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Curvature Ductility of High Strength Concrete Beams

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The ductility of reinforced concrete beams is very important, because it is the property that allows structures to dissipate energy under seismic loading, consequently, the brittle fractures of reinforced concrete structures can avoided. In the modern construction of reinforced concrete structures, the high-strength concrete (HSC) is widely used because of its benefits provided, such as the mechanical and durable properties. But the increasing of the concrete strength can negatively influence the ductility of reinforced beam sections, for this reason, the study of the local ductility of high-strength concrete beams is required. This paper present a numerical parametric study about the influence of different parameters on the curvature ductility of high strength concrete beams as the concrete strength, the yield strength of steel and the ratio of tension and compression reinforcements. In the end, a formula to predict the curvature ductility of HSC beams is proposed. This formula is compared with the Eurocode 2 numerical results and other numerical and experimental results.

1 Introduction

The collapse of many reinforced concrete structures subjected to seismic actions gives an idea of a design badly adapted to energy dissipation in plastic field. In high seismicity zones, the ductility is an important factor in the design of reinforced concrete elements to ensure the structural integrity in the plastic range, where it allows structures to dissipate seismic energy. Under seismic loads, the structures can dissipate the seismic energy by promoting the apparition of the plastic hinges in beams rather than in columns.

Comparing ordinary and high strength concrete, the latter has the advantage of providing a very high resistance to different loads, but it provides a lower ductility for resisting accidental overloads, impacts and earthquakes. From here comes the importance to study the local ductility of high strength concrete members, where some seismic codes like the Eurocode 8[1] require their verification during the design. To study the ductility of high strength concrete members, firstly, we must know the behavior of high strength concrete, Mander *et al* [2], Cusson and Paultre [3], Attard and Setung [4] and

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Razvi and Saatcioglu [5] proved that there are significant differences in the stress-strain relationship between the ordinary and the high strength concrete.

The high strength concrete is increasingly used, and the engineers are simply following the existing rules with little attention given to the provision of sufficient ductility. Now, it is urgent to develop appropriate rules for the ductility design of the HSC members. For this reason, several researches in the recent years have studied the effects of various parameters on the local ductility of HSC members as the concrete strength, the yield strength of steel and the ratios of tension and compression reinforcement, consequently many methods and formulations to deduce the curvature ductility factor are proposed.

Several researches have been done in this area. Based on experimental and numerical results, Pam et *al.* [6] and [7] proposed two formulas to predict the curvature ductility factor, these two formulas are applicable to beams with concrete strength up to 100 MPa. Afterwards, the proposal of Pam et *al.* [7] was improved by Kwan et *al* [8], to make it simpler and more useful. In other numerical investigations, Arslan and Sihanli [9] developed another simplified approach with the variation of the concrete strength up to 110 MPa. Recently, based on the numerical analysis Lee [10] and [11] proposed two approaches to calculate the curvature ductility factor of high strength concrete beams. In experimental studies, Bengar and Maghsoudi [12], Maghsoudi and Sharifi [13] tested in laboratory different series of reinforced concrete beams, where the validation of the obtained results is made with the ACI [14] and CSA [15] provisions. Also Shohana et *al* [16] and Mohammadhassani et *al* [17] tested beams in laboratories with high strength concrete.

Although the mentioned studies have used different constitutive laws of materials, the basic consideration in these researches is to take into account the balanced reinforcement ratio adopted by the ACI [14] code as a basic element, but the researchers who use the Eurocode 2 [18] are not allowed to use this ratio. Seen the importance accorded by the Eurocode 8[1] to take into account the curvature ductility factor during the design of structural elements (beams, columns and …), and this by the requirement of admissible curvature ductility factor. Accordingly, it is necessary to have a simplified relation that allows verifying the local ductility condition according to Eurocode 8[1] and takes into account parameters in accordance with Eurocode 2[18], in particular the constitutive laws of materials (concrete and steel).

This paper presents the calculation method of curvature ductility factor according to the Eurocode 2[18]. In order to predict an approach for directly calculating the curvature ductility factor in a simple and explicit manner and takes into account the Eurocode provisions, a numerical parametric study on the parameters affecting the local ductility of ordinary and high strength concrete beams will be conducted.

2 Materials

2.1 Concrete

According to Eurocode 2 [18**],** the stress-strain curve of unconfined concrete under compression is shown in the Fig. 1.

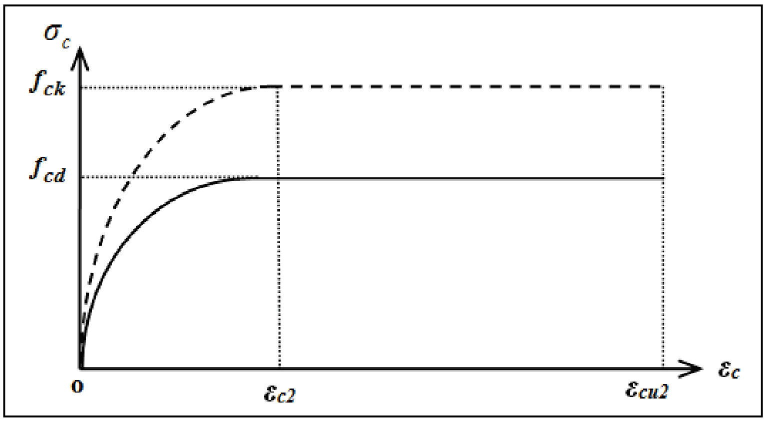

Fig. 1 - Parabola-rectangle diagram for unconfined concrete under compression Eurocode 2 [18]

