TECHNIQUE FOR CALCULATING THE RESULTANT FORCE AND MOMENT OF SOIL RESISTANCE TO CUTTING BY BLADE OF ROTARY TILLAGE TOOL
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МЕТОДИКА РАСЧЕТА РЕЗУЛЬТИРУЮЩЕЙ СИЛЫ И МОМЕНТА СОПРОТИВЛЕНИЯ ПОЧВЫ РЕЗАНИЮ ЛЕЗВИЕМ ЛОПАСТНОГО РОТАЦИОННОГО РАБОЧЕГО ОРГАНА

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ABSTRACT
The objective of this study was to develop a technique for calculating the resultant force and moment of soil resistance to cutting by the blade of the earlier proposed rotary tillage tool as functions of the rotation angle on the basis of the developed mathematical model of soil-blade interaction. The dependences taking into account the constructive and functional parameters were obtained and analyzed. It was shown that the blade is subjected to repeated loads during the soil cutting, so the equivalent resultant force of soil reactions acting on the blade changes its magnitude, direction and the point of action periodically with changing the rotation angle. The obtained formulas can be used both for carrying out the long-term strength analysis of the rotary tool and for optimizing its constructive and functional parameters. The proposed technique allows to simplify experiments for determining the rotary tool force characteristics and to reduce their number significantly. Thanks to the commonality of the assumptions it can be transferred mainly to other rotary tillage tools.

РЕЗЮМЕ
Целью данного исследования явилось создание методики расчета результирующего сопротивления почвы и результирующего момента сопротивления почвы резанию лезвием предложенного ранее лопастного ротационного рабочего органа в зависимости от угла его поворота на основе предложенной математической модели взаимодействия лезвия с почвой. Полученные зависимости учитывают влияние конструктивных и режимных параметров. Показано, что при резании почвы лезвие испытывает циклические нагрузки. Равнодействующая сил реакций почвы на лезвие периодически изменяет свою величину, направление и точку приложения. Полученные формулы можно использовать не только для расчета ротационного рабочего органа на длительную прочность, но также и для оптимизации его конструктивных и режимных параметров. Предложенная методика позволяет существенно упростить эксперименты по определению силовых характеристик рабочего органа и значительно уменьшить их объем. Благодаря общности основных исходных положений предлагаемая методика может быть в основном перенесена на другие ротационные рабочие органы.

INTRODUCTION
The problem of finding the resultant of soil reactions on a rotary tillage tool blade is up-to-date, since its solution allows not only to calculate the strength of the blade, but also to determine the rational values of its constructive and functional parameters.

Professor N. Nerli was one of the first who took into account a distribution of elementary soil reactions, when he determined the resultant of soil reactions on the blade of a flat disk, free rotating in soil (*Nerli, 1929*). Later, many researchers solved similar problems with different success (*Sineokov G.N., 1949; Luchinskij N.D., 1977*). The case of the PTO (power take-off) powered flat disk was researched (*Medvedev V.I. et al., 1974*), and the generalized mathematical theory of soil-disk interaction has been developed (*Medvedev V.I. et al., 2001*). This model allowed setting and solving the single criterion optimization problem of the disk constructive and functional parameters (*Akimov A.P. et al., 2008*) and the bicriterion one for the powered disk (*Akimov A.P., Konstantinov Y.V, 2016; Akimov A.P., Konstantinov Y.V, 2017*).

Directions of elementary soil reactions on the blade of a rotary tillage tool depend not only on the position of its instantaneous rotation axis, but also on the rotation angle. So the resultant of soil reactions on the blade and their resultant moment are functions of that angle.

When researchers determined the resultant soil resistance force to cutting by a rotary tiller blade, they did not take into account the distribution of elementary soil reactions on the blade and it did not allow them to calculate the values of resultant soil resistance and resultant moment of soil resistance at an arbitrary rotation angle value of the blade (*Thakur T.C., Godwin R.J., 1989; Thakur T.C., Godwin R.J., 1990; Kalantari D., 2000*). Some researchers have plotted the graphs of such dependences on the basis of special experiments (*Chertkiattipol S., Niyamapa T., 2010; Niyamapa T., Chertkiattipol S., 2010; Matin M.A. et al., 2015*).

