NUMERIC MODEL OF THE GRAIN MIXTURE FLOW IN A CYLINDRICAL SIEVE WHICH REVOLVES AROUND THE INCLINED AXIS

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ДОСЛІДЖЕННЯ РУХУ ЗЕРНОВОЇ СУМІШІ В ЦИЛІНДРИЧНОМУ РЕШЕТІ, ЩО ОБЕРТАЄТЬСЯ НАВКОЛО НАХИЛЕНОЇ ОСІ

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ABSTRACT

A mathematical model of a grain mixture interaction with a cylindrical sieve was proposed. Two options of the model were considered: the axis of the cylindrical sieve rotation is horizontal; the axis is inclined to the horizon at a certain angle.

The calculation scheme of the "a grain heap – a sieve" system assumes that the grain mixtures move integrally affected by the forces of interaction with the sieve, which arise on the contact surface. The forces of weight and the centrifugal forces, which arise during the rotation, also affect the integral motion of the grain mixtures. It is assumed that the consumption of the mixture at the inlet of the sieve is equal to the total grain consumption during the separation and the removed impurities.

The differential equations which describe the motion of the heap along a cylindrical sieve were obtained. These equations can be used both to substantiate the geometric and kinematic characteristics of the sieve, and to choose the rational operating mode.

The graphic illustration of numerical solution of differential equations for a concrete example was given. It allows determining the position of the grain heap in the sieve at any time.

РЕЗЮМЕ

Запропонована математична модель взаємодії зернової суміші з барабаном при очищенні зерна. Розглянуті варіанти для випадків, коли вісь обертання барабана є горизонтальною та для осі нахиленої до горизонту під деяким кутом. Розрахункова схема системи «зерновий ворох – решето» передбачає, що масив зернової суміші рухається як єдине ціле під дією сил взаємодії з решетом, які виникають на контактній поверхні, сил ваги та відцентрових сил, що виникають при обертанні. Приймається, що витрата суміші, що потрапляє в решето дорівнює сумарній витраті зерна при сепаруванні та домішок, які видаляються з масиву під час очистки.

Виведені диференціальні рівняння, що описують рух вороха по циліндричному решету, які можуть бути використаними як при обґрунтуванні геометричних і кінематичних характеристик решета, так і при виборі раціонального режиму його роботи.

Наведена графічна ілюстрація чисельного розв'язку диференціальних рівнянь для конкретного прикладу, яка дозволяє в будь-який момент часу визначати положення зернового вороха в решеті.

INTRODUCTION

Grain production has the biggest impact on the supply of food to the population and the development of forage resources for several agrarian branches (*Matveev et al, 2016*). The creation of new machines is mostly aimed at improving conventional principles, making basic constructions more complex (*Awgichew A., 2017; Werby R., 2010*). The grain postharvest cleaning and separation problem is new challenge for implementing advanced technologies and creating next-generation grain cleaners (*Linenk et al, 2012*). Introducing new grain crops production technologies require high performance of grain cleaning and its compliance with the standards of cleanliness (*Nesterenko et al, 2017*). It is impossible to improve postharvest cleaning and separation of grain without implementing advanced technologies and creating next-generation grain cleaners (*Capov & Shepelev, 2010; Jing Z., 2013)*. There were several attempts to solve the problem at hand by optimizing the rational set of particular operations and parameters of grain cleaners that determine the sequential and step-by-step efficient cleaning schemes (*Eskhozhin et al, 2016; Tyschenko et al, 2010*). Outdated grain cleaners have been replaced by new ones for many years, since no significant progress has been made in the design of new efficient grain cleaners of local manufacture or the development of improved grain cleaning techniques (*Li et al. 2009; Mingjie et al, 2012).* It is necessary to improve separation of grain and create next-generation grain cleaners (*Liang et al 2016*). Dynamics of corn cleaning still remains a complex problem. The commonly used approaches on particle dynamics and trajectory tracing and high-speed photography was adopted to quantify the cleaned grain (*Li J. et al., 1997; Wenqing et al., 2001; Wenqing et al., 2002*). The mainstream efforts of research are mostly of parameterized characteristics, e.g., adjusting physical parameters of processed materials and changing construction and motion to illustrate its cleaning performances (*Li H.et al., 2009*).