The study of the reinforced concrete structures behavior according to Eurocode 2 [18] uses the characteristic compressive strength of concrete f_{ck}. For the high-performance concrete, the maximum value of this strength at 28 days is limited to 90 MPa for a cylindrical concrete Specimens and 105 MPa for a cubic specimens. The design value of the compressive strength of a cylindrical concrete Specimens f_{cd} is defined by:

$$f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_c} \tag{1}$$

Where, γ_c is the partial safety factor for concrete, equal to 1,5 for durable situations and 1,2 for accident situations. α_{cc} is the coefficient taking account of long term effects on the compressive strength and of unfavorable effects resulting from the way the load is applied, its value varies between 0,8 and 1. In the following, the accident situation is fully considered.

The stress σ_c in the concrete is limited by:

$$\sigma_c = \begin{cases} f_{cd}\left[1-\left(1-\dfrac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] & for\ 0 \le \varepsilon_c \le \varepsilon_{c2} \\ f_{cd} & for\ \varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2} \end{cases} \tag{2}$$

Where ε_c is the compressive strain in the concrete and ε_{c2} is the strain at reaching the maximum strength f_{cd}, and is expressed by:

$$\varepsilon_{c2}(‰) = \begin{cases} 2 & for\quad f_{ck} \le 50\,MPa \\ 2{,}0 + 0{,}085\left(f_{ck}-50\right)^{0,53} & for\quad f_{ck} > 50\,MPa \end{cases} \tag{3}$$

And, ε_{cu2} is ultimate compressive strain in the concrete, defined as:

$$\varepsilon_{cu2}(‰) = \begin{cases} 3{,}5 & for\quad f_{ck} \le 50\,MPa \\ 2{,}6 + 35\left(\dfrac{90-f_{ck}}{100}\right)^4 & for\quad f_{ck} > 50\,MPa \end{cases} \tag{4}$$

The exponent n takes the following values:

$$n = \begin{cases} 2 & for\quad f_{ck} \le 50\,MPa \\ 1{,}4 + 23{,}4\left(\dfrac{90-f_{ck}}{100}\right)^4 & for\quad f_{ck} > 50\,MPa \end{cases} \tag{5}$$

2.2 Steel

According to Eurocode 2 [18] the design of reinforced concrete section is performed from a specified class of frames represented by the characteristic value of yield strength f_{yk}. This value of f_{yk} varies from 400 up to 600 MPa.

The stress-strain steel diagram shown in Figure 2, is distinguished by the bilinear elasto-plastic curve, characterized by a inclined branch up to a value of deformation equal to $\varepsilon_{sy,d}$ and a stress in steel equal to f_{yd}, and a top branch supposed horizontal corresponding to a maximum deformation ε_{uk} and a stress in steel equal to f_{yd}, where :

$$f_{yd} = \frac{f_{yk}}{\gamma_s} \tag{6}$$

Where, γ_s is the partial safety factor for steel, equal to 1,15 for durable situations and 1 for accident situations.

$\varepsilon_{sy,d}$: Elastic elongation of steel at maximum load.

$$\varepsilon_{s,yd} = \frac{f_{yd}}{E_s} \tag{7}$$

E_s: Modulus of elasticity of the steel, equal to 200000 MPa.

ε_{uk}: Characteristic strain of reinforcement or prestressing steel at maximum load, this ultimate strain is limited to 5% for class B and 7,5% for class C. The recommended value of ε_{ud} is 0,9ε_{uk}.

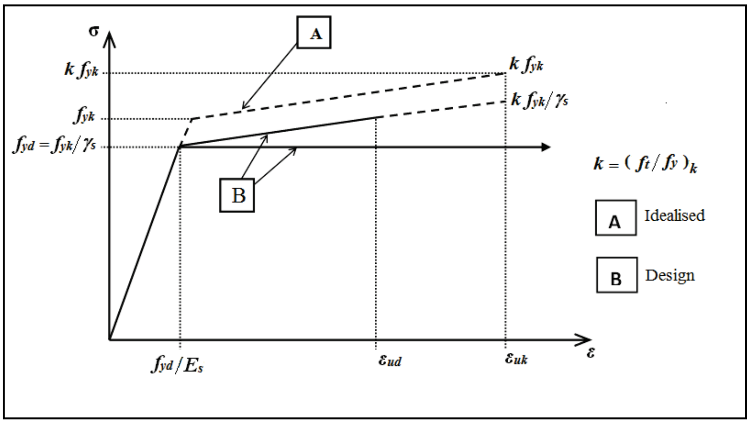

Fig. 2 - Idealized and design stress-strain diagrams for reinforcing steel (for tension and compression) Eurocode 2 [18]

3 Curvature ductility factor

The analysis of reinforced concrete beam cross section in flexure requires a study in the limit states [19]. The evaluation procedure of the curvature ductility factor is adapted according to the Eurocode 2 [18] recommendations [20]. The curvature ductility factor is obtained by the ratio between the curvature determined at the ultimate limit state and the curvature determined at the first yield:

$$\mu_\phi = \frac{\phi_u}{\phi_y} \tag{8}$$

3.1 Curvature at first yield

Figure 3 shows a cross section of a doubly reinforced concrete beam in serviceability limit state, where ξ_y represents the height factor of the compressed zone in the elastic state, d is the effective depth of the section and d' is the distance from extreme compression fiber to centroid of compression reinforcements.

