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An introduction to the theory and applications of SQUID sensors

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Abstract Many physical quantities of interest could be detected and measured by devices that involve superconducting materials. Superconducting quantum interference devices (SQUIDs) are very sensitive sensors that respond to changes in magnetic flux which occur due to variations in external magnetic field. This note is an introduction to the theory and applications of these versatile sensors.

Keywords Superconductors, Joesphson Junction, SQUIDs

Introduction

Superconductivity dates back to Kamerlingh-Onnes, who in 1908 investigated the resistance of various materials [1-2]. By laboratory measurements it was found that the electrical resistance of many metals near room temperature decreases linearly with decreasing temperature. At extremely low temperatures, three possibilities were proposed. The resistance could reach zero ohms with decreasing temperature or a finite minimal value, or thirdly, it could pass through a minimum and rise to infinity as the temperature tends to absolute zero. These are illustrated in Fig. 1. The first possibility is prompted by the decreasing trend of resistance. The second possibility precludes the possibility of infinite current, and the third possibility means that the charge carriers are tightly bound and cannot allow energy transfer as the temperature is reduced to zero degree Kelvin.

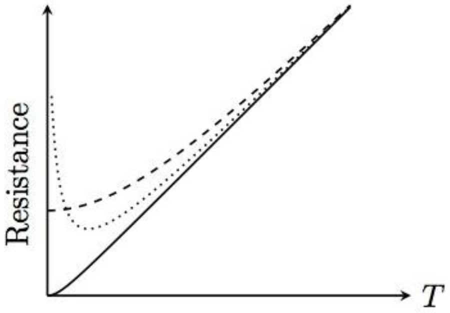


Figure 1: Three proposed possibilities of variations of resistance at extremely low temperatures. solid-1, dashed-2, dotted-3

Kamerlingh-Onnes first oberved the second possibility and found that the residual resistance depended on the purity of the specimen. He also believed that it could be reduced nearer to zero. Later experiments using improved apparatus revealed that there was an abrupt fall in the resistance of mercury to a millionth of its original value at the melting point slightly below 4.2 K, allowing superconductivity. Following this discovery, thousands of superconducting compounds have been identified and still many more are being added to the list.

At atmospheric pressure, niobium is the element with the highest transition temperature of about 9 K. Another characterizing feature of superconductors is that any external magnetic field is annulled inside the volume leaving a thin outer layer suggestive of diamagnetism. The magnetic flux is quantized. The ideal diamagnetism of superconductors was discovered by Walther Meissner and Robert Ochsenfeld in 1933.

A theoretical proof for the possibility of superconductivity was developed by John Bardeen, Leon Neil Cooper, and John Robert Schrieffer in 1957, which is now called the BCS theory. At the onset of superconductivty, they discovered that pairs of electrons condense into a new quantum mechanical state and turn into a matter wave characterized by a crisply defined phase function. The crystal lattice undergoes quantized vibrations where energy is manifested by phonons. The formation of a coherent matter wave, termed as a *macroscopic wave function* is the distinguishing feature of the superconducting state. Such phenomena were noted in other contexts; for example, lasers, superfluids etc., all of which were correctly predicted by Bose-Einstein condensation theory put forth in 1925 by Bose and Einstein. In 1995 such condensates were experimentally brought to existence by special optical and magnetic refrigeration techniques at temperatures below 1μK.

For almost four decades superconductivity remained as an occurrence possible only at very low temperatures. But in 1986 J. G. Bednorz and K. A. M$\ddot{u}$ ller succeeded in making superconductors using copper oxide at about 30 K. Although their work met with some skepticism, they managed to prove their results theoretically and experimentally by showing the presence of Meissner-Ochsenfeld effect which supports the manifestation of paramagnetism of metals at the temperatures maintained. They measured negative magnetic susceptibility which spurred further research.

In late 80s superconductors employing the LaSrCuO compounds with transition temperatures above 40K and those making use of YBaCuO above 80K were produced. At temperatures above 77K and above the boiling point of liquid nitrogen, many significant results were demonstrated. These developments paved way to a whole gamut of high temperature superconductors (HTSs) based on copper oxide. Popular among them are compounds made of $YBa_2Cu_3O_7$ (also YBCO or Y123) and $Bi_2Sr_2CaCu_2O_8$ (also BSCCO or Bi2212) that exhibit superconductivity around 90K. Others have transition temperatures even above 100K. For example, $HgBa_2Ca_2Cu_3O_8$, is known to have a T_c around 135K at atmospheric pressure and 164K at a pressure of 30 GPa.

