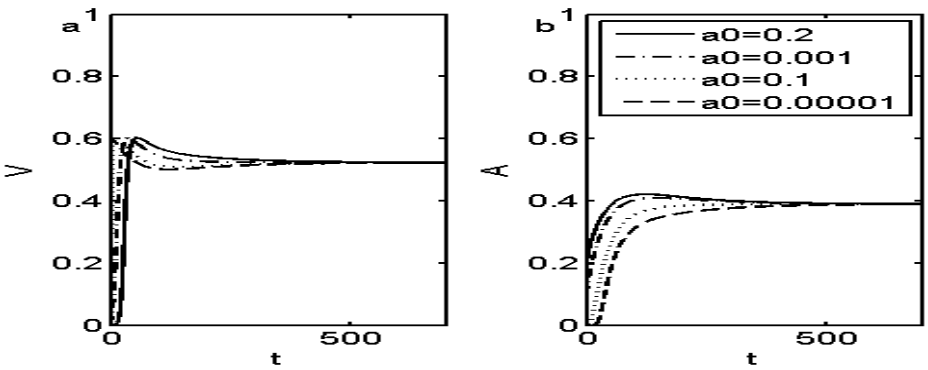


Figure 2: Results showing the effect of small amount of kidnappers on evolution of kidnapping in a kidnap free society, where $t = 1$, represents approximately 10 days in real time. The initial conditions used are $S = 0.9, A = 0.1, V = 0, N = 1$ and the parameter values are given in Table 3.

In Figure $2a$, the proportion of susceptible human population drops while the entire population is gradually increasing in Figure 2d. The fraction of kidnappees peaks and later drops to a steady state in Figure $2c$, resulting from an increase in the proportion of kidnappers as shown in Figure $2b$. In Figure Figure $2c$, more than half of the population are kidnap victims indicating high level of kinapping prevalence. Figure $3a, b, c, d$ show the effect of different values of a_0 on the various fractions of human population. We investigate each of the human sub-populations alongside the entire population as a_0 varies from 0.00001 to 0.2 and the results show that there is a unique steady state for each human compartment irrespective of the value of a_0 whereas the human population is gradually increasing.



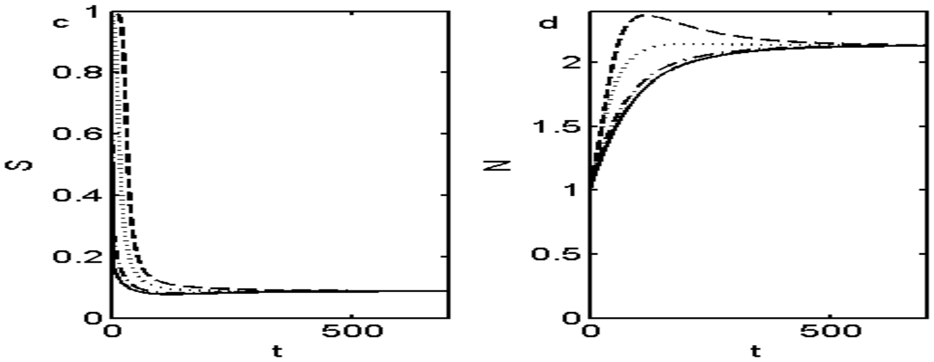


Figure 3: *Results showing the entire population (Figure* $3d$*) and the effect of introducing different amount of kidnappers on the different compartments (Figure* $3a, b, c$*).*

The parameter values used for the simulations are the same as those in (Figure 2) except that for the initial conditions, we have used $S = 1 - a_0, A = a_0, V = 0, N = 1$ with different values of a_0, as shown in the graphs.

