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A fuzzified Industrial warehouse inventory model for deteriorating items with decreasing demand and various fuzzy cost parameters

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Abstract: In this research study, an attempt has been made to develop an industrial warehouse inventory model considering demand as exponentially decreasing function of time. Shortages are not allowed in the model and fuzziness is introduced in the system by assuming the various cost components(holding cost,ordering cost,purchase cost,deterioration cost).In the fuzzy environment cost parameters are taken to be triangular fuzzy numbers.The purpose is to minimize the total cost associated with the inventory system.A numerical example is given to illustrate the model approximately.

Keywords: Inventory model, Deterioration, Decreasing demand, Triangular fuzzy number, Holding cost, Defuzzification.

Introduction: Demand has always been the prime issue in dealing an inventory system.Sometimes the demand of the items may be probabilistic in nature, sometimes it may be static i.e, constant for each time period.Further it may follow the pattern of increasing and decreasing type or constant type.So, due to the change in the market scenario demand varies from time to time.Seasonal factor is one of the major things on which the demand depends.For an example when winter approaches Room Heater,Woolen clothes etc are of high demand and at the end of winter season demand of these items decreases.In this context both increasing and decreasing demands are coming in to the picture.But in an inventory model not only demand but also different cost factors play a vital role as cost parameters(holding cost,ordering cost etc) are known and have definite value with ambiguity.Some of the business situation fit such conditions but in most of the cases due to the change in market scenario these parameters are imprecise.This uncertainty concept can be defined as fuzziness or vagueness.The industrial authority have to decide the quantity to be manufactured.Also deterioration factor must be taken in to consideration as so many physical goods are there which deteriorate during the stock in periods due to different factors like dryness,rusting of iron,damage,spoilage and vaporization.So considering all the factors an inventory model has to be prepared so that the total cost associated with the system is minimum and profit is maximum.

During the last few decades innumerable numbers of inventory model have been prepared. Ghare and Schrader(1963) developed for the first time an inventory model for deteriorating items. Convert and Philip(1973) extended their work. Hartely (1976) first proposed a problem in his book "Operations Research – A Managerial Emphasis. Dave (1988) discussed the two-warehouse inventory models for finite and infinite rate of replenishment .Donaldson (1977) developed an optimal algorithm for solving classical no shortage inventory model.Benkherouf (1997) presented a two-warehouse model for deteriorating items with the general form of time dependent demand under continuous release fashion. Lee and Ma (2000) developed a no-shortage inventory model for perishable items with free form of time dependent demand and fixed planning horizon. In their model, some cycles are of single warehouse system and the remaining is of two-warehouse system.On the other hand, considering two-storage facilities, Yang (2004) developed two inventory models for deteriorating items with uniform demand rate and completely backlogged shortages under inflation. Recently, Yang (2006) extended the models introduced in Yang (2004) by incorporating the partially backlogged shortages.Deb Choudhury.P and Dutta.P (2015) developed a two warehouse inventory model considering demand as cubic function of time. Also Deb Choudhury.P and Dutta.P(2015) have fuzzified the same model. The concept of fuzzy logic was first proposed by Zadeh(1965) .Bellam and Zadeh(1970) discussed the difference between randomness and fuzziness . Zimmermann(1985) gave a review on applications of fuzzy set theory. Park (1987) discussed the EOQ model in which trapezoidal fuzzy numbers are used. Yao and Lee(1999) presented a fuzzy inventory model with and without backorder for fuzzy order quantity with trapezoidal fuzzy number.

As fuzziness means vagueness,certain parameters like various cost parameters are not always measurably properly,so we have assumed these parameters as fuzzy number in fuzzy system. A comparative study between crisp and fuzzy system has been highlighted properly.In the current research study, an inventory model has been prepared considering demand as decreasing function of time.Fuzzification is allowed in the system considering the cost parameters as triangular fuzzy numbers.Signed distance method has been used for defuzzification.

PRELIMINARIES:

FUZZY SET

A fuzzy set $\breve{A}$ in a universe of discourse X is defined as the set of pairs $\breve{A} = \{(x, \mu_{\breve{A}}(x)) : x \in X\}$, where $\mu_{\breve{A}}(x)$:X→ [0,1] is a mapping and $\mu_{\breve{A}}(x)$ is called membership function of $\breve{A}$ or grade of membership of x in $\breve{A}$.

