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**Research Article**

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# Analysis of Rate-Distortion functions for Multi-Hypothesis Motion Compensation (MMC) in Video Compression Systems

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**Abstract** This work presents a computational analysis of distortion functions for multi-hypothesis frames at their respective bit-rates in video compression systems. Theoretical bit assignment for multi-hypothesis frames is achieved with Lagrange multiplier and its cost function for reasonable low cost compression systems at minimum bit- rates. Video signals are compressed at a minimum transmission bit-rate to achieve accurate decoding or reconstruction depending upon the number of hypothesis used in coding the signal. This has made multi-hypothesis motion compensation (MMC) a great process in the prediction of the actual signal through the combination of more than one motion compensated prediction (MCP). The distortion functions were analyzed at bit-rate range 0.0 [bpp] to 1.0 [bpp] using MATLAB in order to optimize the number of hypothesis to be used in decoding for a reasonable signal compression. Results show that a minimum distortion of the range 0.0104-0.0105 was realized with eight numbers of hypotheses frames. This implies that varying number of hypotheses actualizes minimum distortion of signals due to optimization. This also addresses a better prediction due to greater time correlation between hypotheses which consequently helps in the improvement of data compression, coding and reconstruction quality of video signals.

**Keywords** Bit-rate, Motion Compensation, Rate-Distortion Functions, Video Compression

## 1. Introduction

In recent years advances in video signal compression has made it evident for the development of hybrid coders like MPEG, H.263, H264 etc towards noise and redundancy reduction through motion compensation. Therefore, digital video being immune to noise, is easier to transmit and also able to provide a more interactive interface to users. In the video scene, the data redundancy arises due to changes in spatial, temporal and statistical correlation between the frames [1]. Video compression, motion compensation, Transform coding, entropy coding, are used to reduce the temporal, spatial and statistical redundancy between the successive frames in their respective domains [1]. The entropy coding technique (lossless compression technique) is also commonly used in file compression. Therefore efficient compression is achieved, at reduced bit-rates for a certain minimum distortion of video signal. Digital representation of images is important for digital transmission and storage on different media such as magnetic or laser disks [2]. However, pictorial material requires vast amounts of bits if represented through direct quantization, this made compression (coding) of video data evident [3]. Therefore, for easy transmission of video signals, video data are compressed (reduced in size) through several coding algorithms that employs different coding and compression techniques through motion compensation for an acceptable sound and picture quality at the receiver. This also gives rooms for signals to be spread across many frequencies for many benefits, including resistance to jamming and interference, allowing multiple users to send data simultaneously over the same frequency range and also enabling encryption through error detection and correction.

Multi-hypothesis motion compensation has found many applications in video coding such that coders employ motion-compensated prediction signals that are superimposed to predict the original frame for easy reconstruction at the decoder. This term was first used to provide a frame work for overlapped block motion compensation (OBMC). OBMC was introduced to reduce blocking artifacts in motion-compensated prediction [4]. Concatenated block codes have been considered for embedded bit stream transmission with minimum distortion variance and over error-prone memoryless channels [5]. The theoretical motivations for multi-hypothesis motion compensation have also been presented in [6], while the mathematical frameworks on temporal and spatial predictive processing in motion-compensated video compression have also been presented in [7]. A mathematical model was also introduced for rate-distortion functions and coding gain of multi-hypothesis motion compensated video signal [2].

