HEAT AND MASS TRANSFER ANALYSIS OF A GRAIN DRYER
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تحليل انتقال الحرارة والكتلة لمجفف الحبوب

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ABSTRACT

Grain drying is essential for preserving grains. In this work, heat, and mass transfer analysis is done for a thin layer dryer that consists of radiated semi-cylindrical wall and two individual flat plates at the base. The grain plate is a conveyor bed, whereas the other plate is a heater. The radiation heat transfer from the heated plate is used to remove the moisture. The plate temperature and convective heat transfer coefficient are calculated under different operating conditions. At design conditions, the required heat flux for evaporating the moisture content is 0.89 of the radiation heat flux from the plate.

الملخص

إن تجفيف الحبوب عنصر أساسي للحفاظ عليها. في هذا العمل تم إجراء تحليل لانتقال الحرارة والكتلة لمجفف ذي طبقة رقيقة يتكون من جدار نصف اسطواني مشع ولوحين مسطحين فرديين في القاعدة. حيث إن صفيحة الحبوب عبارة عن سرير متحرك بينما الصحيفة الأخرى عبارة عن سخان. حيث يتم نقل الحرارة بالإشعاع لإزالة الرطوبة. وتم حساب درجة حرارة اللوحة ومعامل نقل الحرارة في ظروف التشغيل المختلفة. حيث إنه في ظروف التصميم، يكون تدفق الحرارة المطلوب لتبخير محتوى الرطوبة هو 0.89 من تدفق حرارة الإشعاع .

INTRODUCTION

Grains are the primary food source in the world. Grain storage is incredibly essential in order to prevent rotting and nuclei growth. Therefore, grains must be treated prior to being stored for a long or short time by extracting moisture from grains. Moisture removal is generally done using natural or hot air.

The most common drying methods are deep bed drying, batch drying and thin layer drying. Firstly, a deep bed dryer has a large number of grains that are dried with natural air in a typical bin with a fan. Unheated or slightly heated air (less than 6°C difference) is forced through the grains by a mechanical fan. Grains are dried first at the air inlet. Grains located near the air outlet are dried last. Disadvantages of this type are the maximum limit of initial moisture content that can be effectively dried, and electricity consumed by the fan *(Hellevang, 2013; Mrema et al., 2011)*.

Various computer programs simulate deep bed grain drying; after that, simulation results are compared with experimental results and confirmed within acceptable tolerances. The simulation and study computer programs are: COMSOL Multiphysics, Ansys, Visual Basic, FORTRAN and UNIX operating systems *(Dimitriadis and Akritidis, 2004; Elgamal, Kishk and Elmasry, 2017; Scaar et al., 2016; Reddy et al., 2017; Torki Harchegani et al., 2012; Pieters, Elgamal and Ronsse, 2013; Ranjbaran, Emadi and Zare, 2014; Ngunzi, Mugucia and Kituu, 2014; Satimehin, 2014)*.

Deep bed grain drying is studied under unsteady-state conditions. The effects of humidity, temperature, the velocity of air and grain temperature with a variation of bed height are studied. The partial and ordinary equations are solved by using backward implicit numerical scheme and Runge-Kutta method *(Srivastava and John, 2002)*.

Secondly, batch dryers are divided into bin batch and column batch dryers. The bin batch dryer is similar to deep bed dryer except the grains are placed into the drying bin as layers, and the bin has a fan and a heater. Typical farm size bin diameters range from 5.5 m to 14.6 m. There are several sizes of fans and heaters available to provide a wide range of drying capacities. The main disadvantage of this dryer is handling the grains twice rather than one. Other disadvantages are the considerable moisture variation in the dryer and grain damage, which may occur *(Bridge et al., 1986; Hellevang, 2013)*. This dryer may be used to dry wet green grains to minimize energy losses *(Najjar and Radhwan, 1988; Najjar and Zaamout, 1996; Najjar, Abubaker and El-Khalil, 2015)*.

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Another batch dryer is the column dryer. It consists of two columns surrounding a plenum chamber which has forced hot air. It can be moved from one location to another. The disadvantage of this type is the ineffective heat available in deep bed drying (*Hellevang, 2013*). Also, column dryer cannot be easily automated, so it requires more supervision and labour than other types of dryers.