CONVEX FUZZY SET

A fuzzy set $\breve{A}$ in a universe of discourse is called convex if for all

$x_1, x_2 \in X, \mu_{\breve{A}}(\delta x_1 + (1-\delta)x_2 \geq \min\{\mu_{\breve{A}}(x_1), \mu_{\breve{A}}(x_2)\}, where\ \delta \in [0,1]$.

NORMAL FUZZY SET

A fuzzy set $\breve{A}$ is called normal fuzzy set if there exists at least one $x \in X$ such that $\mu_{\breve{A}}(x_1) = 1$.

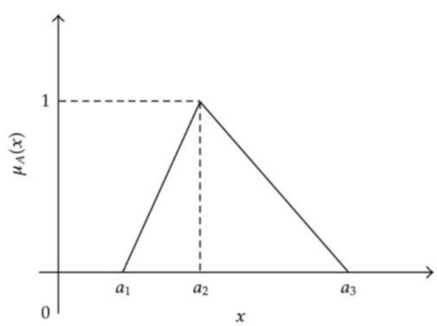
FUZZY NUMBER

A fuzzy number is a special case of a fuzzy set. Different definitions and properties of fuzzy numbers are encountered in the literature .But it actually represents the notation of a set of real numbers 'closer to a' where 'a' is the number being fuzzified. A fuzzy number is a fuzzy set which is both convex and normal.

TRIANGULAR FUZZY NUMBER (TFN)

A triangular fuzzy number $\breve{A}$ is represented by the triplet (a_1, a_2, a_3) and is defined by its continuous membership function where $\mu_{\breve{A}}(x)$:X→ [0,1] is given by

$$\mu_{\breve{A}}(x) = f(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } x = a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

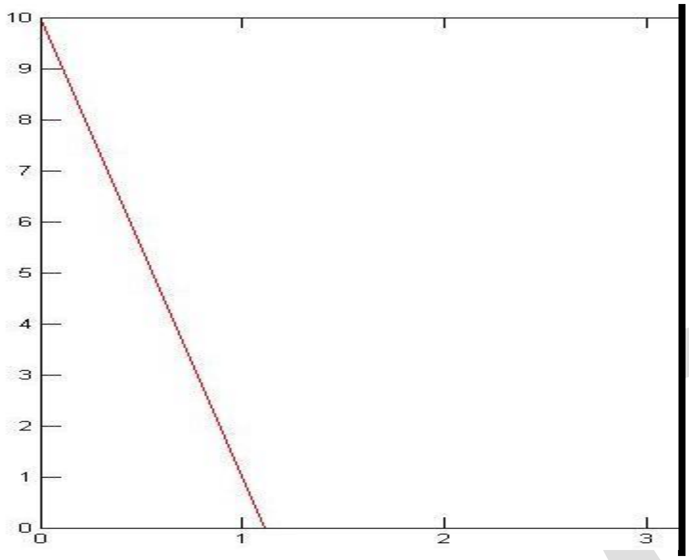


Assumptions and Notation:

The following assumptions have been used in developing the model:

- Replenishment rate is infinite.
- Lead time is constant.
- Shortages are not allowed in the system.
- Costs are considered as triangular fuzzy numbers.
- Model is formulated both in Crisp and Fuzzy system.

- A single item is considered over the prescribed period T units of time, which is subject to variable demand rate.
- Model is considered for imperfect items.
- Deterioration rate is constant.
- Demand is assumed as $D = Ae^{-bt}, decreases\ with\ time\ . A > 0, b > 0$.