## 2. Theory

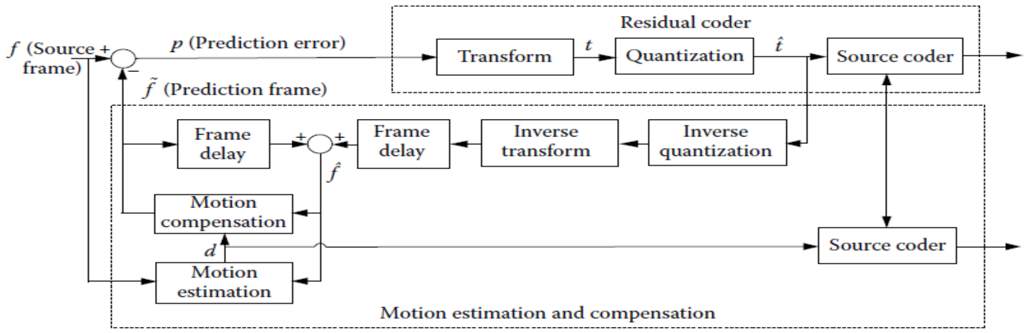


*Figure 2.1: Hybrid video compression system*

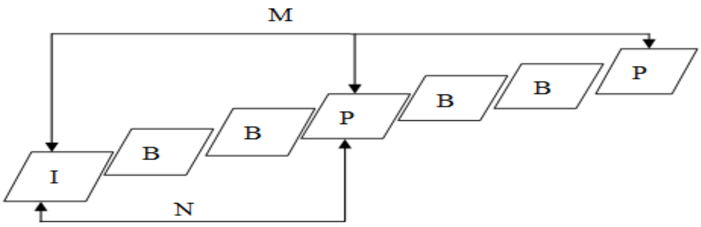


*Figure 1.2: Group of picture (GOP) structure*

Virtually all video compression systems identify and reduce four basic types of video data redundancy: inter-frame (temporal) redundancy, inter-pixel redundancy, psycho-visual redundancy, and coding redundancy [3]. Figure 2.1 shows a typical diagram of a hybrid video compression system. Firstly the current frame is predicted from previously decoded frames by estimating the motion of blocks or objects, thus reducing the inter-frame redundancy. Afterwards to reduce the inter-pixel redundancy, the residual error after frame prediction is transformed to another format or domain such that the energy of the new signal is concentrated in few components and these components are as uncorrelated as possible. The transformed signal is then quantized according to the desired compression performance (subjective or objective). The quantized transform coefficients are then mapped to code words that reduce the coding redundancy [3].

As shown in figure (2.2), all standard hybrid video systems like MPEG-1, MPEG-2, MPEG-4, H.263, H.264 encodes set of image signals to reduce spatial and temporal redundancy by dividing the frames of video signal into group of pictures (GOP). Thereafter, classifying each frame into I, P or B frames (intra, inter and Bi-directional frames). Image data in each frame are split into regions (blocks) for the prediction of a block in the current frame by using reference frames as the previously decoded frames (last I or P-frames). In figure 2.1, the displacement ***d*** of the block is not fixed and must be encoded as side information (SI) in the bit-stream. This gives room for motion compensation and estimation to be block-based in order to minimize the SI and simplify the encoding process. Therefore, by the process of multi-hypothesis motion compensation and estimation discussed, each set of blocks available for prediction (multi-hypothesis) are assigned motion vectors which are transmitted with the prediction error ***p*** between the original and reference frame (signal error) for the final reconstruction of the image at the decoder after undergoing transformation and quantization. M and N represent the distance between core pictures and the number of core pictures respectively. Therefore, M×N=Q which means the total number of pictures in a group of pictures.

**2.1. Quantization**

Quantization is the mapping of vectors or scalars of an information source into a finite collection of codeword for storage or transmission. This involves two processes, the encoding and decoding. It can further be simply defined as means of providing approximations to signals and signal parameters by a finite number of representation levels (mapping a large set of input values to a countable smaller set). This process is irreversible and thus always introduces quantization noise (error) which forms part of lossy compression algorithms (rate-distortion) in video sequence. This error is as a result of the difference in the input and quantized signal of the source encoder. A device or algorithmic function which performs quantization is called a quantizer.