Thirdly, the thin layer dryer consists of a thin grain layer moving along a conveyor bed. The thickness of the layers is about 20 cm. Fundamental theories and modeling of the thin layer are discussed in (Sahari and Driscoll, 2013; Esther Magdalene Sharon, Banuu Priya and Subhashini, 2016). The drying rate and diffusion coefficient are found to increase with increasing air temperature following the Arrhenius equation *(Watson and Bhargava, 1974)*. The obtained system of non-linear partial differential equations is numerically solved by a finite volume method (*Mabrouk, Khiari and Sassi, 2006*).

The thin layer dryer is farther modeled and experimentally studied (*Alibas, 2012*). The measured values are compared with predicted values, which are obtained from the theoretical equation. The drying air temperature and the pellets' size are influencing factors of drying kinetics. Statistical results of ten mathematical models at different drying conditions show that the modified Henderson and Pabis is the best model (*Bassene et al., 2013*). Regression analyses are carried out to investigate the thin layer model. It gives the best drying kinetics of the artificial and natural drying of cocoa beans (*Pandey, Diwan and Soni, 2015*). Then, modeling of thin layer drying kinetics of cocoa is established *(Hii, Law and Cloke, 2008)*.

The justifications for this research in the semi-cylindrical grain dryer are: firstly, thin layer can overcome the significant moisture variation, which exists in the bin batch dryer. Secondly, this dryer overcomes the limitation of grains having initial moisture content that can be effectively dried in deep bed dryers. Thirdly, there is not any research in drying analysis using the heat and mass transfer in the semi-cylindrical grain dryer.

MATERIALS AND METHODS
THEORETICAL ANALYSIS
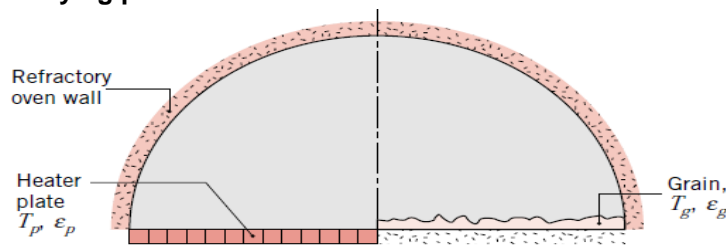#### A. Model of the drying process

Fig. 1 - Front view of the grain dryer

Figure 1 shows the front view of the one-meter long semi-cylindrical grain dryer. The wall of the dryer is a semi-cylindrical reradiated wall, with a one-meter radius. The base surface of the dryer consists of two halves. The first half is an electrically heated plate. The second half is a conveyer belt which moves the grains.

The required heat flux to evaporate the water from the grain is equal to:

$$q = m\, h_{fg} \ [W] \tag{1}$$

Where m is the evaporated water flow rate; h_{fg} is the latent heat of vaporization.

Radiation heat transfer rate q_{rad} is the summation of evaporation and convection heat transfer rate (*Incropera et al., 2007*) as:

$$q_{rad} = q + q_{conv} \ [W] \tag{2}$$

Also, this q_{rad} is derived in Appendix A as:

$$q_{rad} = \frac{A\,(E_p - E_g)}{\frac{1-\epsilon_p}{\epsilon_p} + \frac{1}{F_{pg} + \left(\frac{1}{F_{pR}} + \frac{1}{F_{gR}}\right)^{-1}} + \frac{1-\epsilon_g}{\epsilon_g}} \ [W] \tag{3}$$

Where F_{pR} is the fraction of the radiation that leaves heated plate surface, and it is intercepted by radiated wall surface; F_{gR} is the fraction of the radiation that leaves radiated wall surface, and it is intercepted by grain surface; E_p and E_g are the emissive powers from heated plate and grain surfaces; A is the surface area; ϵ_p and ϵ_g is the emissivity from heated plate and grain surfaces.

$$q_{conv} = hA(T_g - T_{air}) \text{ [W]} \tag{4}$$

Where h is the convection heat transfer coefficient; T is the temperature. The convective heat transfer from the plate to air is assumed to be zero, since air flows over the grains, and not over the plate, and the free convection heat transfer is small.