As an alternative to existing rotary tillage tools for the purpose of improving the quality of soil treatment and simultaneously reducing its specific tilling energy, the rotary tillage tool (RTT) was proposed, which can be used for various agricultural operations (*Medvedev V.I. et al., 1984, Akimov A.P. et al., 2015*). The method to determine the resultant force and the resultant moment of soil reaction forces acting only on the surface of this rotary tillage tool depending on the rotation angle was proposed (*Akimov A.P., Konstantinov Y.V, 2011; Akimov A.P. et al., 2012; Akimov A.P. et al., 2017*). But the soil reactions on the RTT blades have not been taken into account.

The major objective of this study was to develop a technique for determining the resultant force and resultant moment of soil resistance to cutting by a RTT straight rotary blade, which takes into account the distribution of elementary soil reactions and allows determining dependences of these characteristics upon the RTT rotation angle.

MATERIALS AND METHODS

The RTT consists of the flange, to which four identical flat vanes are attached. These vanes are quarters of plate, bounded by an ellipse. The blades of the vane small semi-axes are located in a vertical plane, and the vanes constitute some angle with it. The projection of all the vanes on the plane is a circle of radius *r*, equal to the small semi-axis of the ellipse. A model sample of the RRT and the RRT row on a shaft of rotary plough are shown in Fig. 1 *a*, *b*.

(a)

(b)

Fig. 1 – A model sample of the RRT (*a*) and the RRT row on a shaft of rotary plough (*b*)

This rotary tool can be used also in other rotary machines, which perform various agricultural operations. It can effectively work as a furrow opener in seeders *(Medvedev V.I., Kazakov Y.F., 1984)*, as a ridger in rotary cultivators *(Leshhankin A.I., 1991)* and as an active coulter (power-take-off driven) in ploughing units *(Chatkin M.N., 1990).*

For calculating the RTT strength and optimizing the process of its operation it is necessary to develop a mathematical model of the RTT interaction with soil and, in particular, to determine the resultant force of soil resistance to cutting by this rotary tool.

Let us assume that a tillage machine or a unit moves rectilinearly with a constant speed v_0, the RTT rotates around the rotor axis with a constant angular velocity ω and its blades during work plunge to a maximal depth *h* into homogeneous soil. The RTT operating regime is characterized by two constant dimensionless parameters: by the ratio of rotor peripheral speed to forward velocity of the tillage machine (kinematic parameter) $\lambda = \omega r / v_0$ and the maximal relative depth $\xi = h/r$.

The minor semi-axis blade of a RTT vane cuts the soil, performing a plane-parallel motion in a vertical plane. In the *Oxz* coordinate system with the origin *O* on the RTT axis of rotation and uniformly moving with the RTT, the position of minor semi-axis blade is set by rotation angle α from the vertical (Fig. 2, *a*). A soil cutting speed, varying along the blade edge, is defined by position of the RTT straight blade instantaneous centre of velocity, located at the point $C(0; a)$, $a = r/\lambda$. Let us also connect with the RTT another coordinate system $Ox'z'$, directing the axis Ox' on the major semi-axis of vane, and the axis Oz' – on its minor semi-axis.