The goal of this work is to develop a mathematical model for the interaction of a grain heap with a drum, which can provide a study of the dependence of the sieve performance on the kinematic and geometric characteristics of the system.

MATERIALS AND METHODS

The research of the grain motion in a cylindrical sieve which rotates around the axis is important for substantiating its geometric and kinematic characteristics. These characteristics are necessary to maintain the predetermined operating modes and the throughput of the drum during the separation. The common approach to the calculation scheme "grain heap - a drum" has not been found yet concerning the interaction of the grain mixture with the surface of the cylindrical sieve. In one case the grain heap is considered as “air-fluidized medium” (*Kharchenko S., 2014*). In other cases it can be characterized as a bulk material (*Pershin et al, 2009*). It was described the movement both homogeneous and heterogeneous, granular media, the vibro viscous factor of which is differently dependent on one of the spatial coordinates. The hydrodynamic theory describes the standard vertical movement of pseudo rarefied mixture on the inner surface of the cylindrical vibratory centrifugal sieve, which rotates at a constant angular velocity around a vertical axis (*Tyschenko et al, 2010*). The application of the hydrodynamic theory is impossible for cases in which the “air-fluidized medium” does not occur. In addition, there are some doubts regarding the exactness of models of the grain interaction with a sieve, in which the grain is loaded with a centrifugal force of inertia, which is defined by the angular velocity of the drum. For the moment it has not been mentioned how the model can be used to analyse the motion of the grain heap. In this respect, it is interesting to implement a stationary process of grain separation when the axis of the sieve rotation is horizontal. The scheme of forces acting on the grain heap in the sieve in the equilibrium state is shown in Fig. 1.

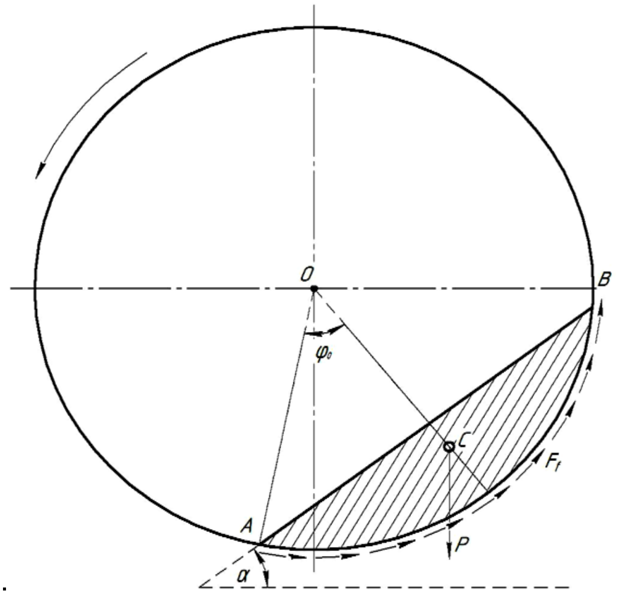

Fig. 1 - The scheme of forces acting on the grain heap in the sieve in the equilibrium state

In this case, the mixture in the sieve is in a state when the free surface of the heap is inclined to the horizon at an angle of natural slope.

For a sieve with horizontal axis of rotation, it is possible to implement a stationary grain separation process in accordance with the scheme shown in Fig. 2.