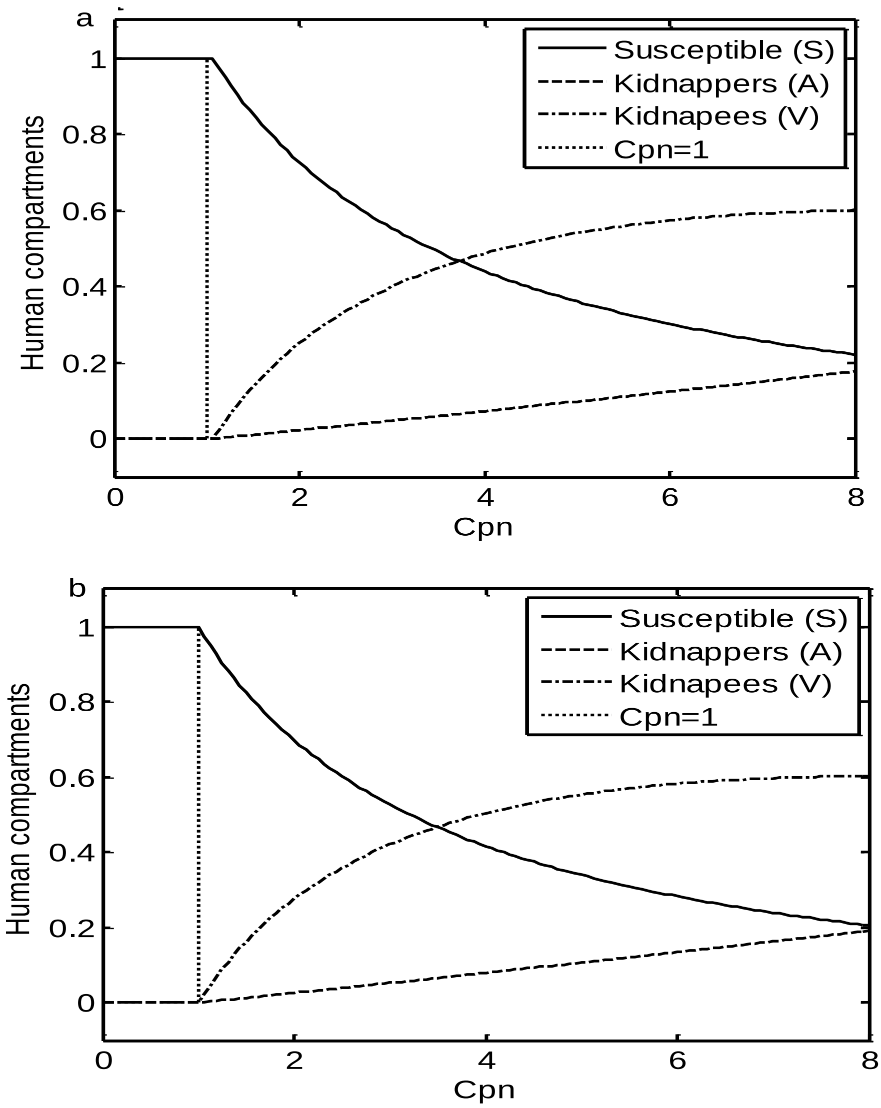


Figure 4: Results showing the kidnap free state when $C_{p_n} < 1$ *and the kidnap persistent state for* $C_{p_n} > 1$ *by varying the value of* C_{p_n} *from* 0 *to* 8*. The parameter values used to obtain these results are given in Table* 3*, except that for Figure* $4b$*,* $g = 0$*.*

We used the parameter, c to change C_{p_n} where $C_{p_n} = 8$ corresponds to $c = 0.128$. Figure 4 shows the relationship between the crime propagation number and the kidnapping crime profile as it affects the population. Especially, in Figure *4b* Kidnapping establishes itself for values of $C_{p_n} > 1$ and dies out if $C_{p_n} < 1$. However, in Figure *4a*, where $g > 0$, kidnapping still prevails for some $C_{p_n} > 1$ .The values of C_{p_n} were obtained by varying c and $C_{p_n} = 1$ corresponds to $c = 0.016$. Figure 5 is a bifurcation diagram showing a switch from a kidnap free state to a kidnap persistent state. The result is obtained by drawing the steady states of kidnap victims against different values of C_{p_n}. In order to demonstrate the impact of the crime propagation number on the dynamics of the system, we plot the steady states of the various compartments against the crime propagation number (C_{p_n}). Figure 5 shows the kidnap free state when C_{p_n} is less than unity and for $C_{p_n} > 1$ kidnappers invade the population. The plot shows a transcritical bifurcation in the vicinity of $C_{p_n} = 1$, as is expected from the analysis. Although some uncertainty still surrounds our quest on whether or not kinappers invade at $C_{p_n} = 1$, the kidnapping free state is stable for values of $C_{p_n} < 1$, but becomes unstable when $C_{p_n} > 1$ whereas, the kidnap persistent state becomes stable as expected.