The convection heat transfer coefficient is equal to (*Incropera et al., 2007*):

$$h = h_m \, \rho_a \, cp \, Le^{2/3} \text{ [W/m}^2\text{K]} \tag{5}$$

The evaporated water mass flow rate is equal to (Incropera et al., 2007):

$$m = h_m \, A(\rho_{vs} - \rho_{va}) \text{ [kg/h.m]} \tag{6}$$

Where h_m is the convective mass transfer coefficient; ρ_a is the air density; $\rho_{v,s}$ and $\rho_{v,a}$ are the vapour density at the grain surface and in the air; cp is the specific heat at constant pressure; Le is the Lewis number.

The electrical energy consumption Q_{ele} from electric resistance heater is equal to radiation heat transfer (*Demirel, 2012*), as:

$$Q_{ele} = q_{rad} \text{ [W]} \tag{7}$$

B. Design point calculations

The given parameters are the evaporated water flow rate, the temperatures of the grain and the air, the emissivity of the grain and plate. Unknown parameters are the radiation and convection heat flux, the plate temperature, the coefficients of convection, and mass transfer.

Maximum recommended moisture contents of most grains for long storage range from 11% to 13% (*Hellevang, 2013*). For wheat, the maximum recommended moisture content for long storage is equal to 13%. Harvesting delays after wheat firstly reaches 15%. Moisture reduces wheat returns in several ways, and increased field shatter reduces yields (*Gardisser and Huitink, 2006*). Evaporated water flow rate, m, for 12.5 cm grain layer equals:

$$m = \rho_w v_g \, \Delta w \text{ [kg/h.m]} \tag{8}$$

where ρ_w is the water density; v_g is grains volume in the dryer; Δw is the change in moisture content percent.

$$m = 2.5 \left[\tfrac{\text{kg}}{\text{h}}. \text{m} \right] \tag{9}$$

This value will be dried in a one-meter-long dryer for one hour.

The value of the evaporated water flow rate is equal to 2.5 kg/h.m. The maximum recommended drying temperature for continuous flow dryer is shown in table 6 (*Hellevang, 2013)*. It ranges between (305 K - 365 K), then the proposed design air and grain temperatures are 300 K and 330 K, respectively. The iron emissivity depends on the temperature and oxidization status. The emissivity of oxidized iron at 300°C is equal to 0.8 (Mikron Instrument Company). The emissivity values for conventional wood products are generally within the range 0.89-0.92 (*Rice, 2004*), and the grain emissivity is chosen to be 0.9.

Design point particulars   **Table 1**

Evaporated water flow rate, kg/h.m	2.5
Grain temperature, K	330
Air temperature, K	300
Grain emissivity	0.9
Plate emissivity	0.8

Evaluating the value of the unknown parameters must be by solving the previous equations. By using thermophysical properties of saturated water, the properties of the saturated water at T = 330 K are v_g = 8.82 m³/kg, h_{fg} = 2.366 × 10⁶ J/kg. By using thermophysical properties of gases at atmospheric pressure table, the properties of the air at $T \approx$ 300 K, ρ = 1.17 kg/m³, cp = 1007 J/kg.K, α = 22.5 × 10⁻⁶ m²/s.

By using binary diffusion coefficients at the one-atmosphere table, the property of the H₂O(v) – air at T = 298 K: D$_{AB}$ = 0.26 × 10⁻⁴ m²/s.
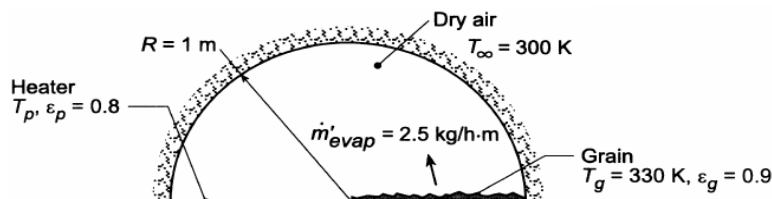