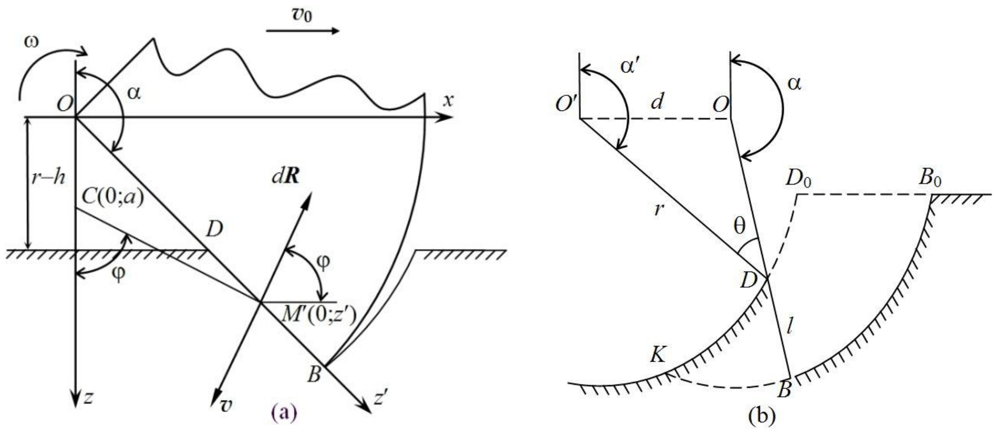

Fig. 2 – Diagram of soil cutting by the first (*a*) and a non-first (*b*) blade of the RTT

On an infinitesimal elementary segment of the blade cutting part, adjacent to a point $M'(0; z')$, with length dz', the elementary soil reaction force *dR* acts, which is directed opposite to the vector of absolute speed v of this point (Fig. 2, *a*). The magnitude of the force is equal to $dR = Q \cdot dz'$, where *Q* is the average specific force of cutting per unit length of the blade [N/m].

Let us denote by φ the angle between the force vector *dR* and the positive direction of *Ox* axis, then $\cos\varphi = (z-a)/[x^2+(z-a)^2]^{1/2}$, $\sin\varphi = x/[x^2+(z-a)^2]^{1/2}$; and $x=z'\sin\alpha$, $z = -z'\cos\alpha$ (Fig. 2, *a*). By integrating the equality $dR_x=Qdz'\cos\varphi$ on the segment of the blade cutting part and passing to the dimensionless variable $u = \lambda z'/r$, we obtain the formula for the horizontal component of the resultant of soil resistance to cutting (RSR)

$$R_x = -\frac{Qr}{\lambda}\int_{\lambda\cdot\zeta_0}^{\lambda}\frac{(u\cos\alpha+1)du}{\sqrt{u^2+2u\cos\alpha+1}}, \tag{1}$$

where $\varsigma_0 = 1-l(\alpha,\xi)/r$, and $l(\alpha,\xi)$ is the length of cutting part of the blade corresponding to the rotation angle value α and the maximal depth of the blade equal to h [m].

Absolutely similarly the formula for the vertical component of this force is obtained:

$$R_z = -\frac{Qr}{\lambda}\int_{\lambda\cdot\zeta_0}^{\lambda}\frac{u\sin\alpha du}{\sqrt{u^2+2u\cos\alpha+1}}. \tag{2}$$

Let us choose as the positive direction of the moments of forces the direction coinciding with the RTT rotation direction, then the moment about the point *O* of an elementary force of soil resistance to cutting will be defined by the equality $dM_O = -z\cdot dR_x + x\cdot dR_z$ (Fig.2,a). By integrating this equality, we find the resultant moment of soil resistance to cutting (MSR)

$$M_O = -\frac{Qr^2}{\lambda^2}\int_{\lambda\zeta_0}^{\lambda}\frac{u(u+\cos\alpha)du}{\sqrt{u^2+2u\cos\alpha+1}}. \tag{3}$$

The integrals in formulas (1) – (3), depending on the variable α and on the parameters λ, ς_0, are easily calculated by the method of undetermined coefficients *(Gradshteyn I.S., Ryzhik I.M., 2007)*. The obtained explicit expressions are cumbersome, and we use a way of sequential designations. Let us introduce three functions of two variables:

$$F_1(\alpha,u) = (u^2 + 2u\cdot\cos\alpha + 1)^{0,5}$$
$$F_2(\alpha,u) = \ln(u + \cos\alpha + F_1(u,\alpha))$$
$$F_3(\alpha,u) = 0,5(u - \cos\alpha)\cdot F_1(u,\alpha) \tag{4}$$