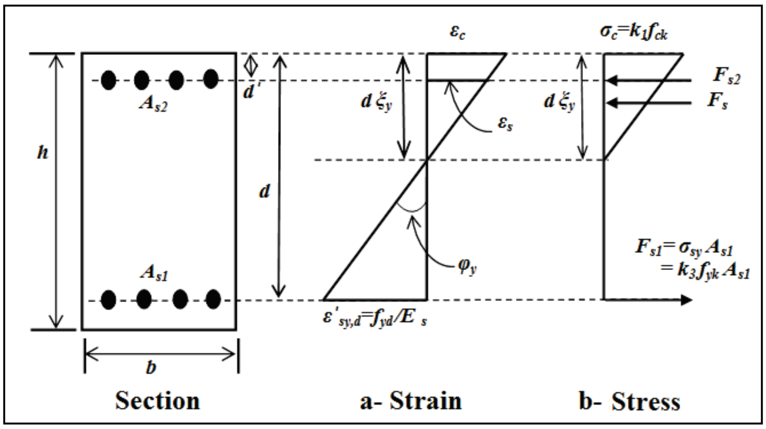

Fig. 3 - Behavior of reinforced concrete beam section in flexure at the serviceability limit state

From Figure 3, the curvature at first yield is expressed by:

$$\varphi_y = \frac{\varepsilon_{sy,d}}{d(1-\xi_y)} \tag{9}$$

And the height factor of the compressed zone in the elastic state is given by:

$$\xi_y = \left(\frac{1}{2} + \frac{k_3 f_{yk}}{k_1 f_{ck}}(\rho + \rho')\right) - \sqrt{\left(\frac{1}{2} + \frac{k_3 f_{yk}}{k_1 f_{ck}}(\rho + \rho')\right)^2 - \frac{2k_3 f_{yk}}{k_1 f_{ck}}\left(\rho + \frac{d'}{d}\rho'\right)} \tag{10}$$

Where $\rho = A_{s1}/bd$ is the ratio of tension reinforcement, and $\rho' = A_{s2}/bd$ is the ratio of compression reinforcement. With $k_1 = 0{,}6$ and $k_3 = 0{,}8$.

The strain in the compressed reinforcement ε_{s2}, is written:

$$\varepsilon_{s2} = \frac{(\xi_y d - d')}{d(1-\xi_y)} \frac{k_3 f_{yk}}{E_s} \tag{11}$$

After determining ε_{s2} expressed by Eq. (11), if $\varepsilon_{s2} \leq f_{yk}/E_s$, we retain the value of ξ_y obtained by Eq. (10). Otherwise, the compression frames A_{s2} are yielding in compression, in this case the height factor of the compressed zone ξ_y become:

$$\xi_y = \frac{2k_3 f_{yk}}{k_1 f_{ck}}(\rho - \rho') \tag{12}$$

3.2 Curvature at the ultimate limit state

Figure 4 shows a cross section of a doubly reinforced concrete beam in the ultimate limit state, where ξ_u represents the height factor of the compressed zone in the ultimate state.

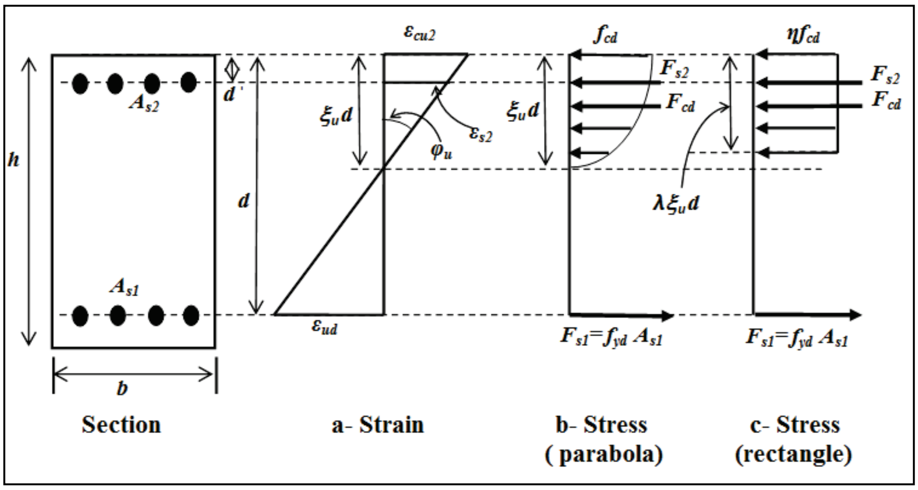

Fig. 4 - Behavior of reinforced concrete beam section in flexure at the ultimate limit state

From this figure, the curvature at the ultimate state is expressed by:

$$\phi_u = \frac{\varepsilon_{cu2}}{\xi_u d} \tag{13}$$

And the height factor of the compressed zone in the ultimate state ξ_u is given by:

$$\xi_u = \frac{(f_{yd}\,\rho - \varepsilon_{cu2} E_s \rho')}{2\lambda\eta f_{cd}} + \frac{\sqrt{(f_{yd}\,\rho - \varepsilon_{cu2} E_s \rho')^2 + 4\lambda\eta f_{cd}\,\varepsilon_{cu2} E_s \rho' \frac{d'}{d}}}{2\lambda\eta f_{cd}} \tag{14}$$