Fig. 2 - Analysis of the semi-circular grain dryer

By using equation (1), the required heat flux to evaporate the water is 1643 W/m. By using equation (3),where A= radius = 1 m and $F_{pR} = F_{gR}$ = 1 and F_{pg} = 0,the radiation heat flux is:

$$q_{rad} = 2.4 \, x10^{-8} \, x \, Tp^4 - 284.62 \quad [W] \tag{10}$$

By using equations (5) and (6) with assumption $\rho_{v,a}$=0, the convection heat and mass coefficients are 6.13×10^{-3} m/s and 6.5 W/m^2.K, respectively. By using equation (4), the convection heat flux is equal to 195 W/m. By using equation (2) and equation (7), the temperature of the plate is equal to 545.34 K

C. Off-design characteristics

Every grain type has the ultimate temperature. The grain will be damaged if its temperature excesses the ultimate point. The amount of evaporated water from grains can be controlled. The emissivity of the plate affects its needed temperature. All these variables will be studied in the off-design characteristic. Table 2 shows the off-design values.

Table 2

The variation of the operating variables

Variable name	Part Load				Design Point	Over Load
Temperature of grain, K	300	313	315	320	330	333
Evaporated water flow rate, kg/h.m	1.5	1.75	2	2.25	2.5	2.65
Plate emissivity	0.6	0.65	0.7	0.75	0.8	0.85

RESULTS

A. Design Point

The design variables are shown in table1. The required evaporation heat, radiation and convection heat transfer, the convection heat and mass transfer coefficients and plate temperature are presented in detail in table 3.

Performance Parameters **Table 3**

Required heat flux for evaporation, W/m	1643
The convection mass transfer coefficient, m/s	6.13×10^{-3}
The convection heat transfer coefficient, W/m^{2}K	6.5
Convection heat flux , W/m	195
Plate temperature, K	545.34 K
Radiation heat flux , W/m	1838

As shown in table 3, the plate temperature is 272°C, and the plate can easily be heated up to this temperature. The convection heat transfer coefficient is 6.5 W/m^{2}K. Referring to table 1.1 *(Incropera et al., 2007)*, the convection heat transfer coefficient can be generated easily.

B. Off-design characteristics

Table 4 shows the value of the plate temperature and the convection heat mass transfer at different grains temperatures.

Table 4

Effect of changing the temperature of the grains

Grains temperature, K	Plate temperature, K	Convection heat transfer coefficient, W/m^2. K
300	529.4	29.19
313	543.3	14.7
315	544	13.31
320	545.1	10.44
330	545.4	6.599
333	545.3	5.787

As shown in figure 3, the needed temperature of the plate will increase as grains' temperature increases until it reaches its maximum point. After that, it will decrease. This relation allows us to use the maximum temperature point to manufacture the heating plate. Due to increasing water vapor density at the grain surface as the grains' temperature increases, the heat transfer coefficient will decrease.
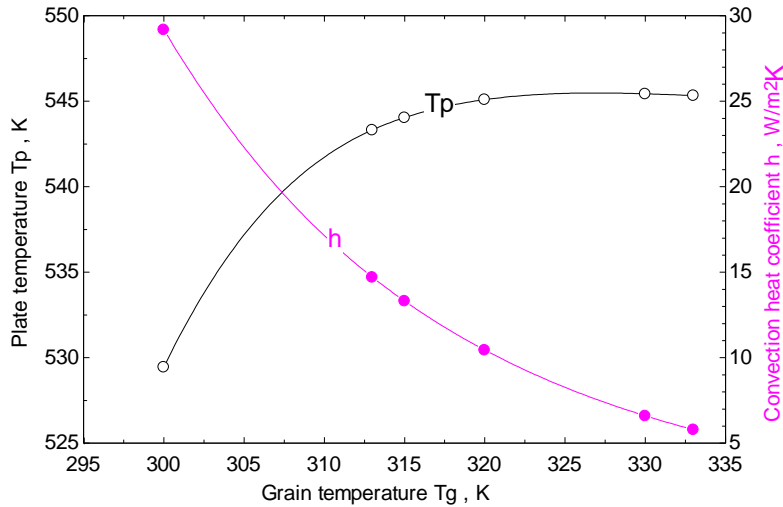

Fig. 3 - Variation of plate temperature and convective heat coefficient with grain temperature

Table 5 shows the value of the plate temperature and the convective heat mass transfer at a different evaporated mass flow rate.