Then the components of resultant of soil reactions and the resultant moment of soil resistance to cutting by the blade will take the following forms:

$$R_x = -Qr\cdot(G_1(\alpha,\lambda,\varsigma_0)\cos\alpha + G_2(\alpha,\lambda,\varsigma_0)\sin^2\alpha)/\lambda,$$
$$R_z = -Qr\cdot(G_1(\alpha,\lambda,\varsigma_0) - G_2(\alpha,\lambda,\varsigma_0)\cos\alpha)\sin\alpha/\lambda,$$
$$M_O = -Qr^2\cdot(G_3(\alpha,\lambda,\varsigma_0) - 0,5G_2(\alpha,\lambda,\varsigma_0)\sin^2\alpha)/\lambda^2, \quad (5)$$

where the functions $G_i(\alpha,\lambda,\varsigma_0) = F_i(\alpha,\lambda) - F_i(\alpha,\lambda\varsigma_0),\ i = 1, 2, 3.$

The sequential expressions (4)–(5) have a simple structure, and are easily programmed. They allow calculating the values of R_x, R_z, and M_O at given values of α, λ and ς_0 with the help of a computer.

The blade, which is the first to enter into soil (further the first blade), is represented in Fig. 2,*a*. From the diagram it follows that for the first blade $l(\alpha,\xi)=r + (r - h)/\cos\alpha$, hence for it

$$\varsigma_0 = (\xi - 1)/\cos\alpha \quad (6)$$

For any blade entering into soil after the first one (further non-first blade), formula (6) is valid only at the initial stage of cutting. The condition $\varsigma_0 = 1$ determines the angle of the beginning of soil cutting process by a blade $\alpha_0 = \arccos(\xi-1)$. The cutting process by the first blade is completed at $\alpha = 2\pi-\alpha_0$ (Fig.2,a). Thus, formulas (4) – (6) determine the resultant force and the resultant moment of soil resistance to cutting by the first vane blade of RTT at the initial stage $(\alpha_0 \le\alpha\le 2\pi - \alpha_0)$.

For a non-first vane blade of the RTT formula (6) is valid only until the moment of contact of the blade edge with the upper surface of the cut soil slice in the point D_0 (Fig.2,*b*). At the final stage of soil cutting the edge of the considered RTT blade crosses the upper surface of the slice in some point *D* of the trochoid formed by the previous blade. To this point there corresponds the angle α' measured from the vertical through the point O' which is located on the horizontal straight line passing through point *O* of the considered blade and lying at the distance *r* from the point *D* (Fig.2,*b*). When the RTT rotation axis moves from the position determined by the point O' to the position determined by the point *O*, the rotary tool rotates through the angle $2\pi/z_b+(\alpha-\alpha')$, and at the same time its rotation axis passes the $O'O$ distance equal to $d=[2\pi/z_b + (\alpha-\alpha')]\cdot r/\lambda$, where $z_b=4$ is the number of rotary blades. The equality (7) follows from the theorem of sines for the triangle $O'DO$ (Fig.2,*b*):

$$[2\pi/z_b + (\alpha-\alpha')]\cdot\lambda^{-1}\cos\alpha + \sin(\alpha-\alpha') = 0 \quad (7)$$

As we can see from the Fig. 2, *b*, $r\sin(\alpha' - \pi/2) = (r - l)\sin(\alpha - \pi/2)$.

Hence:

$$\varsigma_0=\cos\alpha'/\cos\alpha \quad (8)$$

The angle α' is a function of the angle α. This function is defined implicitly by the transcendental equation (7). It is found by means of solving the equation (7) using one of numerical methods. The substitution of this function into the equality (8) allows us to express ς_0 through the angle α. Thus, equalities (7), (8) and (4), (5) at the final stage of cutting determine the resultant force and the total moment of soil resistance to cutting by a non-first RTT blade as the composite functions of the angle α.