Simultaneous quantization of several samples is called vector quantization (VQ), which is a generalization of scalar quantization (SQ). It involves the application of multi-dimensional (vector valued) input signal [8]. Therefore, we map a continuous N-dimensional vector x to a discrete-valued N-dimensional vector according to the rule.

$$X \in C_i \Rightarrow Q[X] = Y_i, \tag{1}$$

where $C_i$ is an N-dimensional cell. The $i = 1,........L$ possible cells are non-overlapping and contiguous to fill the entire geometric space. The vectors $\{y_i\}$ correspond to the representation levels in a scalar quantizer. In a VQ setting, the collection of representation levels is the codebook. The cells $C_i$, also called ''Voronoi regions,'' correspond to the decision regions. In VQ an indirect approach is utilized via a distortion measure d (x, y) to test for the interval which a signal sample belong.

$$Q[X] = Y_i \Leftrightarrow d(x, y_i) \leq d(x, y_j), \tag{2}$$

$$for\ j = 0.............L-1$$

After obtaining the best match $y_i$, the index i identifies the vector and it's therefore coded as an efficient representation of the vector. The receiver (decoder) can then reconstruct the vector $y_i$ by looking up the contents of cell number i in a copy of the codebook. Thus, the bit rate in bits per sample in this scheme is $\log_2 L=N$ when using straight forward bit representation for i.

**2.2. Transform Coding (TC)**

Transform coding constitutes an integral component of contemporary image/video processing applications. Most image and video compression schemes apply a transformation to the raw pixels or to the residual error resulting from motion compensation before quantizing and coding the resulting coefficients[3]. Transform coding relies on the premise that pixels in an image exhibit a certain level of correlation with their neighbouring pixels. Similarly in a video transmission system, adjacent pixels in consecutive frames show very high correlation. Consequently, these correlations can be exploited to predict the value of a pixel from its respective neighbours. A transformation is therefore, defined to map this spatial (correlated) data into transformed (Uncorrelated) coefficients. The transformation sub-block de-correlates the image data thereby reducing or removing inter-pixel redundancy [9].

**2.3. Bit-Rate Lagrange Multiplier (λ)**

By minimizing the average reconstruction error variance$\sigma_r^2$[6] as a distortion we allocate bit-rate utilizing the Lagrange multiplier (λ) method[10], [11]with a Lagrange cost function given by

$$\mathrm{j} = \mathrm{D} + \lambda R_c \tag{3}$$

$$\min \sigma_r^2 = \frac{1}{Q}\left[\sigma_{r,I}^2 + (Q-1)\sigma_{MHP}^2\right]^2 \quad (4)$$

taking into account the constant bit-rate constraints $R_c$ given by

$$R_c = R_I + (Q-1)R_{MHP} - QR, \quad (5)$$

the target function becomes

$$\min\ j = \min\left\{\frac{1}{Q}\left[\sigma_{r,I}^2 + (Q-1)\sigma_{MHP}^2\right] + \lambda\left[R_I + (Q-1)R_{MHP} - QR\right]\right\}. \quad (6)$$

By relating equation (3) and equation (6) the derivative of the function j is given by

$$\frac{dj}{dR} = \frac{dD}{dR} + \lambda = 0 \quad (7)$$

Since

$$j(R_{MHP}, R_I, \lambda) = \frac{1}{Q}\left[\sigma_{r,I}^2 + (Q-1)\sigma_{MHP}^2\right] + \lambda\left[R_I + (Q-1)R_{MHP} - QR\right][10],$$

Then $$\frac{\partial j}{\partial R_{MHP}} = \frac{1-Q}{2Q}\log_2\left[\frac{\sigma_e^2}{\sigma_s^2}\right] + \lambda(Q-1) = 0,$$

$$\lambda = \frac{1}{2Q}\log_2\left[\frac{\sigma_e^2}{\sigma_s^2}\right], \quad (8)$$

Since $R_{MHP} = R + \lambda$ (9)

substituting eqn (8) into eqn (9) we have

$$R_{MHP} = R + \frac{1}{2Q}\log_2\left[\frac{\sigma_e^2}{\sigma_s^2}\right]. \quad (10)$$

