MATHEMATICAL MODEL FOR THE EVOLUTION OF *Chlorella Algae* / *MODEL MATEMATIC PENTRU EVOLUȚIA ALGELOR CHLORELLA*

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ABSTRACT

The paper presents an example of a mathematical model for algal cultures. The model presented is based on an elementary growth equation (more generally, evolution or dynamic, because it includes the phenomenon of decreasing population), represented by a first-order differential equation, over which are grafted (according to the influence parameters model, for example used for the USLE modelling of soil erosion is used), as factors of influence, the parameters that influence the evolution of the algae. We have given some examples of simulation of algal evolution processes and the possible applications, advantages and disadvantages of the model are suggested. References are made and conclusions are drawn on the extremely important role played by experiments both in the construction and in the validation of the mathematical models of the living systems (bio-systems) evolution phenomena. The model is easy to transfer and generalize to crop plants, as well as other bio-systems.

REZUMAT

Lucrarea prezintă un exemplu de model matematic pentru culturile de alge. Modelul prezentat se bazează pe o ecuație elementară de creștere (mai general, evoluție sau dinamică, deoarece include fenomenul de scădere a populației), reprezentat de o ecuație diferențială de ordinul întâi, peste care sunt altoiți (conform modelului parametrilor de influență, folosit spre exemplu, in modelul USLE al eroziunii solului), ca factori de influență, parametrii care influențează evoluția algelor. S-au prezentat câteva exemple de simulare a proceselor de evoluție a algelor și s-au sugerat posibilele aplicații, avantaje și dezavantaje ale modelului. S-a făcut referire și s-au tras concluzii cu privire la rolul extrem de important pe care îl au experimentele atât în construcția, cât și în validarea modelelor matematice ale fenomenelor evoluției sistemelor vii (biosisteme). Modelul este ușor de transferat și generalizat la plantele de cultură, precum și la alte bio-sisteme.

INTRODUCTION

The mathematical modelling of algal growth process is generally approached very often in the literature. Under these conditions, it's both hard and easy to be original. It is very difficult to fundamentally modify a mathematical model or build a new one. It is easy to operate a simple change, thus obtaining a new model, starting from an old one. In both cases, validation is required. However, the most difficult experimental stage is the previous one, and simultaneously the mathematical model.

In the dynamic mathematical modelling of algal crops development, a fairly clear distinction can be made between models that work with a single differential equation of growth, (*Stemkovski et.al., 2016; Thornton et al. 2010; Yang JSh et al. 2011; Yang Zh et al., 2017*), the rest of the parameters being loads or control functions, and respectively, the models working with a system of differential equations, (*Concas et al., 2013; Davidson and Gurney, 1999; Mardlijah et.al., 2017*). In the latter case, some of the process's influence parameters, or all, become unknown functions of the differential equations system, but they also have charging and / or elimination components.

From an experimental point of view, it is simpler to construct and validate a mathematical model in the first category. When experimental possibilities are lower or at an early stage, a simple model from the first category is therefore more appropriate. For these reasons, the experimental model presented in this paper is part of the first category. The development can be done after validation, gradually, taking into account the large variety of mathematical models existing in the literature.

MATERIALS AND METHODS

The results presented in this article are based on hypotheses formulated using data and experimental conclusions by *Blinova et.al. (2015)* and *Nedelcu et.al. (2018 a, b)*.

1) General Algal Evolution Model

The mathematical model presented in this paper is an evolutionary model of the *Chlorella vulgaris* algae crop. We call it a model of evolution because it is not only a growth mathematical model; it is a model that can also describe the decline of the algae population, including the death of the algae colony.

The evolution of the algae population is described by a single ordinary differential equation *(Chen Sh et al., 2009; Surendhiran et.al., 2015 and Thornton et al., 2010)*:

$$\frac{dx}{dt} = \mu(t, x) - \delta(t, x) \tag{1}$$

Mathematical models of type (1) are frequent in the literature dedicated to the evolution of living matter and originate in the classical growth model that only shows the growth rate in the right side. If the growth rate is strictly positive, then the population grows monotonous. The origin of this model is found in (*Malthus, 1798*), in the finite form. The logistic form of growth is formulated by *Verhulst (1838)* and *McKendricka & Kesava (1912),* for example. Many contemporary mathematical models retain the fundamental forms of growth or modify them, more or less *Stemkovski et.al. (2016) and Thornton et al. (20100,* for example.

The names, meanings, and measurement units of all model parameters are given in Table 1.

Table 1

Notation, meanings and units of the mathematical model

Notation	Meanings	Units
x	algae concentration in solution	g/l
t	time	hours
x_0	initial value of algae concentration in solution	g/l
t_0	initial time	hours
μ	the growth rate of the algae population	1/hours=hours^{-1}
δ	the decline rate of the algae population	1/hours=hours^{-1}
μ_0	model parameter to be determined experimentally	1/hours=hours^{-1}
δ_0	model parameter to be determined experimentally	1/hours=hours^{-1}
θ	average temperature in the algae crop solution	Celsius degree
co2	time dependence in carbon dioxide concentration in the solution	%
ph	time dependence of pH in the solution	-
cfIL	minimum degree of illumination	lux
cfILb	degree of illumination with blue light	lux
cfILr	degree of illumination with red light	lux
pl	wavelength of light	nm
s	time dependence of the solution salinity	g/l
Θ	the function of the temperature influence	-
CO_2	the function of influence of the dissolved carbon dioxide concentration in the solution	-
pH	the function of the algae crop solution pH	-
L	the function of the degree of illumination influence	lux
N	the function of the nutrient concentration in the algae crop solution	-
S	the function of the algae crop salinity influence	-
$\delta_{\lim}$	natural factor of algae growth moderation at reaching a limit concentration	-
$x_{\max}$	maximum algal concentration in solution	g/l
δ_1	model parameter to be determined experimentally	dimensionless

Obviously, the model (1) may become a growth and decrease model, even if only the growth rate is considered, assuming that it can move from negative values to positive values, and possibly return to positive values.

2) The modelling of the process influences by multiplication of separate influences

According to (*Blinova et.al., 2015 and Nedelcu et.al., 2018a, b*), is considered the ***hypothesis*** that the physical parameters which influenced the algal evolution, are the following: solution temperature variation,

variation in the amount of dissolved carbon dioxide in the solution, pH variation of the solution, degree of crop illumination, the amount of nutrients introduced into the solution, the salinity of the solution.