Where, the factor λ defining the effective height of the compression zone, according to the Eurocode 2 is expressed by:

$$\lambda = \begin{cases} 0,8 & for \quad f_{ck} \leq 50\,MPa \\ 0,8 - \dfrac{f_{ck}-50}{400} & for \quad 50\,MPa < f_{ck} \leq 90\,MPa \end{cases} \tag{15}$$

And η define the effective strength, expressed by:

$$\eta = \begin{cases} 1,0 & for \quad f_{ck} \leq 50\,MPa \\ 1,0 - \dfrac{f_{ck}-50}{200} & for \quad 50\,MPa < f_{ck} \leq 90\,MPa \end{cases} \tag{16}$$

Suppose the compression reinforcement A_{s2} remain in the elastic state, its deformation ε_{s2}, is given by:

$$\varepsilon_{s2} = \frac{(\xi_u d - d')}{\xi_u d}\varepsilon_{cu2} \tag{17}$$

4 Parametric Study

The essential factors influencing the local ductility of unconfined concrete beams are the concrete strength f_{ck}, the yield strength of steel f_{yk} and the ratio of tension and compression reinforcement (ρ and ρ'). The effect of each factor on the curvature ductility of reinforced concrete beam sections will be presented in the following.

4.1 Effect of the concrete strength (f_{ck})

In order to take into account the high concrete strength, f_{ck} is varied from 20 to 90 MPa. The figure 5 shows the curvature ductility factor μ_φ according to the ratio of tension reinforcement ρ.

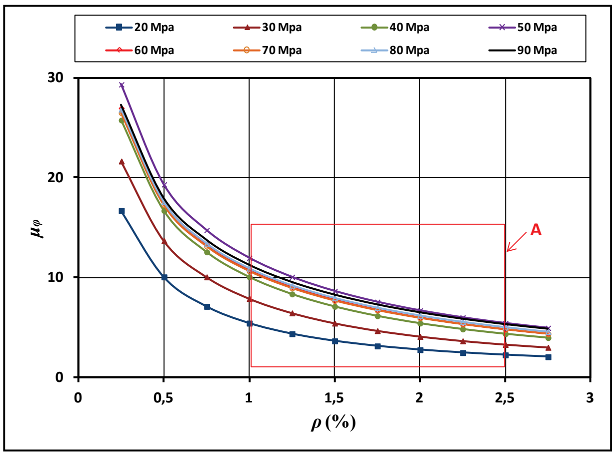

Fig. 5 - Effect of the concrete compressive strength fck for fyk = 400 MPa; ρ'/ρ = 1

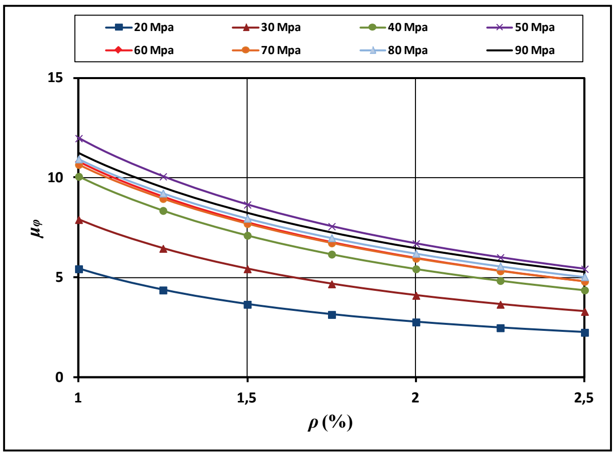

Fig. 6 - A area

From this figure we can divide the variation of the curves $\mu_\varphi = f(\rho)$ in two different cases. For the concrete strengths inferiors to 50 MPa, the level of the curves increases with increasing of the concrete strength f_{ck}, and the level of the curve $\mu_\varphi = f(\rho)$ corresponding to the strength f_{ck} = 50 MPa is the highest among all the curves. For concrete strength superior to 50 MPa, the level of the curves begins to decrease despite the concrete strength increases but with small variation, where the factor μ_φ decreases from 29 to 27 and from 4,9 to 4,4 for ρ = 0,25 and 0,75 % respectively. These findings are very clear in the zoom of the A area in the figure 6. Accordingly, we can deduce that the curvature ductility factor continues to improve with the increase of the concrete strength f_{ck} until strength equal to 50 MPa, after this value, the curvature ductility stops improving and decreases slightly.

4.2 Effect of the yield strength of steel (f_{yk})

In order to examine the effect of the yield strength of steel f_{yk}, we must respect the application domain of the Eurocode 2, f_{yk} are considered from 400 to 600 MPa. The obtained results are shown in figure 7 in form of curves $\mu_\varphi = f(\rho)$.