Hence, where where $\varepsilon^2$ and $R_{MHP}$ are the quantization performance factor and bit rates for the multi-hypothesis frames respectively and R is the overall bit rate. we now also obtain that for the cases of P-frame, 2, 3, 4 and 8 hypotheses respectively given $\left[\frac{\sigma_e^2}{\sigma_s^2}\right]$ in [2]:

$$R_{Pframe} = R + \frac{1}{2Q}\log_2\left[2 - 2\rho_s^{\Delta r}\rho_t^{\Delta t}\right], \quad (11)$$

$$R_{2MHP} = R + \frac{1}{2Q}\log_2\left[\frac{3}{2} - 2\rho_s^{\Delta r}\rho_t^{\Delta t} + \frac{1}{2}\rho_s^{\Delta h}\right], \quad (12)$$

$$R_{3MHP} = R + \frac{1}{2Q}\log_2\left[\frac{4}{3} - 2\rho_s^{\Delta r}\rho_t^{\Delta t} + \frac{2}{3}\rho_s^{\sqrt{3}\Delta r}\right], \quad (13)$$

$$R_{4MHP} = R + \frac{1}{2Q}\log_2\left[\frac{5}{4} - 2\rho_s^{\Delta r}\rho_t^{\Delta t} + \frac{1}{2}\rho_s^{\sqrt{2}\Delta r} + \frac{1}{4}\rho_s^{2\Delta r}\right], \quad (14)$$

$$R_{8MHP} = R + \frac{1}{2Q}\log_2\left[\frac{9}{8} - 2\rho_s^{\Delta r}\rho_t^{\Delta t} + \frac{1}{4}\rho_s^{\sqrt{2}\Delta r} + \frac{1}{8}\rho_s^{2\Delta r} + \frac{1}{4}\rho_s^{\sqrt{2-\sqrt{2}}\Delta r} + \frac{1}{4}\rho_s^{\sqrt{2+\sqrt{2}}\Delta r}\right], \quad (15)$$

### 2.4. Rate-Distortion Function

Rate-distortion function is a function that allows us to calculate performance bounds without consideration of a specific coding method. Rate distortion function describes the trade-off between lossy compression rate and the corresponding distortion. It provides a minimum transmission bit-rate, if a distortion D between the original image at the transmitter and the reconstruction image at the receiver will not exceed a maximum acceptable distortion [2]. The rate distortion function for Gaussian source with a squared error distortion is given by [12].

$$R(D) = \begin{cases} \frac{1}{2}\log\frac{\sigma^2}{D} & 0 \le D \le \sigma^2 \\ 0 & D > 0 \end{cases} \tag{16}$$

Hence this can be rewritten in terms of rate as,

$$D(R) = \sigma^2 2^{-2R} \tag{17}$$

Supposing the overall number of frames tends to infinity and taking into account the optimal bit allocation, the distortion –rate functions D(R) can be obtained in the closed forms [13].

$$D_{MHP}(R) = \varepsilon^2 2^{-2R}\left[\frac{\sigma_e^2}{\sigma_s^2}\right] \tag{18}$$

$$D_{Pframe}(R) = \varepsilon^2 2^{-2R}\left[2 - 2\rho_s^{\Delta r}\rho_t^{\Delta t}\right] \tag{19}$$

$$D_{2MHP}(R) = \varepsilon^2 2^{-2R}\left[\frac{3}{2} - 2\rho_s^{\Delta r}\rho_t^{\Delta t} + \frac{1}{2}\rho_s^{2\Delta r}\right] \tag{20}$$

$$D_{3MHP}(R) = \varepsilon^2 2^{-2R}\left[\frac{4}{3} - 2\rho_s^{\Delta r}\rho_t^{\Delta t} + \frac{2}{3}\rho_s^{\sqrt{3}\Delta r}\right] \tag{21}$$