A second ***hypothesis*** we introduce is the structure by of product, of the growth rate:

$$\mu(t,x)=\mu_0\cdot x\cdot\Theta\cdot\left(1-CO_2\right)\cdot pH\cdot L\cdot N\cdot S \tag{2}$$

and of the decrease rate:

$$\delta(t,x)=\delta_0\cdot\left(1-pH\right)\cdot x\cdot\delta_{\lim}(x) \tag{3}$$

where the form of the moderation factor, by ***hypothesis***, is the following:

$$\delta_{\lim}(x)=\begin{cases}1, if\ \ x<x_{\max}\\ \delta_1\cdot\dfrac{x-x_{\max}}{x_{\max}}, if\ \ x\geq x_{\max}\end{cases} \tag{4}$$

The parameters that influence and form the structure of the growth and decrease rates take values between 0 and 1. These functions, the influence parameters, are defined in the following subchapter.

3) Bump functions

The ***bump functions*** are functions of one or more real variables, with values in the set of real numbers, which is both smooth (in the sense of having continuous derivatives of all orders) and compactly supported, *Muthukumar, (2016); Thornton et al., (2010), Tu, (2008) and Fry and McManus, (2002),* for example.

In this paper we used *bump functions* to model the action of factors that influence the evolution of algae, particularly Chlorella. The parameterization of curves which is the graph of the bump functions (of a single variable) is given in Figure 1.

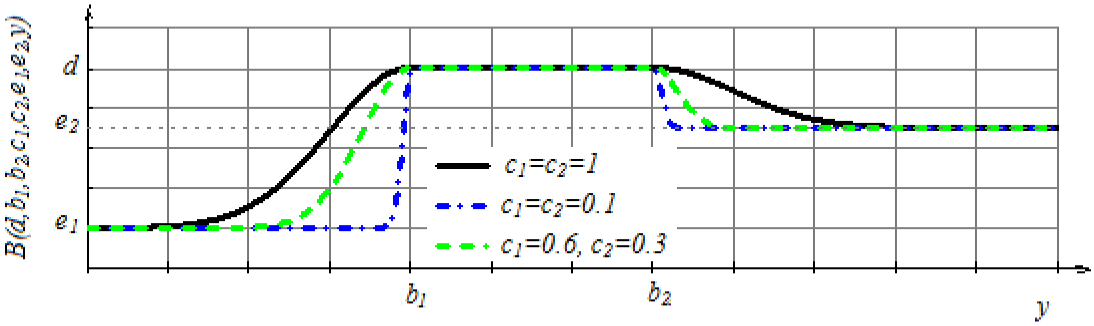

Fig. 1 - Parameterization of bump function, which is used to model the influence of factors involved in the algal evolution process

The phenomenological interpretation of a bump function consists of filtering an optimal zone of influence or, on the contrary, an area unfavourable to the phenomenon. This area is geometrically visible by the constant height *d*, bounded by the abscissa b_1 and b_2. The curve in Figure 1 corresponds to a bump function that models a parameter with only one optimal interval. The linear summation of such functions yields models for influence factors with several optimal ranges. Influence factors must not necessarily be annulled outside the optimum range. Influence factors modelled by *bump functions* can take non-zero values outside the optimum range. Outside of optimal interval, the *bump function*, asymptotically converge to the values e_1 and e_2 (can be, or not be zero).

To define the bump function, first define the Gaussian function:

$$G(p,q,r,s,y)=p\cdot\exp\left(\frac{-(y-q)^2}{2r^2}\right)+s \tag{5}$$

where *p*, *q*, *r* and *s* are parameters and *y* is the real variable. Using the Gaussian function (5), the function of the bump is defined by the formula (6):

$$B(d,b_1,b_2,c_1,c_2,e_1,e_2,y)=\begin{cases}G(d-e_1,b_1,c_1,e_1y), if\ \ y<b_1\\ d, if\ \ y\in\left[b_1,b_2\right]\\ G(d-e_2,b_2,c_2,e_2y), if\ \ y>b_2\end{cases} \tag{6}$$

The bump functions can be extended to forms that are not constant on the centre of optimal range.

4) The final description of the model

Taking into account the observations (in *Blinova et.al., 2015*) regarding the influence of temperature on algae evolution, the function of the temperature influence is defined by the following formula[1]:

$$\Theta(\theta) = B(1{,}16{,}27{,}2{,}3{,}0.005{,}0.05{,}\theta) \tag{7}$$

Experimental observations in *Blinova et.al., 2015*, lead to assumptions that result in the following influence functions for algae growth:

$$CO2(co2) = 1 - B(0.4{,}0.015{,}0.055{,}0.01{,}0.01{,}0.1{,}0.1{,}co2) \tag{8}$$

for the CO2 concentration influence function (see also *Mardlijah et.al., 2017; Singh SP and Singh P, 2014*),

$$pH(ph) = B(1{,}7{,}9{,}2{,}4{,}-0.5{,}-0.5{,}ph) \tag{9}$$

for the pH influence[2] function,

$$L(pl) = cfIL[cfILb \cdot B(1{,}400{,}500{,}1{,}1{,}0.5{,}0.5{,}pl) + cfILr \cdot B(1{,}600{,}700{,}1{,}1{,}0.5{,}0.5{,}pl)] \tag{10}$$

for the light influence[3] function,

$$S(s) = B(1{,}0.022{,}0.026{,}1{,}1{,}0.2{,}0.2{,}s) \tag{11}$$

for the salinity influence function,

$$N(\chi) = B(1{,}0.006{,}0.009{,}1{,}2{,}1{,}0.0{,}\chi) \tag{12}$$

for the nutrition influence function. Establishing the nutrient requirement for Chlorella culture requires two essential aspects: the amount of nutrients needed and the nutrient quality. The two aspects can be introduced into the influence function (12), as follows: the limits b_1 and b_2 denote the minimum and maximum values of the feed requirement per algal concentration in the solution, and the value of the constant ceiling *d* will be expressed according to the load with nutrients and the structure[4] of nutrients administered. The quantitative aspect also requires discussion. The amount of nutrients can be gradually added by progressive loading, depending on the evolution of the biomass concentration in the solution, or it can be discharged into the initial solution at the beginning, taking into account a limit value of the biomass concentration at the time of harvesting. Consideration should be given, in the latter case, to possibilities of nutrient degradation in interaction with the culture medium, and thus losses that can turn into biomass losses at harvest. Also, all these issues need to be discussed considering the use of algae crops (biofuels, food or medication). The final use of harvested biomass dictates the nutrition charge through the final filter of the content in useful substances of the crops. Numerous papers are dedicated to the content of valuable substances from algae, depending on the final use and the adopted cultivation technology, (*Yang JSh et al., 2011; Surendhiran et.al., 2015*).