The point D_0 is located on the soil surface and upper surface of slice. Therefore, if we substitute $\alpha' = \arccos(\xi-1)$ into equation (7), we obtain the transcendental equation for determining the angle $\alpha=\alpha_1$ of the completion of cutting process of the soil surface (initial stage) and the beginning of cutting process of the upper surface of soil slice (final stage). This equation is solved numerically for different values of α.

The value of angle $\alpha = \alpha_2$, corresponding to the completion of soil cutting process by a non-first blade of RTT in the point *K* of cycloids intersection at the bottom of a furrow (Fig. 2, *b*), is also determined by using the equation (7). At this point $\varsigma_0 = 1$, $\alpha' = 2\pi-\alpha_2$, so we have $\sin\alpha_2 = (\pi - \alpha_2 - \pi/z_b)/\lambda$. This transcendental equation is also solved numerically. If we pass from angle α_2 to angle α'_g by $\alpha'_g = \alpha_2 - \pi$, then equation (7) transforms into the well-known equation, determining the location of crests at the bottom of a furrow during the work of rotary tiller *(Sineokov G.N., Panov I.M., 1977)*.

RESULTS

It follows from formulas (1) and (3) that for the first blade the MSR and the horizontal component of RSR depend only on $\cos\alpha$, therefore their graphs versus α for the relative depth ξ=0.5 and the values λ=3; 3.5 and 5 are symmetric relative to the line $\alpha = \pi$ (Fig.3*a*,*c*). The component R_x is nonnegative, that is, the resultant of soil resistance to cutting is the driving forward force. The vertical component R_z (Fig. 3, *b*) is negative for $\alpha < \pi$, that is, the RSR is the lifting up force and for $\alpha > \pi$ this component is positive, and the soil resistance to cutting is the deepening force. As the MSR is always non-positive, it slows down the RTT rotation, and for overcoming the moment of soil resistance, it is necessary to apply to the rotary tool the positive, equal in modulus, torque. Formulas (1)–(3) show, that for $a<r-h$ the maximal values of the MSR modulus and the RSR horizontal component are reached at the point π and are respectively equal to $P = Q \cdot h$ and to $M = P \cdot (r - h/2)$, and the vertical component of RSR at $\alpha = \pi$ is equal to zero. These expressions must be taken into account to carry out the strength analysis of the RTT and the empirical coefficient *Q* can be easily determined experimentally. It also follows from these formulas that at the vertical position of the RTT blade reactions of soil resistance to cutting are replaced by the single resultant with the magnitude *P*, directed horizontally towards the RTT motion and applied to the blade edge at the point lying at distance *h*/2 from its tip. The moduli of the resultant force components and the resultant moment of soil resistance to cutting monotonically increase with increase of the parameter ξ, but with increase of λ only the R_x and MSR moduli slightly increase, while the R_z modulus slightly decreases.

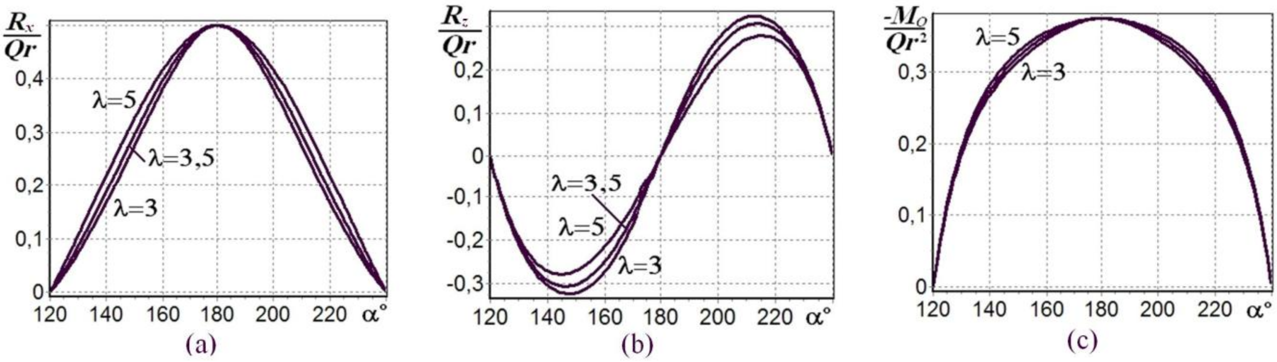

Fig. 3 – Components of the resultant resistance force of soil to cutting on the axis Ox (*a*), Oz (*b*) and the resultant resistance moment (*c*) versus angle α for the first blade