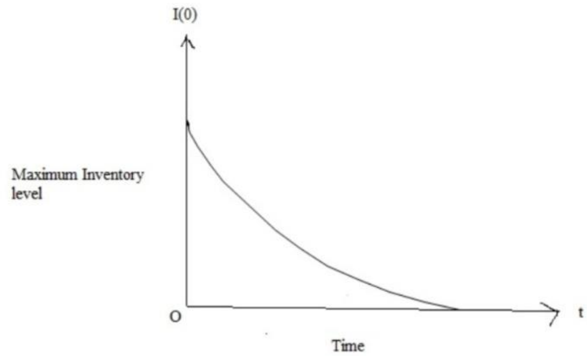


(Demand Vs Time graph, demand is of decreasing type) t→

The following notations have been used in the present model :

- T=total cycle length.
- I(t)=Inventory level in the Industrial warehouse.
- h=holding cost in Industrial warehouse per unit per unit time.
- c =Deterioration rate.
- C_d =Deterioration cost per unit.
- O_c =Ordering cost per unit.
- C_p =Purchase cost per unit.
- Q =Ordering quantity.
- t_1 =time at which the inventory level falls to zero.
- C_h =Inventory holding cost.

Mathematical Model: Let Q be the total amount of inventory purchased at the beginning of each period. Now the demand may be a decreasing function of time but still the inventory level is falling during $[0, t_1]$due to the demand an deterioration and vanishes



Graphical representation of the model

completely at $t = t_1$

Let I(t) be the on-hand inventory level at time t. The differential equations associated with the system is

$$\frac{dI}{dt} + cI = -Ae^{-bt}, I(t_1) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The solution of the equation is given by,

$$I(t) = \frac{Ae^{-bt}}{b-c} - \frac{Ae^{ct_1-bt_1}.e^{-ct}}{b-c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Inventory Holding cost,

$$C_h = \int_0^{t_1} hI(t)dt$$

i.e, $C_h = \frac{Ah}{(b-c)}\left[-\frac{e^{-bt_1}}{c} - \frac{e^{ct_1-bt_1}}{c} + \frac{e^{-bt_1}}{b} + \frac{1}{b}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$

Deterioration cost= $C_d \int_0^{t_1} I(t)dt$

$$= \frac{cC_d Ah}{b-c}\left[-\frac{e^{-bt_1}}{c} - \frac{e^{ct_1-bt_1}}{c} + \frac{e^{-bt_1}}{b} + \frac{1}{b}\right] \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Ordering cost=O_c

Purchase cost= $C_p\left[-\frac{Ae^{ct_1-bt_1}}{b-c} + \frac{A}{b-c}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$

Total cost of the system=$\frac{1}{T}[C_h + C_d + O_c + Purchase\ Cost]$

$$=\frac{1}{T}\left\{\frac{Ah}{(b-c)}\left[-\frac{e^{-bt_1}}{c} - \frac{e^{ct_1-bt_1}}{c} + \frac{e^{-bt_1}}{b} + \frac{1}{b}\right] + \frac{cC_d Ah}{b-c}\left[-\frac{e^{-bt_1}}{c} - \frac{e^{ct_1-bt_1}}{c} + \frac{e^{-bt_1}}{b} + \frac{1}{b}\right] + O_c + C_p\left[-\frac{Ae^{ct_1-bt_1}}{b-c} + \frac{A}{b-c}\right]\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The total cost per unit time is minimum if

$$\frac{\partial TC}{\partial t_1} = 0$$

Fuzzy model:

By using signed distance method we have solved the model in fuzzy environment. We have used triangular fuzzy number for holding costs, deterioration cost, ordering cost, purchase cost.

(i) $h \in [h-\Delta_1, h+\Delta_2], 0 < \Delta_1 < h, 0 < \Delta_1\Delta_2$
(ii) $C_d \in [C_d-\Delta_3, C_d+\Delta_4], 0 < \Delta_3 < C_d, 0 < \Delta_3\Delta_4$
(iii) $O_c \in [O_c-\Delta_5, O_c+\Delta_6], 0 < \Delta_5 < O_c, 0 < \Delta_5\Delta_6$
(iv) $C_p \in [C_p-\Delta_7, C_p+\Delta_8], 0 < \Delta_7 < C_p, 0 < \Delta_7\Delta_8$

The signed distance method of the above fuzzy numbers are as

(i) $d(h,0) = h + \frac{1}{4}(\Delta_2-\Delta_1)$
(ii) $d(C_d,0) = C_d + \frac{1}{4}(\Delta_4-\Delta_3)$
(iii) $d(O_c,0) = O_c + \frac{1}{4}(\Delta_6-\Delta_5)$
(iv) $d(C_p,0) = C_p + \frac{1}{4}(\Delta_8-\Delta_7)$