RESULTS

The mathematical model of evolution of Chlorella algae proposed by the authors is described by equations (1) - (12). This model is complex in the sense that it includes command functions and filters, each with a number of parameters. The complete testing of such a model consumes a lot of time and the results, in detail, would be very bulky. For this reason, only some representative tests, performed on a simplified model, will be given in the results chapter. The simplicity that is made in the tests consists in considering a single rate of "growth", μ, which can be positive (and then the real growth rate) but also negative values, and in this case, the rate of decrease. Simplification can be synthesized by cancelling the parameter δ_0 in the formula (3). Due to this simplification, the asymptotic equilibrium phenomenon, given through the function $\delta_{\lim}$ is also lost.

1) The case of a growth regime with strictly controlled control parameters

A first elementary test of the simplified version of the mathematical model (1) - (12) consists in simulating a strictly controlled evolution regime: temperature, dissolved carbon dioxide concentration, pH and salinity,

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[1] The numerical arguments of these filter functions applied by algae to physical fields in the environment or applied by culture technologies may vary depending on the environmental conditions of each of the other filter functions, or the age of the algae colony, for example. Clarifying such behaviours requires very laborious experiences.

[2] To model the influence of pH on algae growth, we recommend, for example, *Ma et.al., 2017*.

[3] For a more precise formulation of the light algae power filter function, results such as those in *Huesemann et.al., 2016*, should be taken into account in the future.

[4] For example, in *https://www.wikihow.com/Grow-Chlorella-for-a-Food-Supplement*, accessed 07.01.2018, a nutrient recipe is given by the authors as optimal. This refers to Chlorella algae grown for food. It is possible to elaborate a quality coefficient that takes maximum value for the optimal concentrations prescribed by the authors for each component.

constants, and linear loading with nutrients, as it is specified in the commentary of Figure 2. Five constant illumination regimes, also specified in Figure 2, have been applied.

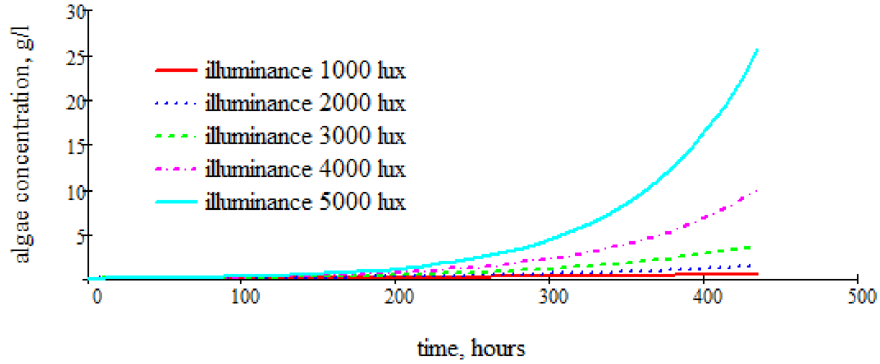


Fig. 2 - The evolution of algal concentration in the solution for a controlled regime: temperature 18 °C, CO_2 - 0.03, pH - 8, progressive loading with nutrients - 0.000001+0.7·t g/l, salinity - 0.023 g/l

It is noted that all the evolution curves of algal concentration in the solution are strictly increasing and preserve the same type of convexity. The curves in Figure 2 are common to growth phenomena, but most experimental results on algae show that their growth curves are closer to logistic growth than exponential growth. Logistic growth curves change the type of convexity in the course of evolution. The effect can also be obtained in model (1) - (12) considering a logistic factor function $\delta_{\lim}$, introduced in the definition of the growth rate, (4).

2) The effect of lowering the pH of the solution

Suppose there is an accidental loss of control of the pH of the solution containing the algae crop, either due to negligence or because of a pollution phenomenon, for example[5].

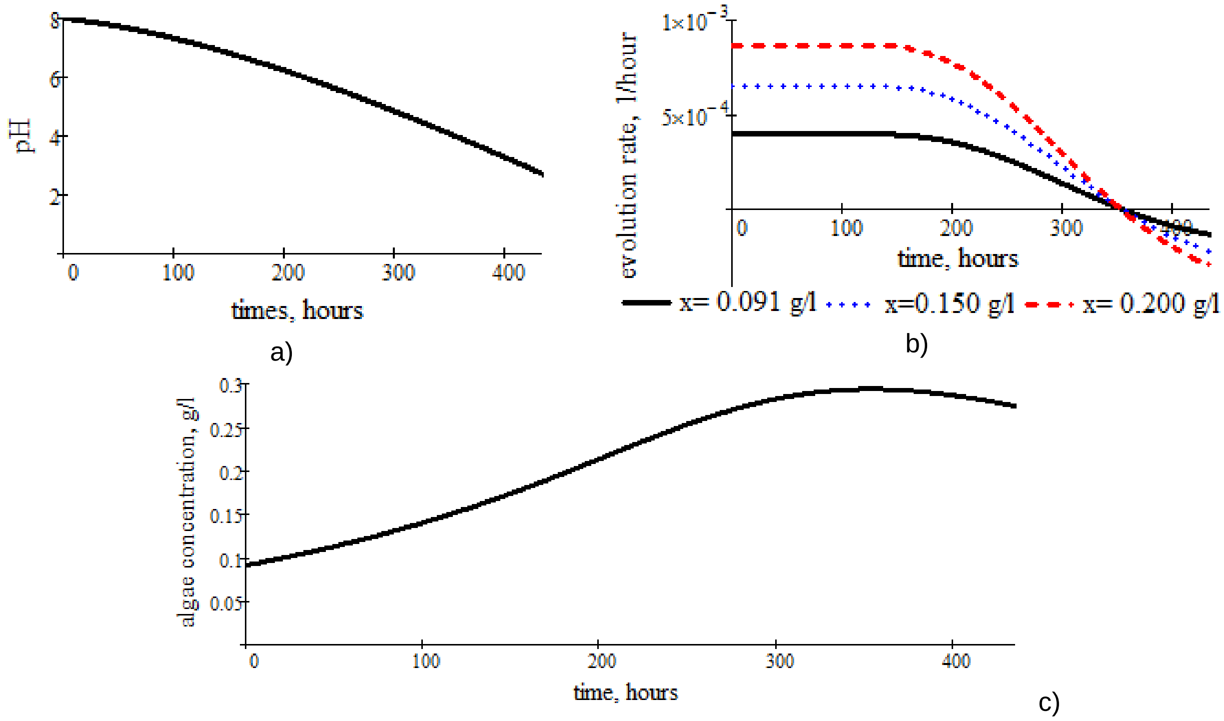


Fig. 3 - Effect of lethal, pH variation on algal concentration in solution

[5] There is also possible an intentional command of the operator, given in order to diminish the algae population, for various reasons: the supportability of the algae density or the qualitative structure of the crop. The problem is that in the case of such processes there are masses of dead algae that can produce positive (in the operator's interest) or negative effects, in which case the dead algal mass should be removed if possible.