$$D_{4MHP}(R) = \varepsilon^2 2^{-2R}\left[\frac{5}{4} - 2\rho_s^{\Delta r}\rho_t^{\Delta t} + \frac{1}{2}\rho_s^{\sqrt{2}\Delta r} + \frac{1}{4}\rho_s^{2\Delta r}\right] \tag{22}$$

$$D_{8MHP}(R) = \varepsilon^2 2^{-2R}\left[\frac{9}{8} - 2\rho_s^{\Delta r}\rho_t^{\Delta t} + \frac{1}{4}\rho_s^{\sqrt{2}\Delta r} + \frac{1}{8}\rho_s^{2\Delta r} + \frac{1}{4}\rho_s^{\sqrt{2-\sqrt{2}}\Delta r} + \frac{1}{4}\rho_s^{\sqrt{2+\sqrt{2}}\Delta r}\right] \tag{23}$$

Also $(\Delta_r)$ represents the radial displacement of each hypothesis. It means the Euclidean distance to zero displacement error vector i.e. $0 \le \Delta_h \le 2\Delta_r$ while $(\Delta_t)$ means the time distance between hypotheses. Therefore, $\rho_s$and $\rho_t$are the spatial and temporal correlation coefficient of a picture element respectively. $\rho_s$=0.93 is the average correlation between horizontally or vertically adjacent picture elements in a typical video signal [14]

## 3. Methodology

In this work, we analyzed the rate-distortion functions which are all polynomial functions (equation 19-23) using MATLAB in order to optimize the number of hypothesis to be used in decoding signals. We used editor's script and command window of MATLAB to run the analysis with the following simulation procedures

- We utilized output functions (rate-distortion functions of multi-hypothesis frames) such as DP, D2MHP, D3MHP, D4MHP, D8MHP and input parameters such as $\rho_t, \rho_s, \Delta_r, \Delta_t$ *and* $R$.
- We generated script for the output functions in the editor's script environment of the MATLAB by defining a poly-function for the input parameters as [DP, D2MHP, D3MHP, D4MHP, D8MHP]= poly3 $(\rho_s, \rho_t, \Delta_r, \Delta_t, R, \varepsilon)$
- In the first case, we set $\rho_t, \rho_s, \Delta_r, \Delta_t, \varepsilon$ *and* $R$ at 0.8, 0.93, 1.0, 0.54, 1.0 and 0.0-1.0 respectively. A script is then used to run a poly function defined on the command window for a reasonable optimization.
- In the second case, we set $\rho_t, \rho_s, \Delta_r, \Delta_t, \varepsilon$ *amd* $R$ at 0.8, 0.93, 0.3, 0.64, 1.0 and 0.0-1.0 respectively. A script is then used to run a poly function defined on the command window for a reasonable optimization.
- In the third case, we set $\rho_t, \rho_s, \Delta_r, \Delta_t, \varepsilon$ *and* $R$ at 0.95, 0.93, 1.0, 0.50, 1.0 and 0.0-1.0 respectively. A script is then used to run a poly function defined on the command window for a reasonable optimization.
- In the last case, we set $\rho_t, \rho_s, \Delta_r, \Delta_t, \varepsilon$ *and* $R$ at 0.95, 0.93, 0.3, 0.95, 1.0 and 0.0-1.0 respectively. A script is then used to run a poly function defined on the command window for a reasonable optimization.
- We obtained the output values of the distortion functions for multi-hypothesis frames with the given values of the input parameters on the command window due to the generated script in each case.

- We plotted a two dimensional (2-D) graph of distortion functions of multi-hypothesis frames against their bit-rates for each case using plot command which contains grid, legend of strings and colors with x and y label for R and the output functions respectively as, **plot(R,DP,'m.',R,D2MHP,'b--',R,D3MHP,'k-.',R,D4MHP,'k-',R,D8MHP,'r:');**
- Finally, the display of the 2-D graphics plot shows the performance limit for distortion of multi-hypothesis frames in signal compression.