Fig. 4 shows the graphs of the resultant force components and the resultant moment of soil resistance to cutting by the edge of a non-first RTT blade versus the angle α and the parts of the corresponding graphs for the blade which is immediately following the non-first one at $\xi = 0.5$ and $\lambda = 3$; 3.5; 5.

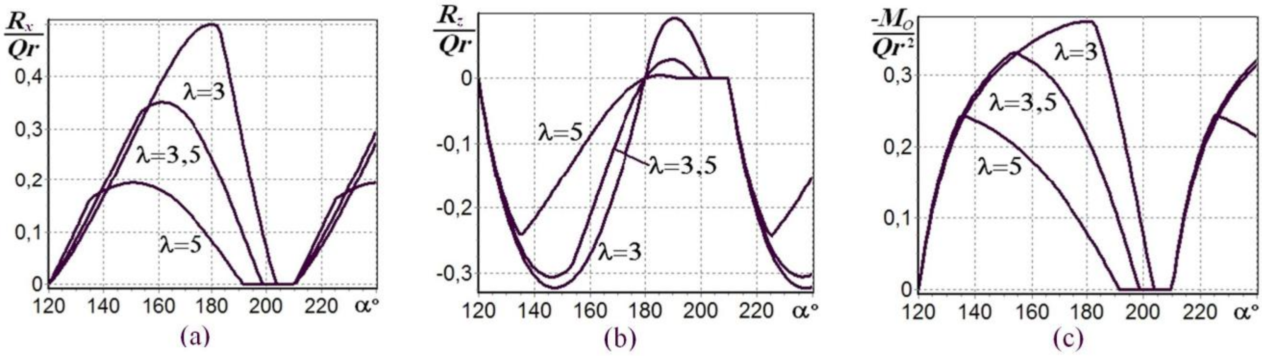

Fig. 4 – Components of the resultant resistance force of soil to cutting on the axis *Ox* (*a*), *Oz* (*b*) and the resultant resistance moment (*c*) versus angle α for a non-first blade

With increasing α the horizontal component R_x increases from zero monotonically and almost linearly at the initial stage of cutting, then continues to increase at the beginning of the final stage (for $\lambda = 3.5$ and 5) until it reaches the maximal value and then it decreases to zero again. The graph of R_x versus the angle α has the breaking point in a passage point from the first stage of cutting to the second one. Through the period $360°/z_b = 90°$ the blade of the following vane of the RTT begins to interact with the soil, therefore the specified change of the component R_x repeats again (Fig.4,*a*). The graph of the vertical component R_z (Fig.4,*b*) versus α is approximately a join of two halves of sinusoids of different amplitudes (for $\lambda = 3$ and 3.5). The amplitude of lifting action on the RTT at the initial stage of cutting is much more than the amplitude of deepening effect at the final stage. The MSR modulus monotonically increases with increase of the angle α at the initial stage of cutting, and it monotonically decreases at the final stage, reaching the greatest value at the breaking point of the graph of the MSR versus α (Fig. 4, *c*). With the increase of the ξ parameter, the maximal values of MRC and the moduli of RSR components significantly increase, and with increase of λ they decrease, that is explained respectively by increase and by decrease of the linear dimensions of a soil slice.

Carrying out the long-term strength analysis of the RTT, it is necessary to take into account that during the interaction of the second and subsequent vanes with soil, the rotary tool is subjected to cyclic loads, the vibration amplitude of which is less than the corresponding greatest values of the forces and the moments, when soil is being cut by the blade of the first vane. At the large values of λ this distinction between the amplitudes is considerable.