The pH variation of the algae-containing solution is based on the scenario described in Figure 3 a). Under these conditions, the pH filter of the algae crop (8) implies the evolution rate of the variation in Figure 3, b). It is noted that there is a time value when the rate of evolution is cancelled regardless of algae concentration. This cancellation is due to the decrease of the pH below a lethal value (solution with unbearable acidity for the algae culture after going through zero, the evolution rate[6] becomes negative and the death process of some of the algae begins. This process is visible in Figure 3, c), in which, starting from the moment of the evolution rate cancellation, the algal concentration variation curve begins to decrease. With the exception of the pH of the algae solution and the illumination, kept constant at 1000 lux, the other control conditions of the process were maintained identical to those specified in the scenario of the first example, conditions specified in the comment in Figure 2.

3) The effect of the thermal variation

In this subchapter, an example of the influence of the thermal field large variations on the algal culture evolution is given. To achieve this aim, we have modified the thermal filter (7) to mark temperature limits beyond which algae may die. The modified thermal filter is given by the formula (13):

$$\Theta(\theta) = B(1,16,27,7,6,-0.5,-0.5,\theta) \quad (13)$$

The temperature dependence of the algae solution is given in Figure 4 a). The small amplitude and high frequency correspond to the variation of the diurnal temperature, and the component with high amplitude of the temperature time dependence corresponds to an accidental variation. This example is inspired by the experiences the processed results of which were exposed in *Nedelcu et.al., 2018b*. In these experiments the thermal control was minimal, applied only in order not to exceed the lethal limits of the algae crop.

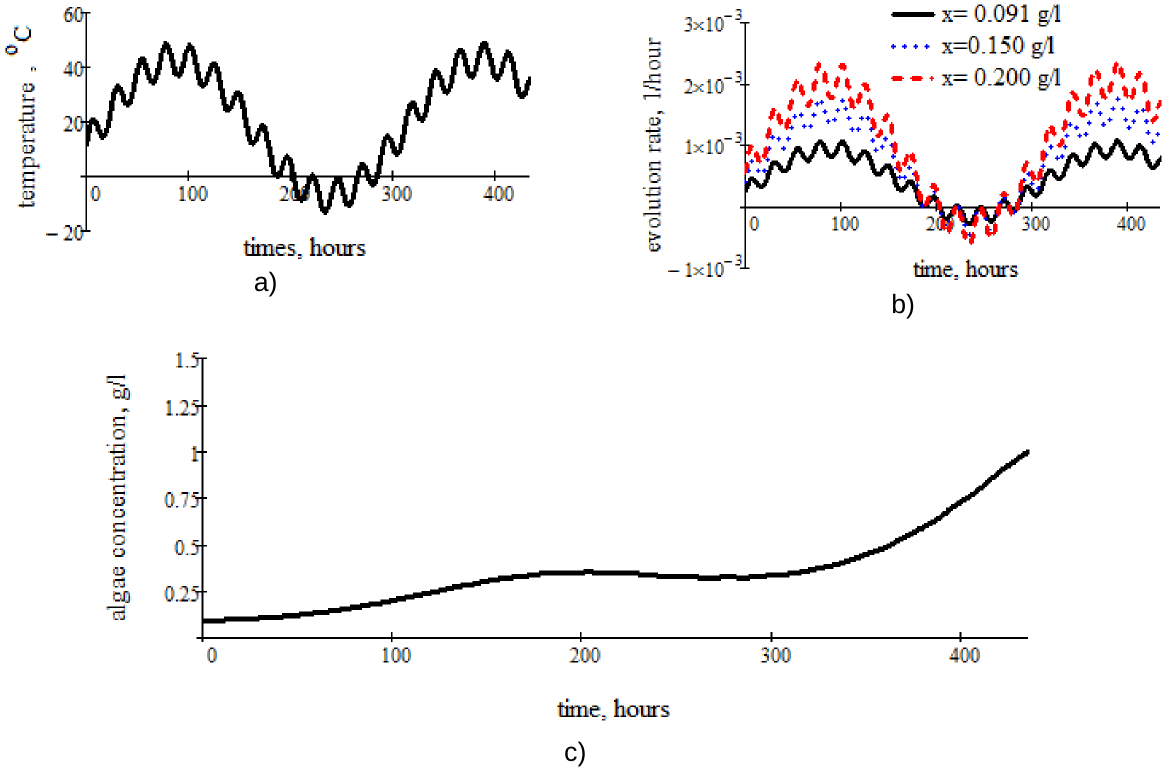


Fig. 4 - Effect of the thermal variation (with random component) on the algae crop biomass concentration

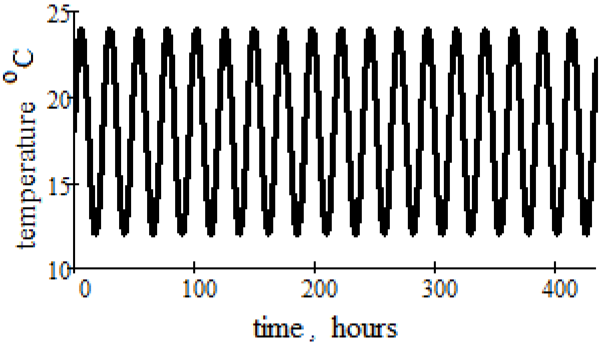
The evolution rate of algae crop varies according to temperature, copying both diurnal and random variation, as it can be seen in Figure 4, b). As we can see, variation in the evolution rate of algae cultures differs for different concentrations of the solution. More precisely, the higher the concentration is, the higher the rate of variation. This topic also deserves an experimental exploration about the existence of a mechanism to limit the growth rate of algal evolution, taking into account the conditions of culture. The consequence of the

[6] In other words, the evolution rate goes from the (positive) state of the growth rate to the (negative) state of the decrease rate.