## 4. Results and Discussions

### 4.1. Results

Setting the values of parameters as given in figure 4.1a, distortion of 8MHP frames is minimal at 0.0663 while that of P frames is minimal at 0.0878. Therefore, a distortion of 0.0215 is collectively obtained for 8MHP frames over P frames.

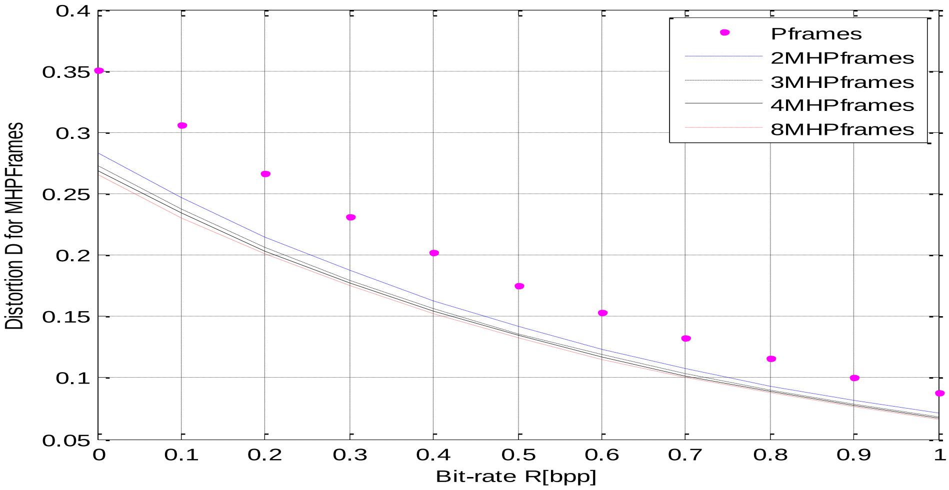


*Figure 4.1a: Distortion functions for multi-hypothesis with* $\rho_t, \rho_s, \Delta_r, \Delta_t, \varepsilon, R$ *at* 0.8,0.93,1.0,0.54,1.0,0.0 − 1.0 *respectively.*

Setting the values of parameters as given in figure 4.1b, distortion of 3MHP frames is minimal at 0.0648 while that of P frames is minimal at 0.0759. Therefore, distortion of 0.0111 is collectively obtained for 3MHP frames over P frames.

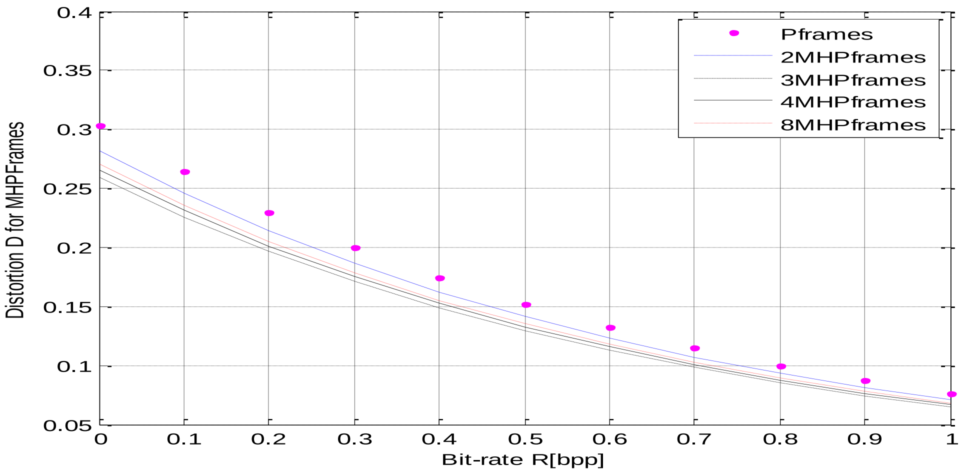


*Figure 4.1b: Distortion functions for multi-hypothesis with* $\rho_t, \rho_s, \Delta_r, \Delta_t, \varepsilon, R$ *at* 0.8,0.93,0.3,0.64,1.0,0.0 − 1.0 *respectively.*

Setting the values of parameters as given in figure 4.1c, distortion of 8MHP frames is minimal at 0.0252 while that of P frames is minimal at 0.0468. Therefore, a distortion of 0.0216 is collectively obtained for 8MHP frames over P frames.