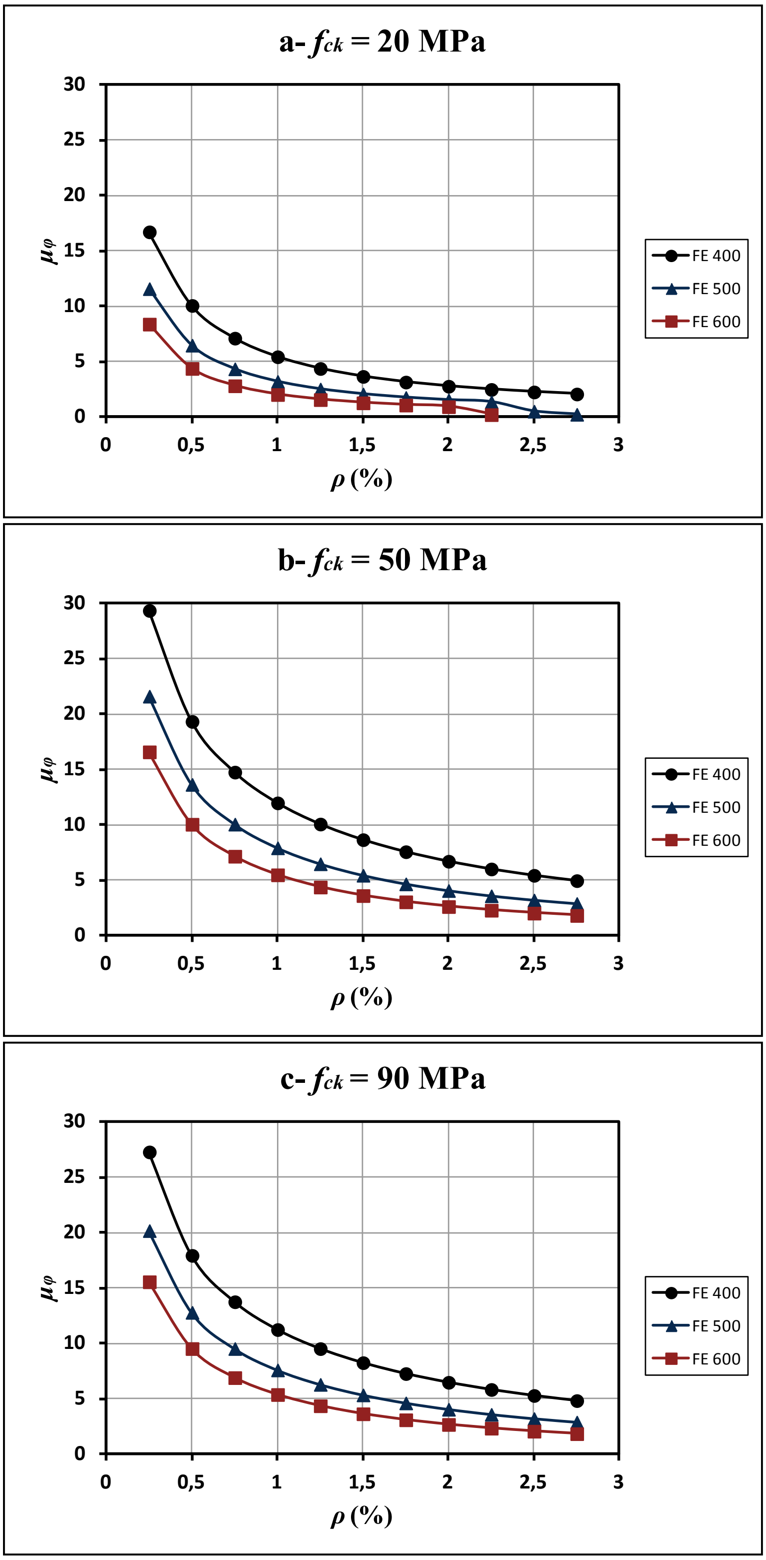

Fig. 7 - Effect of the yield strength of steel f_{yk} for ($\rho'/\rho = 1/2$)

From Figure 7.a, when the concrete strength f_{ck} equal to 20 MPa, we note that the level of the curves $\mu_\varphi = f(\rho)$ decreases with the increase of the yield strength of steel f_{yk}, this finding is clearly distinguished in the figures 7.b and 7.c when the strength f_{ck} is equal to 50 and 90 MPa respectively. Accordingly, it can be said that the increase of the yield strength of steel f_{yk} is very hostile to the curvature ductility of the unconfined reinforced concrete beams.

4.3 Effect of tension reinforcement (ρ)

The action of tension reinforcement is expressed by their ratio ρ, in this study; the ratio ρ will be varied from 0,25 to 4 % in steps of 0,25 %. The collected results are illustrated in the figure 8.This figure shows the curves $\mu_\varphi = f(\rho)$ for different values of the concrete strength f_{ck}. After seen the figure 8.a, we notice that each curve $\mu_\varphi = f(\rho)$ decreases when the percentage of tension reinforcement ρ *(%)* increases whatever the value of the concrete strength f_{ck}, the same remarks are observed when the yield strength f_{yk} increases to 600 MPa in figure 8.b. Accordingly, the repercussion of tension reinforcements is inversely proportional with the curvature ductility factor μ_φ.