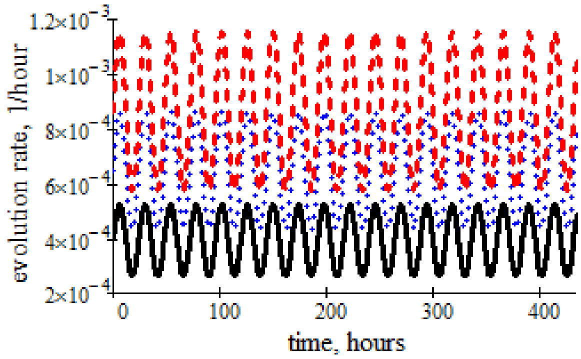
heat field variation on algae culture can be seen in Figure 4 c). The growth of the algae population is not monotonous however, even with the simulated accident in the scenario, culture continues to develop. After accidental temperature variation (around 220 hours), a decreasing portion appears immediately, but with a low intensity and after that population growth continues. The algae concentration in the solution at the end of the 435 hours of observation is 1.001 g/l.

To investigate in depth the behaviour of the model, let's see how the algae culture developed in the absence of accidental (random) thermal phenomenon. The normal heat regime for the algae crop remains, in this case, the one given by the daily variation (about 18 days), the temperature variation over time being shown in Figure 5, a). The evolution rate of the algae crop varies over time as in Figure 5, b). The algal concentration variation over the observation time frame is given in Figure 5, c).

It is observed that algae concentration in the solution increases monotonously, only with small growth oscillations due to the daily temperature variation. The result of this growth regime is interesting, at the end of the 435 hours of observation, the algal concentration being 0.597 g / l. This value is slightly more than half the value the crop, which suffered by the thermal accident, has reached.



a)



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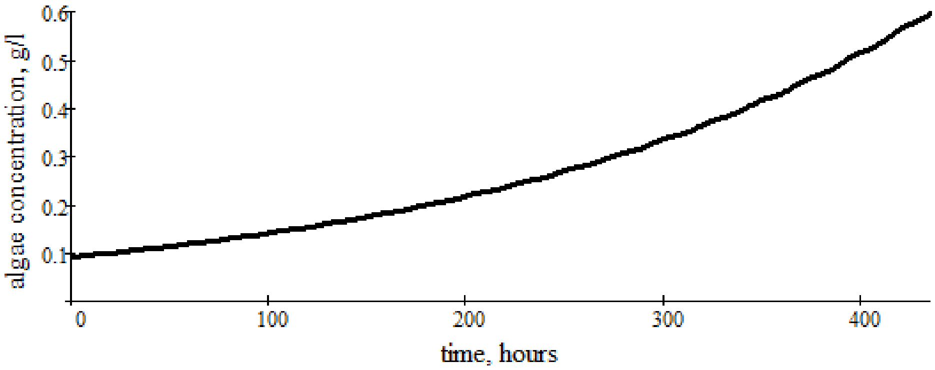

c)

Fig. 5 - Effect of the thermal variation (without the random component) on the algae crop biomass concentration

x= 0.091 g/l ···· x=0.150 g/l - - x= 0.200 g/l

One of the explanations of this phenomenon (a crop that suffers a thermal accident has a higher harvest than the same crop that would not have been subjected to this accident) is that the variation of the random thermal component increased the average evolution rate: from 0.001163 per hour, in the case without the random component, at 0.00209 per hour, if the random thermal component act, i.e. almost double. This example may explain some seemingly paradoxical phenomena in the cultivation of various algae or plants. For the average evolution rate of the crop algae, we used the following definition (14):

$$\bar{\mu} = \frac{1}{T - t_0} \int_{t_0}^{T} \mu(t, x(t))dt \tag{14}$$

4) Simulation of a real evolution

When the first ideas about this model[7] arose, we considered the evolution of an algae crop, recorded in the experiments described in *Nedelcu et.al., 2018b*. Generally, in algae crops, we are accustomed to monotonous growth curves, comfortable in cultivation processes using optimized and partially or completely automated technologies, see Figure 5, c), (or *Blanken et.al., 2016; Huesemann et.al., 2016; Jamsa et.al., 2017; Stemkovski et.al., 2016 and Ma et.al., 2017*). There are also sources that indicate for algae crops, evolutionary curves that are not monotonous (*Dontu N., 2013; Jamsa et.al., 2017; Nedelcu, et.al., 2018,b*). The deviation from the monotonous growth regime of algae biomass draws attention due to several assumptions unfavourable to the cultivation process, which can be formulated: sudden variations of the thermal field or in the pH of the solution, dangerous variations in the carbon dioxide content in air or water, sudden or uncontrolled variations of algae illumination, dangerous variations in the salinity of the solution, and possible nutrient feeding errors. Other types of causes cannot also be excluded: diseases specific to algal cultures, penetration of culture system by the pests, measurement errors and monitoring. Over these causes, there is also the hypothesis that algae, individually or in the colony, develop mechanisms to reject forced growth (with too much growth rate) or even develop tempering functions. In this latter hypothesis we have also introduced the limitation of algal density (limit concentration), which was not used in these examples, however. Because the temperature of the solution, pH, salinity, illumination were well monitored (abstracted from possible errors), we focused on less well-monitored parameters. We have not been able to obtain the desired variations in the algal concentration using the carbon dioxide concentration variation within the limits prescribed (by *Blinova et.al., 2015*, for example).