The soil-cutting reactions by a blade are generally equivalent to the main vector of forces applied at point *O* with components, which are given by the expressions (1), (2) and to the couple with the main moment determined by the formula (3). This system of forces can be replaced with the single equivalent resultant force, applied at the point of blade edge, lying from the RTT rotation axis at the distance $r_0 = -|M_O|/[R\cdot cos(\alpha + \psi)]$, where $R = (R_x^2 + R_z^2)^{0,5}$, and $\psi = -arcsin(R_z/R)$ is the angle between the resultant and the positive direction of the *Ox* axis.

The graphs of the ratio r_0/r versus the angle α for the first blade at a relative depth $\xi = 0.5$ practically coincide for the values of λ=3, 3.5 and 5 (Fig. 5, *a*). For $\alpha < \pi$ the distance r_0 monotonically decreases from the greatest value $r_0 = r$ at the moment of contact of the blade edge with the soil surface to the least value $r_0 = r - h/2$ at $\alpha = \pi$, and then increases to the value equal to r at the moment, when the edge goes out from soil. On the contrary, the modulus of the resultant R, almost not depending on the parameter λ, at the above mentioned values of ξ and λ, for $\alpha < \pi$ increases monotonically from the least value $R = 0$ to the greatest value P at the point π and then decreases again to zero for $\alpha > \pi$ (Fig. 5, *b*). The angle ψ, weakly depending on λ, decreases monotonically with increase of α according to an almost linear law (Fig. 5, *c*).

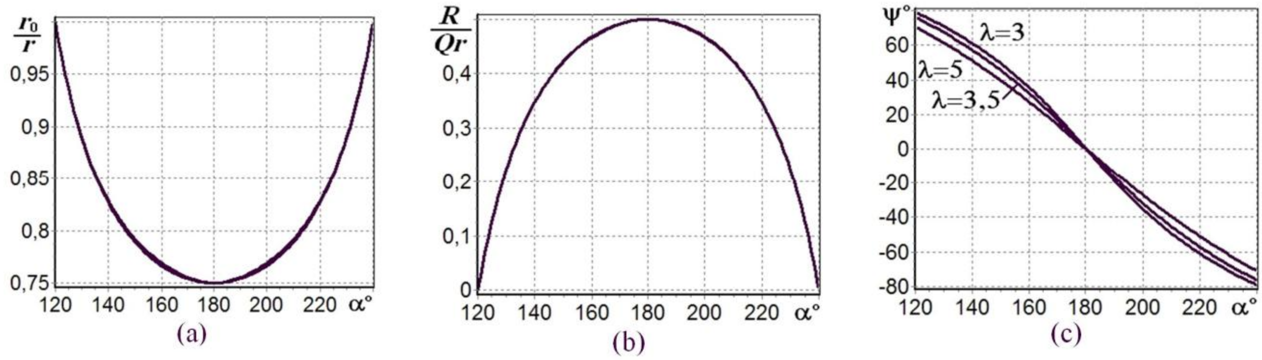

Fig. 5 – The relative distance from rotation axis to point of action of the resultant resistance force of soil to cutting (*a*), the RSR **modulus (*b*) and the** RSR **tilt angle (*c*) versus angle α for the first blade**

At $\alpha = \pi$ elementary forces of soil resistance to cutting acting on the blade are equally directed (horizontally) and equal in magnitude, so they are equivalent to the single resultant force with the point of action in the middle of the cutting part of the blade edge. If the instantaneous centre of velocity is located above the soil surface, then for the non-first blade it follows from the formulas (1) – (3), that $R_x = Qr(1 - \varsigma_1)$, $R_z = 0$, $M_O = -Qr^2(1 - \varsigma_1^2)/2$, where $\varsigma_1 = -cos(\alpha'(\pi))$. Therefore, the ratio $r_0/r = (1 + \varsigma_1)/2$.