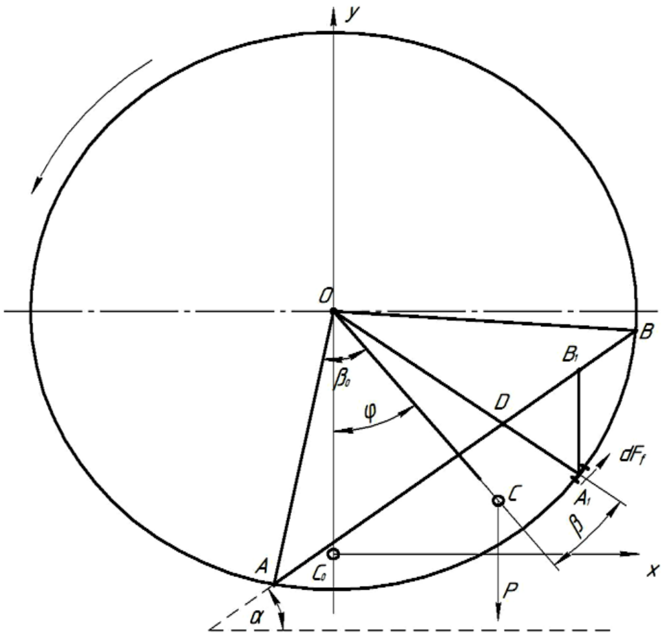

Fig. 2 - Scheme for analyzing the motion of a grain heap in a cylindrical sieve

The friction force Fm and the weight force P are distributed along the interaction area from below, when the material is slid in the sieve. It is assumed that the grain heap, the cross section of which is bounded by a chord AB above and an arc AB below, moves in a sieve with the rotation without deforming from the lower position, in which the axis of symmetry (OC in Fig. 1) occupies the vertical position OC_0 .

The differential equation of the rotational motion of the heap about the sieve axis has the form (*Yablonsky and Nykyforova, 1966*):

$$I_0\ddot{\varphi} = M_0\left(F_{\text{f}}\right) - P \cdot OC \sin\varphi \tag{1}$$

where

I_0 – is the polar moment of inertia of the heap, the cross section of which is considered to be unchangeable, $kg \cdot m^2$;

$\ddot{\varphi}$ – an angular acceleration of the heap during the rotation, s^{-2} ;

$M_0\left(F_{\text{f}}\right)$ –the moment of friction forces that arise between the heap and the sieve about the sieve axis, $N \cdot m$;

P –the weight of the heap, N ;

OC – the distance from the centre of the sieve cross-section to the centre of the weight of the heap cross section, m ;

φ – an angle of rotation, rad .

In determining the frictional forces, we take into account that they will arise from the hydrostatic pressure of the heap on the sieve and from the centrifugal forces which act on the heap during the rotation.

The component of the friction forces, which depends on the hydrostatic pressure of the material at an arbitrary point A (Fig. 1), determined by the angle $(\varphi + \beta)$, is calculated as:

$$\boldsymbol{dF_{\text{fP}} = \gamma \cdot f \cdot H \cdot l \cdot ds} \tag{2}$$

where γ – specific weight of the material, N/m^3;

H – the distance from point A_1 to the free surface of the heap cross section (A_1B_1 in Fig. 2);

l – the length of the sieve, m ;

ds – elementary section of the arc of the sieve cross section, in the centre of which is the point A_1 , m .

Taking into account that $ds = Rd\beta$, for frictional force on the section ds, we obtain:

$$dF_{\mathrm{fP}} = f \cdot \gamma \cdot H \cdot l \cdot R \cdot d\beta \tag{3}$$

For the height H of the triangle ADB we obtain:

$$H = R\left(1 - \frac{\cos\beta_0}{\cos\beta}\right) \cdot \frac{\cos\beta}{\cos\varphi} = R\left(\frac{\cos\beta}{\cos\varphi} - \frac{\cos\beta_0}{\cos\varphi}\right) = \frac{R}{\cos\varphi}\left(\cos\beta - \cos\beta_0\right) \tag{4}$$

Then

$$dF_{\mathrm{fP}} = f\gamma l R^2 \left(1 - \frac{\cos\beta_0}{\sin\beta}\right) \cdot \frac{\cos\beta}{\cos\varphi} d\beta = \gamma f l R^2 \frac{\cos\beta - \cos\beta_0}{\cos\varphi} \tag{5}$$

The centrifugal force that falls on the arc *ds* is determined by the scheme shown in Fig. 3.