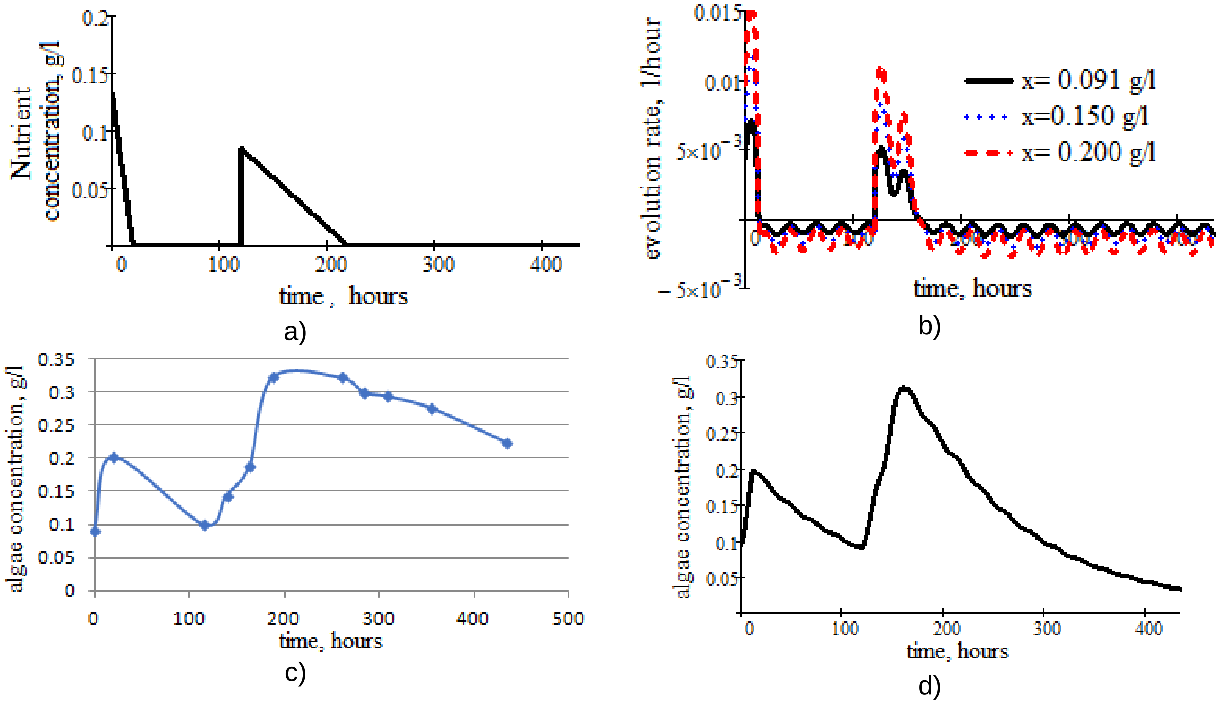

Fig. 6 - Simulation of a real evolution of a *Chlorella vulgaris* algae crop, recorded in the experiments described in *Nedelcu et.al., 2018 b*

a) Nutrient control function (based on nitrogen); b) Time dependence of the evolution rate of algae biomass in solution, at different concentrations; c) The evolution of algae concentration in solution, experimentally recorded; d) Evolution of algae concentration in solution, simulated using the mathematical model described in this article.

Then, from the set of parameters chosen for the influence of algal growth, there was still nutrient supply available.

Using the nitrogen nutrient load given in Figure 6, a) and a slightly modified nutrient filter function, formula (15), (after *Rowley W.M., 2010*), as:

$$N(\chi) = B(2.,0.069,0.137,0.01,0.01,-0.5,0.1,\chi) \tag{15}$$

[7] This model is neither the best, nor the nicest possible. It has advantages and disadvantages. Some of the advantages and disadvantages of this mathematical model will be discussed in the conclusions. But it can be the starting point for more performing models.

we obtained the algal concentration variation in Figure 6, d). How well approximates the prediction of the mathematical model (Figure 6, d), the experimental result (Figure 6, c), at least qualitatively, can be observed by comparing the two curves. The time dependences of the parameters that influenced the evolution of algae, for the example describe in this subchapter, are given graphically in Figure 7. Using these variations of process parameters, a theoretical response that satisfactorily approximates the actual response could be obtained. Due to experimental inaccuracies, however, nutrient loading monitoring cannot be used for comparison with theoretical load. Among these, first, it is important to monitor each parameter that influences the process at the resolution of the shortest (in time) process of significant and planned variation of any of the parameters.

In order to complement the image of the proposed mathematical model, we will give an example, exactly under the same conditions mentioned in this third case examined. The further simulated aspect is the action of the moderation factor, according to formula (16), on the simplified form.

$$\mu(t,x) = \mu_0 \cdot x \cdot \Theta \cdot (1 - CO_2) \cdot pH \cdot L \cdot N \cdot S \cdot \delta_{\lim}(x) \tag{16}$$

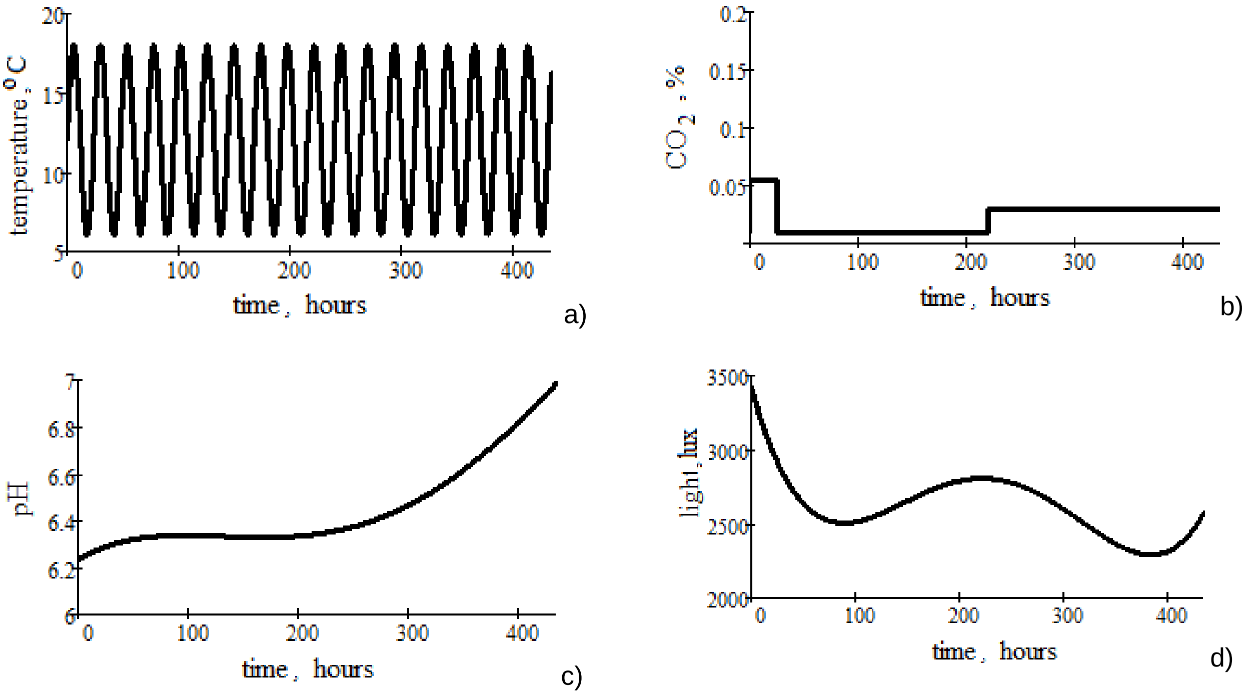

Fig. 7 - Time dependence of the process parameters considered.
Salinity was considered to be constant, 0.023 g /l

The evolution of the algae culture under the conditions in which the moderating factor acts as in (16), and the parameters that influence the process behave as in Figures 6 and 7, is graphically represented in Figure 8.