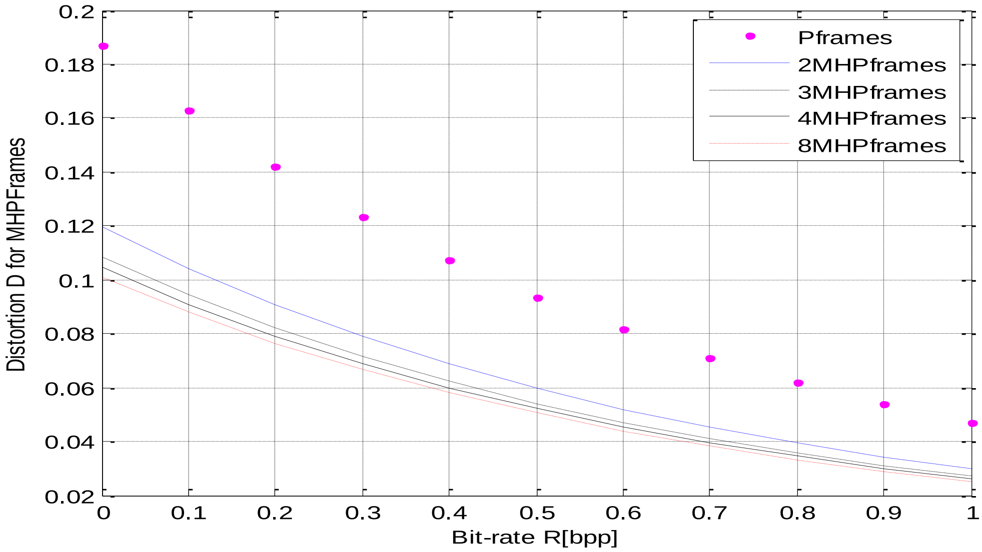


*Figure 4.1c: Distortion functions for multi-hypothesis with* $\rho_t, \rho_s, \Delta_r, \Delta_t, \varepsilon, R$ *at* 0.95,0.93,1.0,0.5,1.0,0.0 − 1.0 *respectively.*

Setting the values of parameters as given in figure 4.1d, distortion of 3MHP frames is minimal at 0.0229 while that of P frames is minimal at 0.0340. Therefore, a distortion of 0.0111 is collectively obtained for 3MHP frames over P frames.