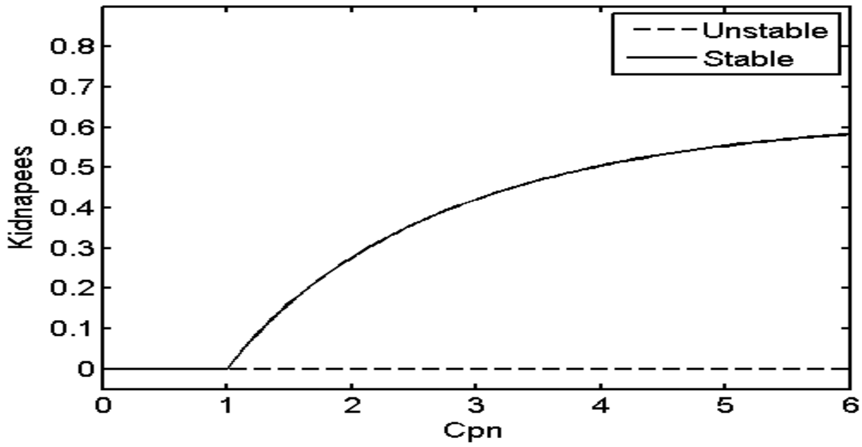


Figure 5: Crime propagation number (C_{p_n}) bifurcation diagram.

The curve shows a transcritical bifurcation obtained by drawing the steady states of kidnappees against different values of C_{p_n} ranging from 0 to 6. Parameter values are the same as those in Figure 4 except that $g = 0$

5. Discussion

Our model describes a typical situation of kidnapping as a social malaise. It is an attempt to replicate infectious disease dynamics in criminology. Just as infection occurs when an infectious individual comes in contact with a susceptible individual so as a crime (kidnapping) is committed when a kidnapper interacts with vulnerable individuals in society. Our model demonstrates a situation of persistent kidnapping which is supported by the value of C_{p_n} given as 19.0, obtained from data using (3.7). There is a high proportion of kidnapees as can be found in the numerical solution. Figure 2 shows that more than half of the population will become kidnapped victims within a period of twenty years whereas 38% of the population would be attracted to the criminal act of kidnapping. This does not only constitute a threat to global security and productivity but creates situations where victims pass through unimaginable ordeals in the form of physical, mental and emotional torture culminating in lack of trust and interpersonal relationship, which are inimical to the spirit of nation building. In order to bring a crime under control in a population of varying size, we need to reduce the crime reservoir represented by C_{p_n} below a threshold value with increasing time. This is demonstrated by the numerical solution in Figure 4 and Figure 5 as corroborated by the results of our analyses in section 3.3. It is worth noting that since c is significantly greater than b, then viable antikidnapping strategies targeting recruitment of kidnappers (reducing γ) and enhancing rescue operations by security agents (increasing w) could potentially be effective since this would drastically reduce c far beyond the value of b. This attests to some level of reliability in the values even though the assumption on some of the parameters may affect the accuracy of our result. However, collection of data and estimation of parameters in mathematical models on kidnapping would lead to better results. Rescue strategies may be jeopardized if some security agents become members of the kidnapping syndicate as reported in some national daily newspapers in some developing countries. Thus recruitment of security agents should be properly done in such a manner that only patriotic men and women of character are selected. The results also suggest that increasing rescue operations alone cannot lead to the eradication of kidnapping but could only help in the management and control of the crime.

6. Conclusion

In this model we have made a bold attempt to use a mathematical model to proffer solution to the problem of ransom kidnapping. The key information we obtained from the analysis and simulations of the model is that there is the possibility of eradicating kidnapping by discouraging the recruitment of kidnappers and enhancing prompt rescue of kidnapped victims. Most of the kidnapping cases are perpetrated by armed robbers under the influence of hard drugs. Therefore illegal hard drug dealers should be treated as great enemies to society. It would be difficult to persuade hard drug addict from committing crime. Lemma 3.2 demands for caution in applying rescue operation strategies in that for the kidnap free state to be globally asymptotically stable, there has to be an additional condition, $g = 0$, apart from $C_{p_n} < 1$. This signifies that there should no casualty of kidnap victims during rescue operations otherwise families of kidnappees would not have confidence in the rescue strategies and rather pay ransom to release their relations. This would likely encourage perpetrators to continue in the crime.

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