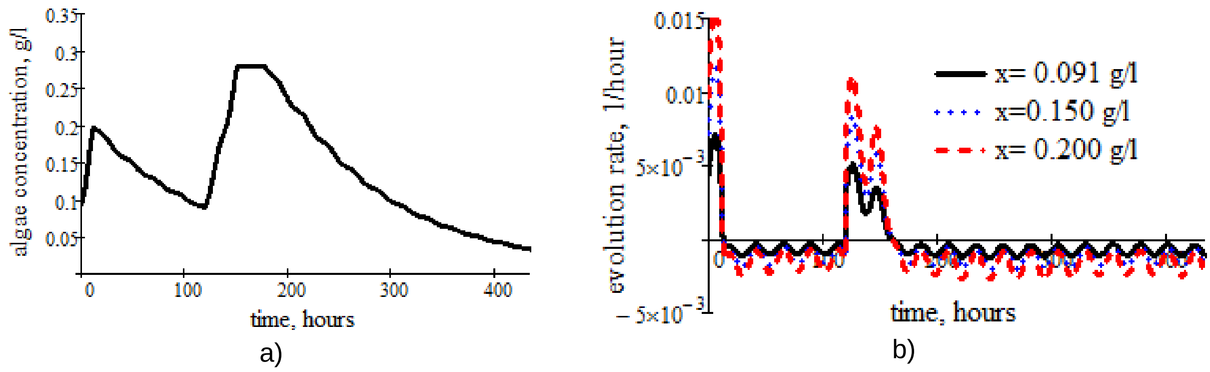

Fig. 8 - Time dependence of the process parameters considered, when the moderation factor (4) acting

Given that 0.28 g/l (chosen intentionally to highlight the consequences) was taken for the upper limit accepted by the culture, it is observed the cutting of the global peak, which is limited to a horizontal ceiling at the biomass limit value. The cut-off effect of growth at the given value corresponds to a population that "does not support excessive crowding", the high density of individuals. In the case of a culture with this type of behaviour, the optimal time of harvesting can be taken as the starting point of the horizontal ceiling. The response of the algae, dictated by the moderation factor, is similar to the behaviour of the mathematical models (*Davidson and Gurney, 1999*), in relation to yield-limited batch culture notion.

CONCLUSIONS

C1) Is recommend the increasing the resolution (in time) of the monitoring to the value of the smallest, planned, significant variation range for each of the process parameters.

C2) Is recommend the increasing the quality of monitoring through extended automation, avoiding as far as possible the laboratory analyses in which the human factor intervenes (labourers manipulating classical devices, typical of chemical laboratories).

C3) The mathematical modelling of algal evolution processes (generally of any plant or animal, of any life form), regardless of the purpose for which it is made, must include as many aspects of its life, meaning important moments: birth, growth, maturation, aging, death (natural or not), diseases, etc. Any measured parameter can give information, or it can be connected with other important parameters. That is why everything which can be measured must be measured and stored (regardless of resolution, given the extreme complexity of the experiences made with living matter).

C4) Deepening experimental research may include accumulation mathematical operators that, depending on feed and environmental parameters introduced, reflect the accumulation of valuable components for transformation into biofuels, food, or medication in algae. Such operators can be, for example, those introduced by *Cârdei P (2000)*, to model the phenomenon of materials fatigue. These operators are by integral type and transform the evolutionary equation of living matter into an integral-differential equation. For this kind of equations, numerical methods are more complex.

C5) The mathematical model considered is simple. The model of influence parameters of a process was used, in part, as in the mathematical model of soil erosion, called the Universal Soil Loss Equation (USLE), (*Wischmeier and Smith, 1960*). This type of model allows estimating the effect of parameters on process evolution, but not vice versa. For this reason, variance equations of each of the parameters that influence the process must be introduced into the model. Thus, the influence of biomass[8] growth on pH, carbon dioxide, temperature, etc. can be emphasized. Creating such mathematical models requires very complex experiences, the costs of which cannot be said to be justified, at least for now.

C6) The influence factors method can also be applied to complex mathematical models of evolution (dynamic, kinetic) defined by systems of differential equations or even equations and systems of partial derivative equations. To achieve this goal, it is sufficient to convert some of the constants or model parameters into a product by factor functions, which express the contribution of each parameter to the value of a particular model parameter.

C7) Mathematical models of crops' evolution, using influence factors, allow for detailed expertise of the losses of some crops, high probability to identifying, of the causes, and imagining real-time loss recovery scenarios.

As a conclusion to this modelling attempt in the field of algae growing (and generally living matter), we will set out some requirements for experiments to clarify the behaviour of algae and not only of them.

In order to better model the influence of physical fields (temperature, pH, concentrations of carbon dioxide, light, nutrients, radiation, etc.) on the studied biological entities, one should, for example, study the variation of the growth rate with the intensity of these fields between the extreme values, more precisely, those values at which the growth rate is cancelled and, if possible, determine the values at which the phenomenon of putrefaction occurs.

It is also necessary to know the extreme values of algal concentrations or their density: the minimum concentration from which development can begin (if any) and the maximum concentration at which the crop yet develops (if there is this limit, especially for crops bounded in space).

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[8] Living or dead biomass can also influence the environmental status.

„Researches regarding the development of an innovative technology for obtaining advanced biofuels from non-food bio-resources”.

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REFERENCES

[1] Blanken W., Postma P.R., de Winter L., Wijffels R.H., Janssen M., (2016), Predicting microalgae growth. *Algal Research*, vol.14: pp. 28-38, Ed. Elsevier, London / U.K;

[2] Blinova L., Bartosova A., Gerulova K., (2015), Cultivation of microalgae *(Chlorella vulgaris)* for biodisel production. *Research Papers Faculty of Materials Science and Technology Slovak University of Technology*, Vol.23, No.36, pp. 87-95, Bratislava/Slovakia;

[3] Cârdei P., (2000), Models for bodies with unsteady quality, *Revue Roumaine des Sciences Techniques. Mecanique Appliquee*, tome 45 (1), pp.97-109, Bucharest / Romania.

[4] Chen Sh., Chen X., Peng Y., Peng K., (2009), A mathematical model of the effect of nitrogen and phosphorus on the growth of blue-green algae population. *Applied Mathematical Modelling*. Vol.33, no.2, pp.1097–1106, Ed. Elsevier, London / U.K;

[5] Concas A., Pisu M., Cao G., (2013), Mathematical Modelling of *Chlorella Vulgaris* Growth in Semi-batch Photobioreactors Fed with Pure CO_2, *AIDIC Conference* Series 11: 121-130 DOI: 10.3303/ACOS1311013.