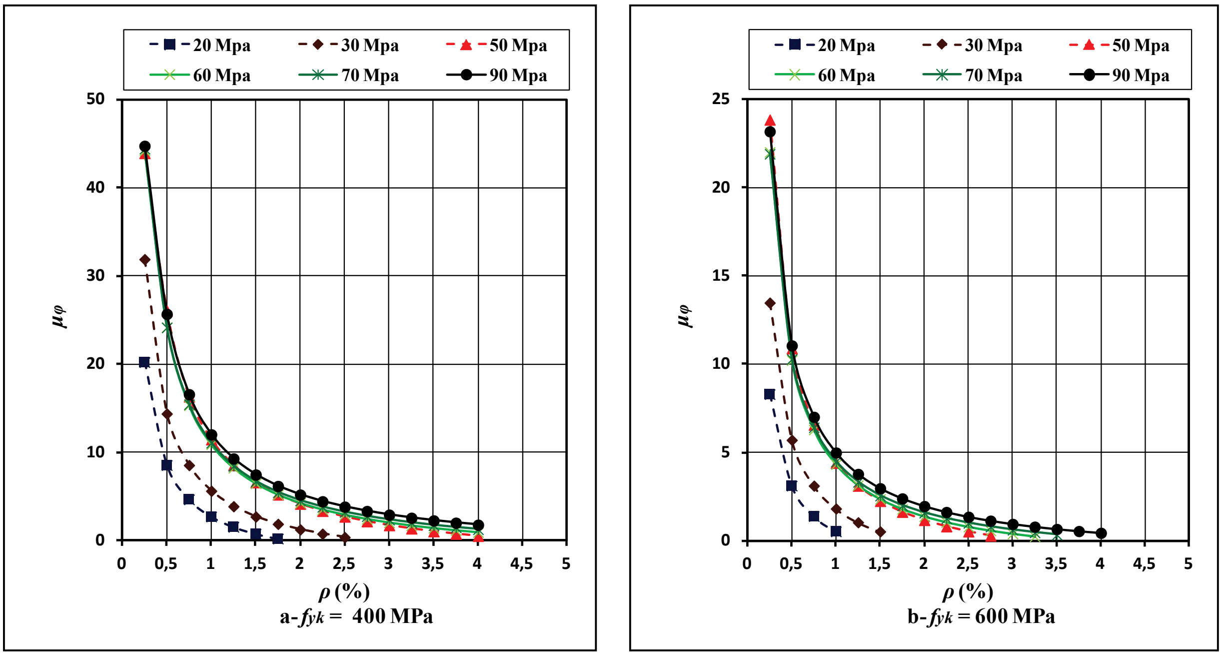

Fig. 8 - Effect of tension reinforcement on the curvature ductility for ρ'/ρ = 0

4.4 Effect of compression reinforcement (ρ')

The effect of compression reinforcement ratio ρ' on the curvature ductility is expressed by the ratio of compression to tension reinforcement *(ρ'/ρ)*. The compression reinforcement ratio will be varied from zero to the value of tension reinforcement ρ, so ρ'/ρ varies from 0 to 1. The figures 9.a, 9.b show the impact of this ratio on the curvature ductility for different cases of concrete strength f_{ck} equal to 50 and 90 MPa respectively.

In Figure 9.a we can see three intervals of variation of curves $\mu_\varphi = f(\rho + \rho')$:

- For $\rho + \rho' < 0,6\%$, the curves decrease with the increasing of the ratio ρ'/ρ;
- For $\rho + \rho' = 0,6\%$, the influence of compression reinforcement is negligible, all curves coincide for any value of the ratio ρ'/ρ;
- For $\rho + \rho' > 0,6\%$, the effect of the ratio ρ'/ρ becomes very advantageous. The curves increase with increasing of the ratio ρ'/ρ;

The same observation is observed in figure 9.b when the concrete strength increases to 90 MPa, with a slight displacement of the coincidence point.

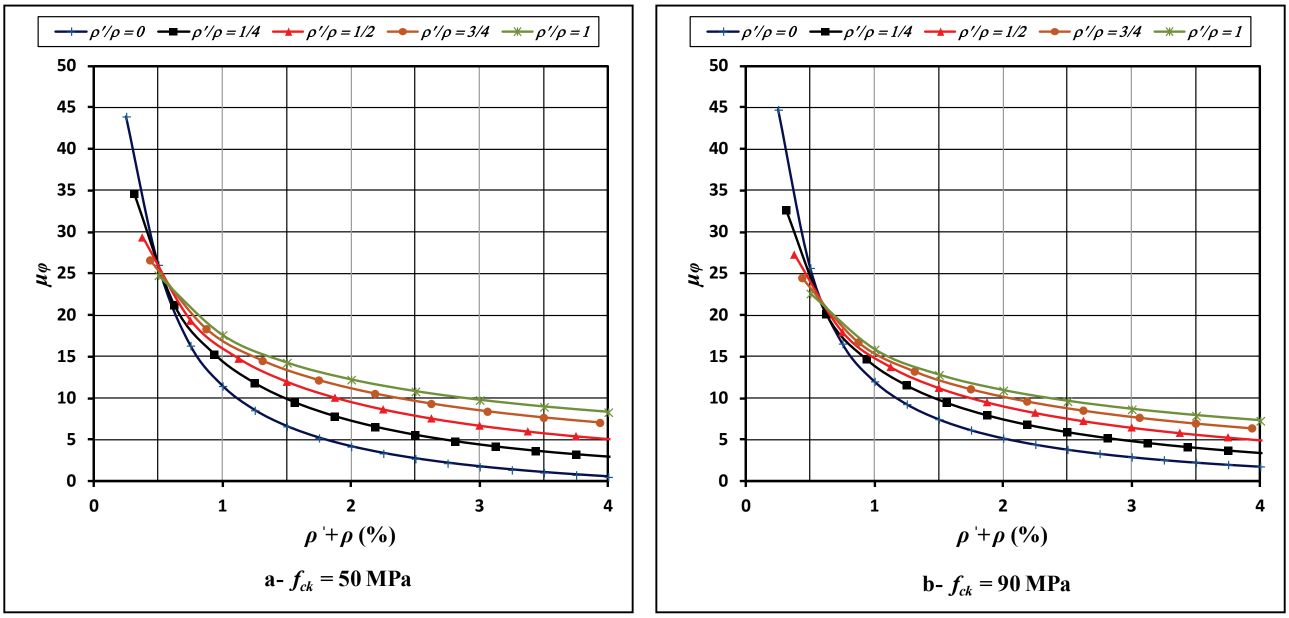

Fig. 9 - Effect of compression reinforcement on the curvature ductility for f_{yk} = 400 MPa