When a specimen is cooled to the transition temperature, ideally, resistance is reduced to zero, and current should be extremely high and should continue to flow without attenuation. But one should note that practical considerations limit these values. AC resistance increases with frequency and even DC resistance is lower bounded by the magnitude of magnetic flux and also the physical geometry of the material. In order for superconductors to abound, cooling technologies must grow. Latest cryo-coolers are capable of temperatures of the range 30K, and in some cases even 4.2K and lower. Cooling with the aid of liquid nitrogen has become the de facto procedure in industry. That implies, superconductivity will no longer be confined to laboratories, but would soon become a commonplace. It is expected that even the energy engineering and microelectronics are going to be prospective fields where this phenomenon would be exploited.

Applications of superconductors are numerous. Generation of high magnetic fields in nuclear-spin tomographs using liquid helium as cooling agent is known in medical field. Superconducting motors are employed for ships. In communication technology superconducting filters made of $YBa_2Cu_3O_7$ are are found to be promising. Detection of minerals and nondestructive testing of materials can be carried out with the aid of superconductors. High temperature superconductors help in magnetic levitation and suspension.

2. Josephson Junction

If two superconductors are sandwiched one upon the other and if a thin non-superconducting barrier is formed between them, this arrangement is called Josephson junction. If the barrier is sufficiently thin of the order of few nanometers, electrons pass from one layer to the other, despite the presence of the non-conducting layer between the two metals. This form of conduction is attributed to the quantum mechanical tunneling. The probability wave function describing the location of an electron, spreads out from one metallic region allowing the electron to tunnel into the second region and a current is constituted across the junction. On account of this tunneling phenomenon, the two superconductors and the associated wave functions are coupled. A supercurrent called the Josephson current, flows across the barrier. The properties of this current, as predicted by Josephson in 1962, are determined by the phase of the macroscopic wave function. In fact this current denoted by I is proportional to the sine of the phase difference of the wave functions of the two layers on either side as:

$$I = I_c \sin(\delta) \tag{1}$$

$$\delta = \varphi_2 - \varphi_1 - \frac{2\pi}{\Phi_0}\int_1^2 Adl \tag{2}$$

Here δ is the gauge invariant phase difference function related to the applied voltage V according to

$$\frac{d\delta}{dt} = \frac{2\pi}{\Phi_0}V \tag{3}$$

Also the path integral is taken over the magnetic vector potential.

The Josephson junction can be used in a SQUID device shown in Fig. 1(a) The working principle is discussed in the next section. Equivalent circuit of such a junction with shunt resistance and capacitance is shown in Fig. 1(b). The terminal current I can be written as

$$I = I_c \sin\delta + \frac{V}{R} + C\frac{dV}{dt} \tag{4}$$

where R is the resistance and C is the capacitance. Using (3) we can write the above as

$$I = I_c \sin\delta + \frac{\Phi_0}{2\pi R}\frac{d\delta}{dt} + \frac{\Phi_0 C}{2\pi}\frac{d^2\delta}{dt^2} \tag{5}$$

Let us introduce the variables

$$\tau = \frac{2\pi I_c R}{\Phi_0}t \tag{6}$$

$$i = \frac{I}{I_c} \tag{7}$$

which permit us to write

$$i = \sin\delta + \dot{\delta} + \frac{2\pi I_c R^2}{\Phi_0}\ddot{\delta} \tag{8}$$

where the dot notation is differentiation wrt τ . Next we denote the coefficient of $\ddot{\delta}$ with β_c as

$$\beta_c = \frac{2\pi I_c R^2}{\Phi_0} \tag{9}$$

which is known as the Stewart-McCumber damping parameter. Hence we arrive at the nonlinear differential equation

$$i = \sin\delta + \dot{\delta} + \beta_c\ddot{\delta} \tag{10}$$

It is found that when $\beta_c > 1$, the volt-ampere characteristic exhibits hysterisis. Otherwise we find a hyperbolic relation as described below. Suppose $\beta_c = 1$, we can approximate Eq. (10) as

$$i = \sin\delta + \dot{\delta} \tag{11}$$

By rewriting Eq. (11) as

$$d\tau = \frac{d\delta}{i - \sin\delta} \tag{12}$$

and by integrating we express

$$\delta = 2\tan^{-1}\left\{\frac{1+\sqrt{i^2-1}\tan\left(\frac{\tau\sqrt{i^2-1}}{2}\right)}{i}\right\} \tag{13}$$

This is periodic with a period τ_p given by

$$\tau_p = \frac{2\pi}{\sqrt{i^2-1}} \tag{14}$$

The time averaged voltage $\tilde{V}$ in terms of the normalized current i also denoted by $\tilde{I}$ is now given by

$$\tilde{V} = \sqrt{\tilde{I}} \tag{15}$$

Fig. 2 depicts this relation in addition to the ohmic relation which acts as its asymptote.