[6] Davidson K., Gurney W.S.C., (1999), An investigation of non-steady-state algal growth. II. Mathematical modelling of co-nutrient-limited algal growth, *Journal of Plankton Research*, vol.21, no. 5, pp. 839-858, Oxford University Press;

[7] Dontu N., (2013), Cultivation of *Chlorella Vulgaris Beijer, Synechocystis Salina Wisl., Phromodium Foveolarum (Mont.) Gom.* and *Tribonema Viride Pasch*. Algae in Medium with urban waste water addition, *Studia Universitatis Moldaviae. Scientific Journal of State University in Moldova*, vol.6, no.66;

[8] Fry R., McManus S., (2002). Smooth bump functions and the geometry of banach spaces: A brief survey, *Expositiones Mathematicae*. Vol. 20, no. 2, pp. 143-183, Ed. Elsevier, London/U.K;.

[9] Huesemann M., Crowe B., Waller P., Chavis A., Hobbs S., Edmundson S., Wigmosta M., (2016), A validated model to predict microalgae growth in outdoor pond cultures subjected to fluctuating light intensities and water temperatures, *Algal Research*, vol.13, pp. 195-206, Ed. Elsevier, London/U.K;

[10] Jamsa M., Lynch F., Santana-S. A., Laaksonen P., Zaitsev G., Solovchenko A., Allahverdiyeva Y., (2017), Nutrient removal and biodiesel feedstock potential of green alga UHCC00027 grown in municipal wastewater under Nordic conditions. *Algal Research*, vol.26, pp. 65-73, Ed. Elsevier, London/U.K.

[11] Ma M., Yuan D., He Y., Park M., Gong Y., Hu Q., (2017), Effective control of *Poterioochromonas malhamensis* in pilot-scale culture of *Chlorella sorokiniana* GT-1 by maintaining CO2-mediated low culture pH. *Algal Research*, vol.26, pp. 436-444, Ed. Elsevier, London/U.K;

[12] Malthus T. R., (1798), An Essay on the Principle of Population. *Library of Economics and Liberty*, London. https://www.econlib.org/library/Malthus/malPop.html?chapter_num=1#book-reader.

[13] Mardlijah M., Jamil A., Hanafi L., Sanjaya S., (2017), Optimal control of algae growth by controlling CO2 and nutrition flow using Pontryagin Maximum Principle, *ICoAIMS 2017*, Journal of Physics Conference Series 890 (1): pp. 1-6, Kuantan, Pahang, Malaysia;

[14] McKendricka A.G., Kesava P.M. (1912), XLV.-The Rate of Multipliation of Micro-organisms: A Mathematical Study, *Proceedings of the Royal Society of Edinburgh*. Vol.31, pp. 649–653.

[15] Muthukumar T, (2016), Sobolev Spaces and Applications. accessed 19.09.2018 http://home.iitk.ac.in/~tmk/courses/mth656/main.pdf.

[16] Nedelcu A., Ciuperca R., Popa L., Gageanu I., Pruteanu A., (2018), Research on Algae Growing in Open System with Cascade-Type Installation, *Proceedings of The 17th International Scientific Conference Engineering for Rural Development.*, pp 412-418, Jelgava / Letonia;

[17] Nedelcu A., Cardei P., Ciuperca R., (2018), Researches on the cultivation of *Chlorella vulgaris* algae in a laboratory installation aimed at designing a real-scale installation. Accessed 20.09.2018. *https://www.researchgate.net/publication/322211066_Researches_on_the_cultivation_of_Chlorella_vulgaris_algae_in_a_laboratory_installation_in_aim_to_design_a_real-scale_installation?showFulltext=1&linkId=5a4bca7d0f7e9b8284c2e2ab*

[18] Rowley W.M., (2010), *Nitrogen and phosphorus biomass-kinetic model for chlorella vulgaris in a biofuel production scheme*, Thesis, Air Force Institute of Technology, Ohio.

[19] Singh S.P., Singh P., (2014), Effect of CO2 concentration on algal growth: A review, *Renewable and Sustainable Energy Reviews,* vol. 38, pp.172–179, Ed. Elsevier, London/U.K;

[20] Stemkovski M., Baraldi R., Flores K.B., Banks H.T, (2016), Validation of a Mathematical Model for *Green Algae* (Raphidocelis Subcapitata), Growth and Implications for a Coupled Dynamical System with Daphnia Magna. *Applied Science*, vol.6, no. 5: p.155. DOI:10.3390/app6050155.

[21] Surendhiran D, Vijay M, Sivaprakash B, Sirajunnisa A, (2015). Kinetic modeling of microalgal growth and lipid synthesis for biodiesel production. *3 Biotech*, vol.5, no.5, pp.663-669. Springer International Publishing;

[22] Thornton A.R., Weinhart T., Bokhove O., Zhang B., van der Sar D.M., Kumar K., Pisarenco M., Rudnaya M., Savcenco V, Rademacher J., Zijlstra J., Szabelska A., Zyprych J., van der Schacs M., Timperio V., Veerman F., (2010), Modeling and optimization of algae growth. *Proceedings of the 72nd European Study Group Mathematics with Industr*y, pp. 54-85, Amsterdam: C.W.I;

[23] Tu L.W., (2008), An Introduction to Manifolds, Springer;

[24] Verhulst P.F., (1838), Notice sur la loi que la population suit dans son accroissement. *Correspondance mathematique et physique*, volume 10, pp. 113-121.

[25] Wischmeier W.H., Smith D.D., (1960), A universal soil-loss equation to guide conservation farm planning. *Transactions of the 7th International Congress of Soil Science,* vol.1: pp. 418-425;

[26] Yang J.Sh., Rasa E., Tantayotai P., Scow K.M., Yuan H., Hristova K.R., (2011), Mathematical model of Chlorella minutissima UTEX2341 growth and lipid production under photoheterotrophic fermentation conditions. *Bioresource Technology*, vol.102, pp. 3077-3082, Ed. Elsevier, London/U.K.;

[27] Yang Zh, Zhao Y, Liu Zh, Liu C, Hu Zh, Hou Y, (2017), A Mathematical Model of Neutral Lipid Content in terms of Initial Nitrogen Concentration and Validation in Coelastrum sp. HA-1 and Application in Chlorella sorokiniana, *BioMed Research International*, vol. 2017, Article ID 9253020, 10p. https://doi.org/10.1155/2017/9253020.

[28] ***https://www.wikihow.com/Grow-Chlorella-for-a-Food-Supplement, accessed 07.01.2018.