Table 5

Effect of changing the evaporated mass flow rate

Evaporated water flow rate, kg/s	Plate temperature, K	convection heat transfer coefficient, W/m^2.K
1.5	490.4	3.96
1.75	505.9	4.62
2	520.1	5.279
2.25	533.2	5.939
2.5	545.4	6.599
2.65	552.4	6.995

As shown in figure 4, the needed temperature of the plate and the heat transfer coefficient will increase as evaporated water flow rate increases. Table 6 shows the values of the plate temperature and the convection heat mass transfer at different plate emissivity.
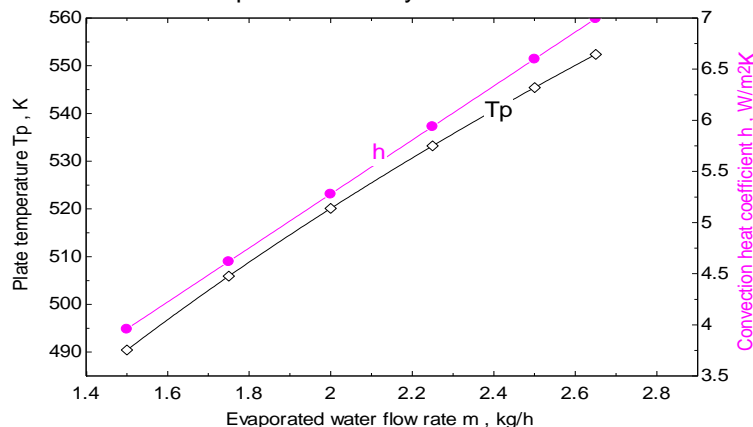

Fig. 4 - Variation of plate temperature and convection heat coefficient with evaporated water flow rate

Table 6

Effect of changing the plate emissivity for different materials

Plate emissivity	Plate temperature, K
0.6	565.2
0.65	559.3

Table 6
(continuation)

Plate emissivity	Plate temperature, K
0.7	554.2
0.75	549.6
0.8	545.4
0.85	541.7

As shown in figure 5, the needed temperature of the plate will decrease as plate emissivity increases. That means that when we use a plate with suitable emissivity, we reduce energy consumption. The heat transfer coefficient does not depend on the plate emissivity.
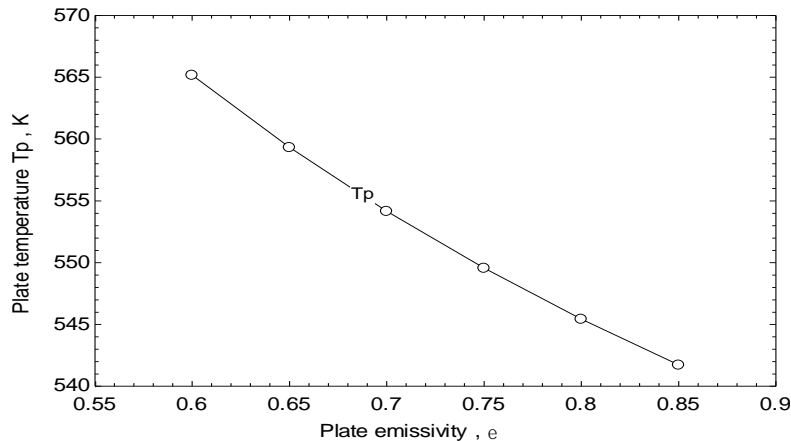

Fig. 5 - Variation of plate temperature with Plate emissivity

C. Energy consumption optimization

The main objective of this optimization is to minimize the energy consumption for evaporating water from grains at a rate of 2.5 kg/h.m. involving convection heat flux, plate temperature and required heat flux for water evaporation. The optimization can be carried out by defining:

Minimize $Q_{ele}(T_g, T_{air}, \varepsilon_p, \varepsilon_g)$, subject to:

$273 < P_{air} < 350$ and $273 < T_g < 350$, $0 < \varepsilon_p < 1$ and $0 < \varepsilon_g < 1$

The results are shown in table 7. The optimum value of the electrical consumption is 1632 W, at specific operating variables. Also, the value of the plate temperature and the convection heat transfer coefficient are evaluated.