5 Proposed formula for beams with HSC (f_{ck} > 50 MPa)

The parametric study of different factors namely: the concrete strength f_{ck}, the yield strength of steel f_{yk} and the ratios of tension and compression reinforcements (ρ and ρ') showed their considerable effect on the local ductility of unconfined high strength concrete beams. In this section, we try to give a formulation that represents the specific effect of each parameter on the curvature ductility factor μ_φ.

5.1 *Effect of tension steel ratio (ρ)*

According to the various curves $\mu_\varphi = f(\rho)$ illustrated in figure 8, the curvature ductility μ_φ can be expressed in the following form:

$$\mu_\varphi = A\rho^{-B} \tag{18}$$

Where, A and B are constants can be determined based on the parameters previously studied (f_{ck}, ρ, ρ'/ρ and f_{yk}).

To facilitate the calculations, the constant B is fixed by the value -0,5 and the constant A can be written based on the studied parameters as follow:

$$A = C(f_{ck}) * D(f_{yk}) * G(\rho'/\rho) \tag{19}$$

Where, C, D and G are functions with the variables f_{ck}, f_{yk} and ρ'/ρ respectively.

5.2 *Effect of the concrete strength (f_{ck})*

In the case of high strength concrete the effect of concrete strength is practically negligible, but for more accuracy it is necessary to take it into account. The function which represents the effect of the concrete strength ($C(f_{ck})$) is expressed by:

$$C(f_{ck}) = \frac{1}{-0{,}0004 f_{ck}^{2} + 0{,}06 f_{ck} - 1} \tag{20}$$

Or

$$C(f_{ck}) = \frac{-1}{0{,}0004 * \left[(f_{ck} - 75)^2 - 55{,}5^2\right]} \tag{21}$$

5.3 Effect of the yield strength of steel (f_{yk})

The function which represents the effect of the yield strength of steel ($D(f_{yk})$) is expressed by:

$$D(f_{yk}) = -17,4*10^4 * f_{yk}^2 \tag{22}$$

5.4 Effect of compression reinforcement ratio (ρ')

In the last step, the function ($G(\rho'/\rho)$) can be expressed as the following form:

$$G(\rho'/\rho) = \left[20(\rho'-\rho) + \left(1,16 - 0,16\frac{\rho'}{\rho}\right)\right] \tag{23}$$

5.5 Curvature ductility factor

Finally, the combination of the different functions (Eqs. 18, 21, 22 and 23) has allowed writing the curvature ductility factor μ_φ as follow:

$$\mu_\varphi = \frac{-\left[20(\rho'-\rho) + \left(1,16-0,16\frac{\rho'}{\rho}\right)\right]*4,35*10^8}{\left[(f_{ck}-75)^2-55,5^2\right]*f_{yk}^2}\rho^{-0,5} \tag{24}$$

6 Evaluation of the proposed formula

The proposed formula in Equation (24) brings together the effect of the concrete strength f_{ck}, the yield strength of steel f_{yk} and the ratios of tension and compression reinforcements (ρ and ρ') on the curvature ductility. The results obtained by this formula (24) are firstly compared with the Eurocode 2 theory, and secondly with the numerical and experimental results of other researchers.

The table 1 shows a comparison between the results obtained by the proposed formula ($\text{Prop}_{\text{Eq.(24)}}$) and other numerical results of Eurocode 2 (Numerical). The mean values (MV) and the standard deviations (SD) of the ratios $\frac{\text{Prop}_{\text{Eq.(24)}}}{\text{Numerical}}$ are presented in table 1, the MV and the SD are calculated for concrete strength from 51 to 90 MPa and a ratio of reinforcements (ρ + ρ') from 1 to 4%. From this table, in the case of f_{yk} = 400 MPa, the MV and the SD of 90 ratios $\frac{\text{Prop}_{\text{Eq.(24)}}}{\text{Numerical}}$ are equal to 0,92 and 0,11 respectively, this observation means to there is a good agreement between the proposed formula and the numerical method of Eurocode 2. This agreement continues with the increase of the yield strength of steel to 500 and 600 MPa, where, the MV and the SD are calculated based on 90 ratios $\frac{\text{Prop}_{\text{Eq.(24)}}}{\text{Numerical}}$ for each case.