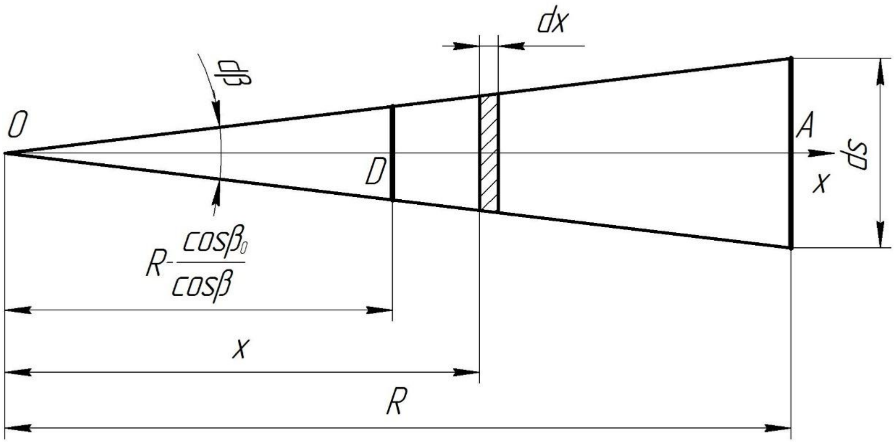

Fig. 3 - Scheme for determining the centrifugal force falling on the elementary arc *ds*

A centrifugal force acts on an element of mass with dimensions (in Figure 3 its cross-section is shaded):

$$\Delta dF_V = \frac{1}{g}\gamma l x d\beta dx \omega^2 x \tag{6}$$

Integrating this expression from point D to point A we obtain dF_V:

$$dF_V = \frac{1}{g}\int_{R\frac{\cos\beta_0}{\cos\beta}}^{R} \gamma l \omega^2 d\beta x^2 dx = \frac{1}{3g}\gamma l \omega^2 R^3 \left(1 - \frac{\cos^3\beta_0}{\cos^3\beta}\right) d\beta \tag{7}$$

Thus, the total frictional force that falls on a section of width ds is defined as

$$dF_{\mathrm{f}} = f\gamma l R^2 \left[\frac{R\omega^2}{3g} \cdot \left(1 - \frac{\cos^3\beta_0}{\cos^3\beta}\right) + \frac{\cos\beta - \cos\beta_0}{\cos\varphi}\right] d\beta \tag{8}$$

For the moment of frictional forces, distributed over the interaction area of the heap with a sieve, it is obtained:

$$M_0\left(F_{\mathrm{f}}\right) = f\gamma l R^3 \left\{\frac{R\omega^2}{3g}\left[2\beta_0 - \sin\beta_0\cos\beta_0 - \frac{\cos^3\beta_0}{2}\left(\ln\left|tg\left(\frac{\pi}{4} + \frac{\beta_0}{2}\right)\right| - \ln\left|tg\left(\frac{\pi}{4} - \frac{\beta_0}{2}\right)\right|\right)\right] + 2\frac{\sin\beta_0}{\cos\varphi} - 2\frac{\cos\beta_0}{\cos\varphi}\beta_0\right\} \tag{9}$$

According to the known specific weight of the bulk material γ, the square of the heap cross section S and the length of the sieve l, for the moment of the heap weight and for the moment of inertia of the heap about the sieve axis of rotation, respectively:

$$P \cdot OC \cdot \sin\varphi = \gamma R^2 \left(\beta_0 - \frac{1}{2}\sin 2\beta_0\right) \cdot \frac{4}{3}\frac{R\sin^3\beta_0}{\left(2\beta_0 - \sin 2\beta_0\right)} \cdot \sin\varphi = \gamma \cdot \frac{2}{3}R^3 \sin^3\beta_0 l \cdot \sin\varphi \tag{10}$$

$$I_0 = \frac{\gamma l R^4}{g}\left(\beta_0 - \frac{1}{3}\sin 2\beta_0 - \frac{1}{12}\sin 4\beta_0\right) \tag{11}$$