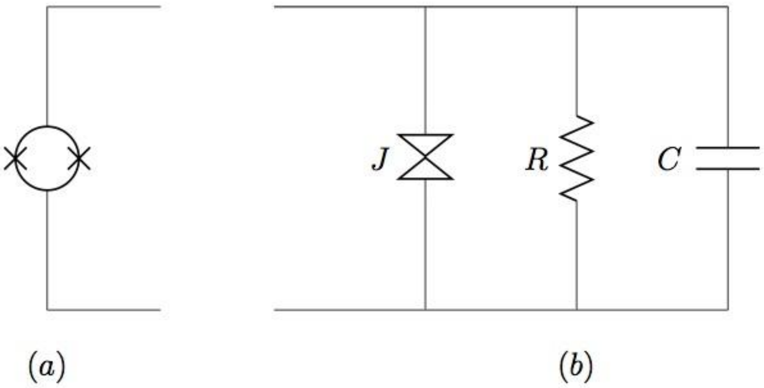


Figure 2: (a) Schematic for SQUID device (b) Equivalent Circuit of a Josephson junction

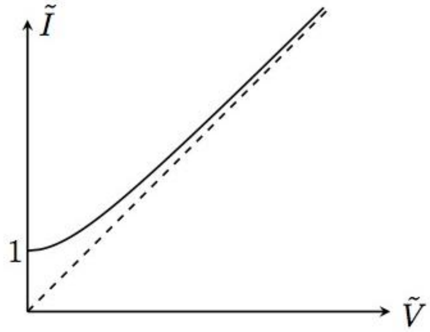
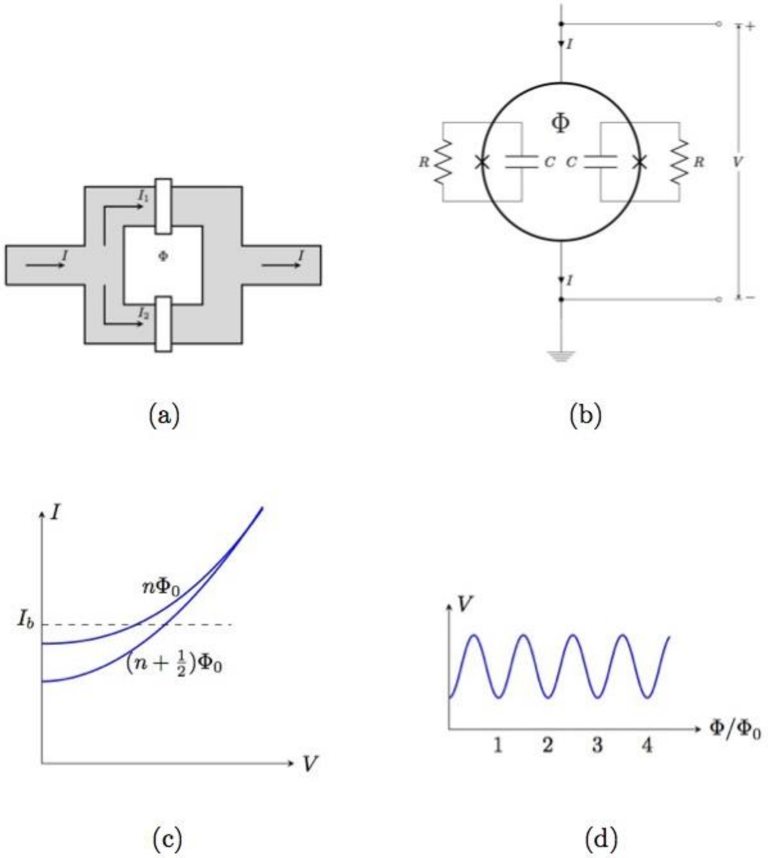


Figure 3: Volt-Ampere characteristics in terms of normalized quantities $\tilde{V}$ and $\tilde{I}$. The dashed line indicates ohmic relation.

3. SQUIDs

SQUIDs are Superconducting QUantum Interference Devices [3-6]. They are the most sensitive detectors of magnetic flux presently known. They are quite versatile, capable of measuring any physical quantity that can be eventually converted to magnetic flux; for example, magnetic field, or the gradient thereof, voltage, current, displacement, magnetic susceptibility and so on. It is a highly sensitive magnetometer capable of measuring magnetic fields as feeble as even 5 X 10^{-18} T. The need for accurate measurement of magnetic field arises in various applications like geomagnetism, biomagnetism, magnetic microscopy, space magnetometry, magnetocardiography, nuclear magnetic resonance (NMR) or low magnetic field magnetic resonance imaging (MRI), metrological application, magnetic microscopy, nondestructive testing or evaluation and magnetic anomaly detection etc. It is hypothesized that certain animals generate tiny levels of magnetic flux from their brain in order to navigate. SQUIDs greatly help in such investigations. Environmental magnetic noise is likely to corrupt the magnetic field measurements unless the measuring device has a high figure of merit; hence, SQUIDs are proposed as viable option.