Table 7

Optimization results

OPERATING VARIABLES	Optimum values of operating variables
Grains temperature, K	335
Air temperature, K	335
Grain emissivity	1
Plate emissivity	1
Energy consumption, W	1632
Plate temperature, K	524
Convection heat transfer coefficient, W/m^2.K	4.765

CONCLUSIONS

1. Due to the relatively low temperature of the heating plate (about 200 ºC), It will be relatively more comfortable to handle the supply.
2. The generated heat rate from the plate that is required to evaporate the moisture content from the grains is equal to 1841 W/m. This is equivalent to 113% of the evaporation rate of the grains.
3. 11% of radiation heat is lost as convection heat losses due to the air passing over grains surface.
4. This type of grain dryer could be used in practice for drying a multitude of grain types: wheat, barley, maize, etc.

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Appendix A

Equation Three Derivation

The term q_i, which is the net rate at which radiation leaves surface *i*, represents the net effect of radiative interactions occurring at the surface. It is the rate at which energy would have to be transferred to the surface by others. It is equal to the difference between the surface radiosity and irradiation and may be expressed as:

$$q_i = Ai (E_i - \alpha_i G_i)$$

By using $\rho_i = 1 - \alpha_i = 1 - \varepsilon_i$. For an opaque, diffuse, and gray surface, the previous equation will be:

$$J_i = \varepsilon E_{bi} + (1 - \varepsilon_i) G_i$$

Solving for G_i and substituting into the first equation, it follows that :

$$q_i = \frac{E_{bl} - J_i}{(1 - \varepsilon_i)/\varepsilon_i A_i}$$

The irradiation of surface *i* can be evaluated from the radiosities of all the surfaces in the enclosure. In particular, from the definition of the view factor, it follows that the total rate at which radiation reaches surface i from all surfaces, including *i*, is:

$$A_i G_i = \sum_{J=1}^{N} A_i F_{ij} J$$

Cancelling the area A_i and substituting into the first equation for G_i, then

$$q_i = \sum_{J=1}^{N} A_i F_{ij} (J_i - J_j) = \sum_{J=1}^{N} q_{ij}$$

This result equates the net rate of radiation transfer from surface i, q_i, to the sum of components q_{ij} related to radiative exchange with the other surfaces.

The heat transfer among three surfaces enclosure with one reradiating surface is equal to :

$$q_1 = -q_2 = q_{12}$$

For $q_i = 0$, $G_i = J_i = E_{b_i}$. We can find the heat transfer by using the heat transfer network of the enclosure surfaces between three walls, so the radiation heat transfer is equal to:

$$q_{12} = \frac{E_{bl\,2} - E_{bl\,1}}{\text{resistance}}$$

Surface R is presumed to be well insulated, and convection effects are assumed to be negligible. Hence, with $q_R = 0$ the net radiation transfer from surface 1 must equal the net radiation transfer to surface 2.

Then,

$$q_1 = -q_2 = q_{12} = \frac{A (E_1 - E_2)}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{12} + \left(\frac{1}{F_{1R}} + \frac{1}{F_{2R}}\right)^{-1}} + \frac{1-\epsilon_2}{\epsilon_2}}$$

The first and second surfaces are the plate and the surfaces of the grains, respectively. Then:

$$q_1 = -q_2 = q_{12} = \frac{A (E_p - E_g)}{\frac{1-\epsilon_p}{\epsilon_p} + \frac{1}{F_{pg} + \left(\frac{1}{F_{pR}} + \frac{1}{F_{gR}}\right)^{-1}} + \frac{1-\epsilon_g}{\epsilon_g}}$$