Substituting the expressions for the geometric characteristics of the heap cross section and for the load acting on the heap, after the obvious transformations of the formulas (9), (10) and (11) in equation (1) we obtain:

$$\frac{R}{g}\left(\beta_0-\frac{1}{3}\sin\beta_0-\frac{1}{12}\sin 4\beta_0\right)\cdot\ddot{\varphi}=$$
$$=f\left\{\frac{R\omega^2}{3g}\left[2\beta_0-\sin\beta_0\cos\beta_0-\frac{\cos^3\beta_0}{2}\ln\frac{tg\left|\left(\frac{\pi}{4}+\frac{\beta_0}{2}\right)\right|}{tg\left|\left(\frac{\pi}{4}-\frac{\beta_0}{2}\right)\right|}\right]+2\frac{\sin\beta_0}{\cos\varphi}-2\frac{\cos\beta_0}{\cos\varphi}\beta_0\right\}- \tag{12}$$
$$-\frac{2}{3}\sin^3\beta_0\sin\varphi.$$

RESULTS

The differential equation (12) makes it possible to analyse the motion of a heap in a sieve, the axis of which is horizontal. The motion of the heap will be considered as composition of a rotational motion around the axis and translational along the x - axis, when the sieve is rotated about the axis, inclined to the horizon at angle ψ (Fig. 4).

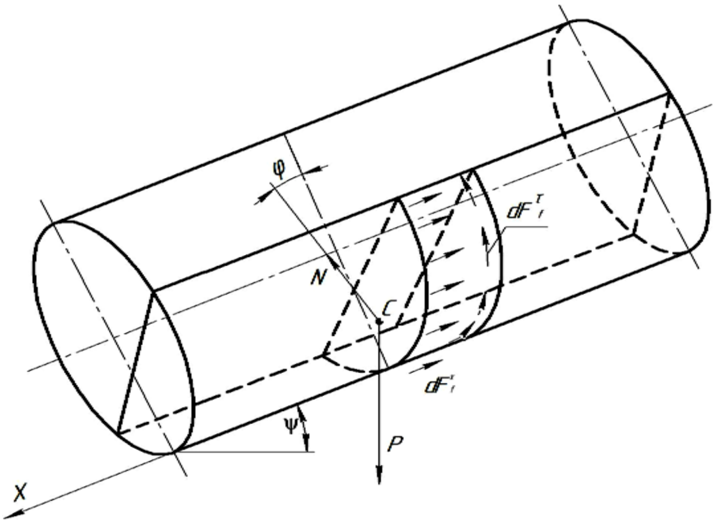

Fig. 4 - The scheme of frictional forces acting on the elemental volume of the heap in the sieve with an inclined axis of rotation to the horizon

The differential equation of rotational motion will have the form:

$$I_0=M_0\left(F_f^{\tau}\right)-P\cdot OC\cdot\sin\varphi\cos\psi \tag{13}$$

where ψ − an angle of inclination of the sieve, rad; $M_0\left(F_f^{\tau}\right)$ − moment of friction forces, $N\cdot m$.

The differential equation of motion of the heap along the sieve has the form:

$$m\ddot{x}=P\sin\psi-F_f^{x} \tag{14}$$

where m – mass of the heap, kg; $\ddot{x}$ − projection of acceleration on the x-axis; F_f^{x} − resultant frictional forces directed along the axis of the sieve, N.

Taking into account the information mentioned above, for differential equations of the heap motion we obtain:

- for the rotational component movement

$$\frac{R}{g}\left(\beta_0-\frac{1}{3}\sin\beta_0-\frac{1}{12}\sin 4\beta_0\right)\cdot\ddot{\varphi}=$$
$$=f\left\{\frac{R\omega^2}{3g}\left[2\beta_0-\sin\beta_0\cos\beta_0-\frac{\cos^3\beta_0}{3}\ln\left|\frac{tg\left(\frac{\pi}{4}+\frac{\beta_0}{2}\right)}{tg\left(\frac{\pi}{4}-\frac{\beta_0}{2}\right)}\right|\right]+2\frac{\sin\beta_0-\beta_0\cos\beta_0}{\cos\varphi\cos\psi}\right\}\times$$
$$\times\frac{R\omega}{\sqrt{\dot{x}^2+R^2\omega^2}}-\frac{2}{3}\sin^3\beta_0\sin\varphi\cdot\cos\psi; \tag{15}$$