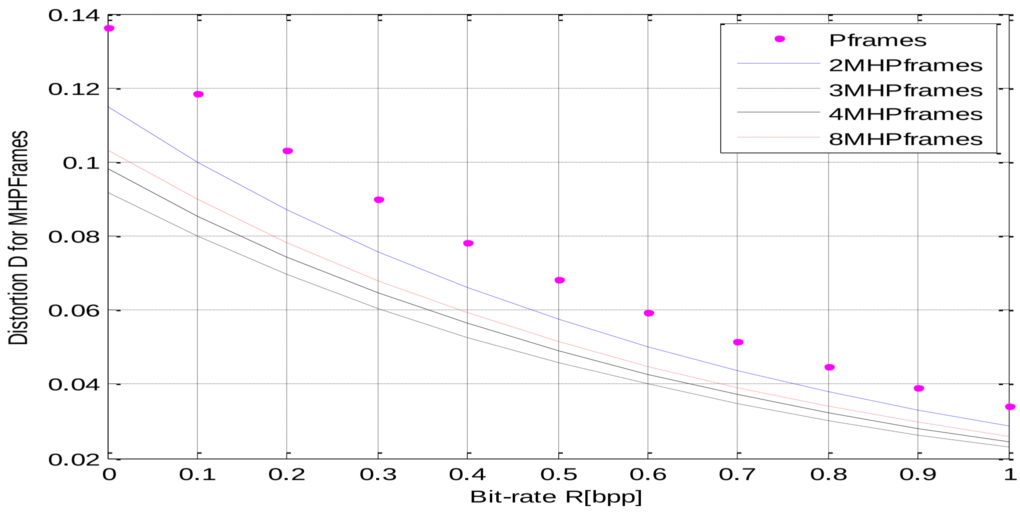


*Figure 4.1d: Distortion functions for multi-hypothesis with* $\rho_t, \rho_s, \Delta_r, \Delta_t, \varepsilon, R$ *at* 0.95,0.93,0.3,0.95,1.0,0.0 − 1.0 *respectively.*

**4.2. Discussions**

The Figures above (4.1a, 4.1b, 4.1c and 4.1d) show the effect of distortion for multi-hypothesis frames (MHP-frames) using inter-frame video coding at minimum bit-rates with the values of the parameters such as time correlation coefficient($\rho_t$), radial displacement ($\Delta_r$), time distance between pictures ($\Delta_t$) and spatial correlation coefficient ($\rho_s$)suggested by video sequence [15]. A MATLAB program is used for the simulation of the parametric equation of rate-distortion functions of the multi-hypothesis frames (19 to 23). The distortions of the multi-hypothesis frames are varied over bit-rates with the radial displacement ranging from 0.3 to 1.0. The lowest rate-distortion value is obtained with three hypotheses from the figures (4.1a, 4.2a, 4.3a and 4.4a) which signify a better prediction and reconstruction of frames using up to eight hypotheses in order to achieve improved compression of video signal transmission.

In Figures 4.1a and 4.1b, a collective distortion value of 0.0215 and 0.0111 is obtained for 8MHP frames and 3MHP frames at a minimum distortion value of 0.0663 and 0.0643 respectively over P frames all at the same bit-rate [0.0bpp-1.0bpp]. Therefore the distortion decreases for about 0.0104 when time correlation coefficient $(\rho_t) = 0.8$ and radial displacement $(\Delta_r)$ ranges from 1.0 to 0.3 at a greater time interval $(\Delta_t)$ of 0.54 to 0.64. In Figures 4.1c and 4.1d, a collective distortion value of 0.0216 and 0.0111 is obtained for 8MHP frames and 3MHP frames at a minimum distortion value of 0.0252 and 0.0229 respectively over P frames all at the same bit-rate [0.0bpp-1.0bpp]. Therefore the distortion decreases for about 0.0105 when time correlation coefficient $(\rho_t) = 0.95$ and radial displacement $(\Delta_r)$ ranges from 1.0 to 0.3 at a greater time interval $(\Delta_t)$ of 0.5 to 0.95. We can also observe that a greater time correlation coefficient $(\rho_t)$ of the range 0.8-0.95 between picture elements and low distortion rate of range 0.0104-0.0105 implies a better video compression.

## 5. Conclusion

Rate-distortion function for multi-hypothesis frames has been theoretically and computationally analyzed in order to deduce the performance bounds without consideration of a specific coding method. Also, the trade-off between lossy compression rate and the corresponding distortion has been described to provide a minimum transmission bit-rate, for an acceptable non-exceeding maximum distortion between the original image at the transmitter (encoder) and the reconstruction image at the receiver (decoder). A low distortion rate of range 0.0104-0.0105 is achieved at minimum transmission bit-rates of the range 0.0bpp to 1.0bpp with eight hypotheses due to optimization. This shows a greater performance of a codec for motion compensated video compression with applications in both digital storage and communication services

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