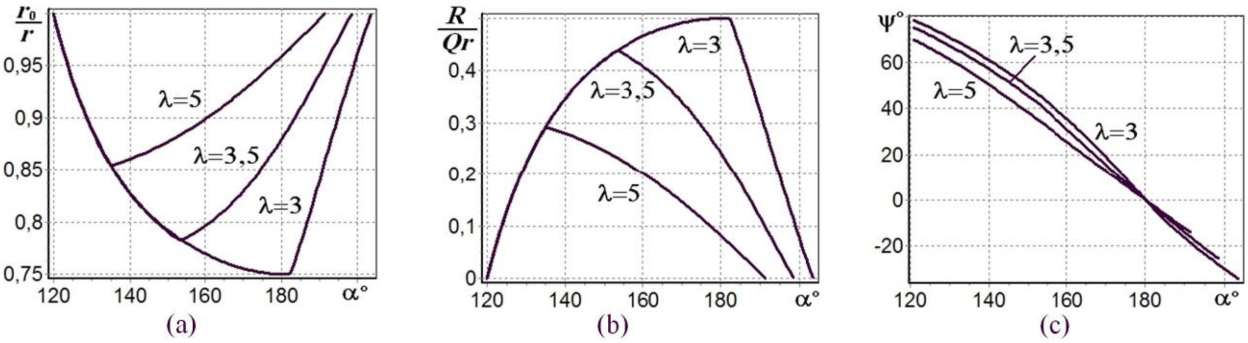

Fig. 6 – The relative distance from rotation axis to point of action of the resultant resistance force of soil to cutting (*a*), the RSR **modulus (*b*) and the** RSR **tilt angle (*c*) versus angle α for a non-first blade**

The graphs of the ratio r_0/r versus the angle α for a non-first blade plotted for the values of the ξ and λ parameters mentioned above show, that the ratio r_0/r decreases at the initial stage of cutting and increases on the final one (Fig. 6, *a*). When soil is being cut, the point of action of the equivalent resultant force changes its position from the most remote one from the RTT rotation axis at the beginning and at the end of the cutting process, to the nearest position at the transition point from the first stage of cutting to the second one at $\alpha=\alpha_1$. With increase of angle α the change character of the modulus of the resultant of soil cutting reactions by non-first blade is similar to the change character of the MSR (Fig. 6, *b*). The angle ψ monotonically decreases with increase of angle α (Fig. 6, *c*), and the graphs of ψ versus α practically coincide with the corresponding parts of graphs of the angle ψ for the first blade (Fig. 5, *c*).

CONCLUSIONS

With the help of the developed mathematical model of soil-blade interaction it is shown that the RTT blade is subjected to repeated loads during the soil cutting. The equivalent resultant force of soil reactions acting on the blade changes its magnitude, direction and the point of action periodically with changing the RTT rotation angle. The proposed technique allows calculating the resultant force and the resultant moment of soil resistance to cutting by the RTT blade as a function of the rotation angle and the RTT parameters.

It was found that the maximal values of the RTT driving force, of the MSR and of the RSR modulus monotonically increase with increasing relative depth ξ for any value of kinematic parameter λ. The angle, which the resultant resistance of soil to cutting constitutes with the positive direction of horizontal Ox axis, decreases almost linearly with increasing rotation angle α.

The obtained formulas can be used both for carrying out the long-term strength analysis of the RTT and for optimizing the RTT constructive and functional parameters.

The developed technique of calculation of the resultant force and the resultant moment of soil resistance to cutting by the RTT blade as functions of the rotation angle and the RTT parameters is valid also for the case of soil cutting by the straight parts (legs) of blades of rotary tillers, and also with some changes it can be transferred to calculating the force characteristics of blades of other rotary tillage tools. This technique allows simplifying the experiments necessary for determining the resultant force and the resultant moment of soil reactions on blades of rotary tillage tools and reducing the number of these experiments considerably.

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