- for the translational component motion of the heap along the axis of rotation

$$\ddot{x}\frac{l\gamma R^2}{g}\left(\beta_0-\frac{1}{2}\sin 2\beta_0\right)=$$
$$=f\gamma lR^2\left\{\frac{R\omega^2}{3g}\left[2\beta_0-\sin\beta_0\cos\beta_0-\frac{\cos\beta_0}{3}\ln\left|\frac{tg\left(\frac{\pi}{4}+\frac{\beta_0}{2}\right)}{tg\left(\frac{\pi}{4}-\frac{\beta_0}{2}\right)}\right|\right]+2\frac{\sin\beta_0-\beta_0\cos\beta_0}{\cos\varphi\cos\psi}\right\}\times \tag{16}$$
$$\times\frac{\dot{x}}{\sqrt{\dot{x}^2+R^2\omega^2}}+l\gamma R^2+\left(\beta_0-\frac{1}{2}\sin 2\beta_0\right)\sin\psi.$$

The considered mathematical model allows us to investigate the motion of the grain heap in the sieve during its rotation. The numerical solution of the system of differential equations, (15) and (16) is carried out by the Runge - Kutta method (*Porshnev and Belenkova, 2005*) according to the initial data given in the table 1.

Table 1

Output data to a numerical solution

R [m]	β_0 [rad]	ψ [rad]	f	g [m/s^1]	l [m]	φ_0 [rad]	ω_0 [rad/s]	V_0 [m/s]	Δt [s]
0.45	0.8	0.1	0.3	9.81	1.2	0	0	0	0.01

In integration it was assumed that the grain heap, the cross section of which is determined by the radius of the sieve R and the loading angle β_0, moves from the rest state, with $\varphi=0;\ w=0;\ V=0$.

The integration process with the step $\Delta t=0.01\,s$ was completed at the passage of weight centre of the heap cross section along the entire length of the sieve ($l=1.2m$). The graphic illustration of the solution shown in fig.5 makes it possible to determine: the position of the centre of the sieve cross section along the sieve length for any moment of its movement (curve 1); the angle of deviation in the normal section of the sieve (curve 3); the angular velocity at any moment of time (curve 2).

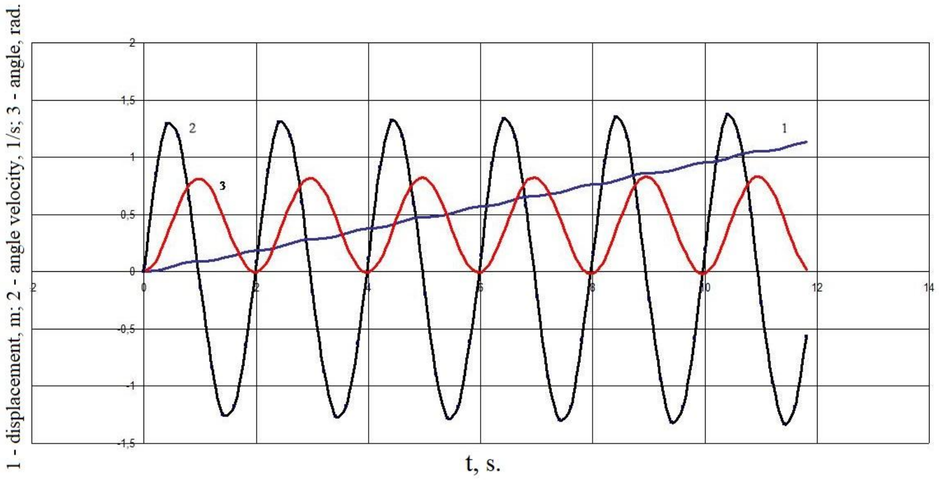

Fig. 5 - Kinematic characteristics of the grain heap during the interaction with the sieve

The numerical solution of the equations above, (15) and (16), allows, based on the analysis of the motion of the grain mixture in the sieve, to assign the most efficient operating mode for grain separation.

CONCLUSIONS

The mathematical model of interaction of the grain heap with a drum, which rotates around an axis inclined to the horizon, has been suggested.

The solution of differential equations of motion allows determining the speed and time of the egress of grain during the separation in a sieve.

The given mathematical model of a grain heap interaction with a drum can be used in substantiating the geometric and kinematic characteristics of the sieve, as well as in choosing a rational mode of its operation.

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