Now, $T\check{C} = (TC_1, TC_2, TC_3)$

$$TC_1 = \frac{1}{T}\left\{\frac{A(h-\Delta_1)}{(b-c)}\left[-\frac{e^{-bt_1}}{c}-\frac{e^{ct_1-bt_1}}{c}+\frac{e^{-bt_1}}{b}+\frac{1}{b}\right]+\frac{c(C_d-\Delta_3)A(h-\Delta_1)}{b-c}\left[-\frac{e^{-bt_1}}{c}-\frac{e^{ct_1-bt_1}}{c}+\frac{e^{-bt_1}}{b}+\frac{1}{b}\right]+(O_c-\Delta_5)\right.$$
$$\left.+ \quad (C_p-\Delta_7)\left[-\frac{Ae^{ct_1-bt_1}}{b-c}+\frac{A}{b-c}\right]\right\}$$

$$TC_2 = TC$$

$$TC_3 = \frac{1}{T}\left\{\frac{A(h+\Delta_2)}{(b-c)}\left[-\frac{e^{-bt_1}}{c}-\frac{e^{ct_1-bt_1}}{c}+\frac{e^{-bt_1}}{b}+\frac{1}{b}\right]+\frac{c(C_d+\Delta_4)A(h+\Delta_2)}{b-c}\left[-\frac{e^{-bt_1}}{c}-\frac{e^{ct_1-bt_1}}{c}+\frac{e^{-bt_1}}{b}+\frac{1}{b}\right]+(O_c+\Delta_6)\right.$$
$$\left.+ \quad (C_p+\Delta_8)\left[-\frac{Ae^{ct_1-bt_1}}{b-c}+\frac{A}{b-c}\right]\right\}$$

The total inventory cost per unit time by signed distance method is

$$d(T\check{C}) = TC + \frac{1}{T}\left\{\frac{A(\Delta_2-\Delta_1)}{(b-c)}\left[-\frac{e^{-bt_1}}{c}-\frac{e^{ct_1-bt_1}}{c}+\frac{e^{-bt_1}}{b}+\frac{1}{b}\right]+\frac{(\Delta_4-\Delta_3)A(\Delta_2-\Delta_1)}{b-c}\left[-\frac{e^{-bt_1}}{c}-\frac{e^{ct_1-bt_1}}{c}+\frac{e^{-bt_1}}{b}+\frac{1}{b}\right]+(\Delta_6-\Delta_5)+(\Delta_8-\Delta_7)\left[-\frac{Ae^{ct_1-bt_1}}{b-c}+\frac{A}{b-c}\right]\right\} \quad \text{.................................}(7)$$

Numerical Example: In order to illustrate the above system of equations connecting total cost of the system , consider an inventory system with the following data and compute in both crisp and fuzzy system.

Crisp Model:

Consider $c = 0.05, h = 0.4, C_d = 0.5, C_p = 0.6, O_c = 2000$ $then$ $t_1 = 1.2$ and $TC = 1669.04$.

Fuzzy Model:

Consider $c = 0.05, h = (0.3, 0.4, 0.6), C_d = (0.4,0.5,0.7), C_p = (0.5,0.6,0.8), O_c = (1900,2000,2200)$ $then$ $t_1 = 1.15$ and $TC = 1690.88$.

Conclusion:

We have developed an industrial warehouse inventory model for deteriorating items having time varying exponentially decreasing demand.The model has been formulated with the practical assumption of demand rate for seasonal products.More precisely during the end of particular seasons like winter the demand rate of room heaters ,woolen clothes, etc decreases so taking in to account these quantities the system has been solved.Also deterioration of products is a natural phenomenon as almost all the products undergo decay during the course of time so deterioration factor has played a vital role in the inventory model.The model we have solved is highlighted in both crisp and fuzzy system.As cost parameters are imprecise and sometimes it is not possible to get the suitable result so fuzziness is used.Well known triangular membership function is used for all the fuzzy numbers.Signed distance defuzzification method is also used to formulate the resulting equations in fuzzy system.

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