(a) (b)

(c) (d)

Figure 4: (a) The structure of a DC SQUID (b) Schematic and the shunt resistive load (c) V-I Characteristic for $\Phi = n\Phi_0$ *and for* $\Phi = (n+\frac{1}{2})\Phi_0$ *cases d) Voltage-flux response characteristics*

There are two central ideas in the development of SQUIDs: (i) magnetic flux quantization and (ii) Josephson tunneling. Just as fundamental charge of an electron is $e = 1.6022\times10^{-19}$ C, magnetic flux is quantized in steps of $\Phi_0 = \frac{h}{2e} = 2.0678$ femto webers, where h is the Planck's constant. The Josephson tunneling effect is the phenomenon of supercurrent, a current that flows indefinitely long without any voltage applied across a junction, which consists of two superconductors coupled by a weak link. The weak link may be composed of a thin insulating barrier, known as a superconductor-insulator-superconductor junction, or SIS, a short section of non-superconducting metal or SNS, or a physical constriction that weakens the superconductivity at the point of contact. SQUIDs are made of a superconducting loop with Josephson junctions. There are two types of SQUIDs - (i) direct current SQUIDs or DC SQUIDs and (ii) radio frequency SQUIDs or RF SQUIDs. Comparatively, RF SQUIDs are less sensitive than DC counterparts, as they employ only one Josephson junction and for this reason they are less expensive.

Essentially, a DC SQUID, as depicted in Figure 4(a), consists of two junctions that are connected in parallel on a superconducting loop of inductance L. Each junction is resistively shunted to eliminate hysteresis on the volt-ampere characteristic. Figure 4(b) shows the schematic with shunted resistive loading and Figure 4(c) shows a typical characteristic for flux of the form $\Phi = n\Phi_0$ and $\Phi = (n+1/2)\Phi_0$, where Φ is the applied magnetic flux, and n is an integer. If we bias the SQUID with a constant current above a threshold, the voltage across the SQUID oscillates with period Φ_0, as we steadily increase Φ, as indicated in Figure 4(d). The SQUID is generally operated on the steep part of the V-Φ curve where the slope is maximum. Thus, the SQUID produces an output voltage in response to even a small input flux, and is effectively a flux-to-voltage transducer. A DC SQUID employing two resistively shunted Josephson tunnel junctions as shown in Figure 4(a) is typically constructed from thin films of superconductors of low transition-temperature T_c. The SQUID is coupled to an integrated superconducting coil carrying a signal source. A bias current entering the SQUID splits into the two parallel branches containing a Josephson junction each. In the absence of any external magnetic field, the input current splits into the two branches equally. If a small external magnetic field is applied to the superconducting loop, a screening current begins to circulate in the loop that generates a magnetic field canceling the applied external flux. The induced current is in the same direction as the incoming bias-current in one of the branches of the superconducting loop, and is opposite to it in the other branch. As soon as the current in either branch exceeds the critical current of the Josephson junction, a voltage appears across the junction.For a given constant biasing current into the SQUID device, the measured voltage oscillates with the changes in phase at the two junctions, which depends upon the change in the magnetic flux. Thus one might estimate the change in magnitude of the incident flux in terms of the voltage alterations.

The RF SQUID involves a single Josephson junction interrupting the current flow around a superconducting loop and is operated with a radio frequency flux bias. In both cases, the output from the SQUID is periodic with period in the magnetic flux applied to the loop. One generally is able to detect an output signal corresponding to a flux change of extremely small magnitude. Instruments based on low critical temperature SQUIDs include magnetometers, magnetic gradiometers, voltmeters, susceptometers. amplifiers, and displacement sensors; their applications vary from neuromagnetism and magnetotelluric sounding to the detection of gravity waves and magnetic resonance. The applications of SQUIDs are wide ranging, from the detection of tiny magnetic fields produced by the human brain and the measurement of fluctuating geomagnetic fields in remote areas to the detection of gravity waves and the observation of spin noise in an ensemble of magnetic nuclei.

4. Conclusion

SQUIDs can detect magnetic flux density of the order of 5 quintillionths of a tesla or even less. Capable of measuring such minuscule magnetic fields is useful for many an application including cosmic geophysical and archeological surveys, nondestructive testing of materials and devices, and imaging the brain, heart, and other body parts. Many materials that can be used in the preparation of SQUID devices are being identified and are found to offer greater prospects than semiconductors in the future. At first the invention of the SQUID device held out no obvious benefit but after few decades they are finding widespread use. In this brief note we have presented their working principle and potential applications.

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