Table 1 - Comparison between the proposed formula and the numerical method used (Eurocode 2 numerical results) for high strength concrete

	f_{yk} = 400 MPa	f_{yk} = 500 MPa	f_{yk} = 600 MPa
Average $\frac{\text{Prop}_{\text{Eq.(24)}}}{\text{Numerical}}$	0,92	0,93	0,96
Standard deviation	0,11	0,18	0,22

In the same context, Table 2 shows a comparison between the results obtained by the proposed formula ($Prop_{Eq.(24)}$) and other results obtained by the prediction of Lee [10] ($Prop_{Lee[10]}$). The mean values (MV) and the standard deviations (SD) of the ratios $\frac{Prop_{Eq.(24)}}{Numerical}$ are calculated for concrete strength from 51 to 90 MPa and a ratio of reinforcements ($\rho + \rho'$) from 1 to 4%. This table presents the MV and the SD of 540 ratios $\frac{Prop_{Eq.(24)}}{Numerical}$, the obtained results have shown that the proposed formula (24) is also in a good agreement with the prediction of Lee [10].

Table 2 - Comparison between proposed formula (24) and the prediction of Lee [10]

	f_{yk} = 400 MPa	f_{yk} = 500 MPa	f_{yk} = 600 MPa
Average. $\frac{Prop_{Eq.(24)}}{Prop_{Lee[10]}}$	1,06	1,00	0,99
Standard deviations	0,11	0,13	0,16

Concerning the experimental validation, Tables 3 shows a comparison between the proposed formula (24) and the experimental results of Bengar and Maghsoudi [12] and Maghsoudi and Sharifi [13]. From this Table, it is observed that the results obtained by our proposal are acceptable compared with the results obtained by experimental works and other results obtained by ACI [14] and CSA [15] provisions. If we neglect some cases when the percentage ρ is greater than 4 %, our results are perfectly acceptable. On the other hand, Tables 4 shows also a comparison between the proposed formula (24) and other experimental results of Zaki et *al*. [21], Carmo et *al*. [22] and Mousa [23]. From this table, it can be noted that all errors calculated are between 0,96 and 25,75 %, except two cases (beam 7 & 10), where the errors equal to 53 and 88 %. The average of the errors out of these two cases decreases from 24 to 14 %, accordingly, the reliability of our formula continues with these experimental results. From this comparison, it can be concluded that the proposed formula (24) has a good performance if the ratio of tension reinforcement is between 1 and 4 % and if the ratio ρ'/ρ is between 0,25 and 1.

Table 3 - Comparison between the proposed formula Eq. (24) and experimental results [12], [13]

	N° Beam	f_{ck} (MPa)	h (mm)	b (mm)	ρ (%)	ρ' (%)	f_{yk} (MPa)	μ_φ				Errors (%)		
								Exp	ACI [14]	CSA [15]	Eq. (24)	Exp	ACI	CSA
[13]	1	73,65	300	200	4,103	2,0515	400	4,33	2,75	3,51	2,92	48,29	5,82	20,21
	2	66,81	300	200	4,773	2,3865	400	-	2,07	2,65	2,49		16,87	6,43
	3	77,72	300	200	5,851	2,9255	400	3,38	1,76	2,18	1,81	86,74	2,76	20,44
[12]	4	56,31	300	200	0,61	0,61	398	11,84	9,89	11,91	12,87	8,00	23,15	7,46
	5	63,48	300	200	1,25	0,61	401	6,84	6,68	8,13	7,83	12,64	14,69	3,83
	6	63,21	300	200	2,03	1,01	373	5,75	5,53	6,87	6,54	12,08	15,44	5,05
	7	71,45	300	200	2,51	1,24	401	5,6	4,75	5,87	4,60	21,74	3,26	27,61

Table 4 - Comparison between the proposed formula Eq. (24) and experimental results [21], [22] and [23]

	N° Beam	f_{ck} (MPa)	*h* (mm)	*b* (mm)	ρ (%)	ρ' (%)	f_{yk} (MPa)	μ_φ Exp	μ_φ Eq. (24)	Errors (%)
[21]	1	60,5	150	150	0,806	0,29	400	9,55	10,54	9,39
	2	60,5	150	150	1,74	0,516	400	6,29	6,23	0,96
	3	60,5	150	150	2,9	0,806	400	3,34	3,88	13,92
	4	60,5	150	150	3,48	0,806	400	2,22	2,99	25,75
[22]	5	53	270	120	0,54	0,2	545	6,14	7,93	22,57
	6	53	270	120	1,62	0,2	545	3,04	3,79	19,79
	7	52	270	120	2,27	0,21	545	1,31	2,79	53,05
	8	63	270	120	1,12	0,21	545	5,52	4,47	23,49
	9	63	270	120	2,96	0,21	545	1,57	1,74	9,77
[23]	10	52	200	150	0,84	0,582	427-498	19,14	10,18	88,02
	11	52	200	150	1,49	0,582		7,70	7,017	9,73
	12	52	200	150	0,914	0,634		10,87	9,71	11,95
									Average 1	24,03
									Average 2	14,73

7 Conclusion

Based on the parametric study performed, a simple equation is recommended for predicting the curvature ductility of high strength concrete beams. The formula proposed in this paper generally includes the parameters affecting the local ductility of a reinforced concrete beam such as, the concrete strength, the yield strength of steel and the tension and compression reinforcements.

The credibility of the proposed formula is validated with the Eurocode 2 numerical results and other numerical and experimental results of different researchers. The proposed formula is applied to the practical conditions of doubly reinforced concrete beams with HSC. Generally, it is valid for the following practical values: $50 < f_{ck} \leq 90$ MPa, $1\% \leq \rho \leq 4$ %, $0,25 \leq \rho'/\rho \leq 1$ and $400 \leq f_{yk} \leq 600$ MPa.

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