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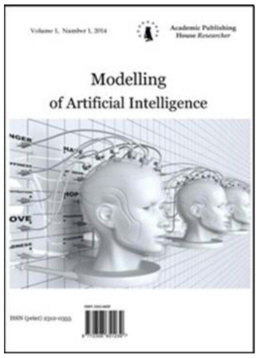

Gas Storage Valuation based on Spot Prices

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Abstract

In the paper we present an algorithmic approach for gas storage valuation. The gas price term structure is described by Andersen's commodity model in Carmona-Ludkovski framework. We first derive Hamilton-Jacobi-Bellman equation for this case, and then switch to algorithmic approach to find the optimal solution under Bellman condition.

Keywords: Hamilton-Jacobi-Bellman equation, Gas storage valuation, Commodity futures.

1. Introduction

The present paper continues the investigations begun in Kechejian, Ohanyan, 2015, Kechejian, Ohanyan, Bardakhchyan, 2016a and Kechejian, Ohanyan, Bardakhchyan, 2016b, dealing with pricing of derivative products based on futures price term structure described according Andersen's model (Andersen, 2008).

Gas storage valuation methods are generally modelled as maximal expected value of optimally controlled value process (Kechejian, Ohanyan, 2012).

2. Results

As in Carmona, Ludkovski, 2006 we approach the optimal gas storage management as a switching process, described by three states or equivalently by three value of control; $u(t) \in \{-1;0,1\}$, where 1 is the state when we withdraw some gas from the storage, -1 when we inject gas into the storage, and 0 when nothing is done, hence capacity changes are described by the sign of control u.

Here we stay the formulations of Carmona and Ludkovski, with some subtle modifications. We denote the capacity of storage by $C(u(t),t)$, the gas spot price by $G(t)$, direct costs by $K\big(u(t),t,C(u,t)\big)$, and we also allow for fuel and commodity charges when injecting or withdrawing gas. We have the following description of instantaneous income process $\psi\big(u(t),t,C(u,t),S(t)\big)$.

$$\begin{cases} \psi(-1,t,C,S) = -G(t)b_{-1}(t) - K\big(-1,C(t)\big); & and \quad dC(t) = a_{-1}(t)dt \\ \psi(0,t,C,S) = -K\big(0,C(t)\big); & and \quad dC(t) = a_0(t)dt \\ \psi(1,t,C,S) = S(t)b_{-1}(t) - K\big(1,C(t)\big); & and \quad dC(t) = a_1(t)dt \end{cases}$$

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Hence the general formulation of the control problem is:

$$V(t,s,c,u) = \sup_{u\in U} E_t\left[\int_t^T e^{-r(v-t)}\psi\big(u(v),v,C(u,v),S(v)\big)\,dv \ \mid G(t)=g; C(.,t)=c\right]$$

Compare this with original formulation in Carmona, Ludkovski, 2006. Here r is discount rate, and U is a set of all adapted processes.

Hamilton-Jacobi-Bellman (HJB) equation for Anderson spot prices models

We assume that the commodity future process is described by Andersen's model (Andersen, 2008). The general model is described by stochastic differential equation (SDE) constituting a Markov process.

$$\frac{dF(t,T)}{F(t,T)} = \sigma_1(t,T)dW_1(t) + \sigma_1(t,T)dW_2(t),$$

where

$$\sigma_1(t,T) = e^{b(T)}h_1e^{-k(T-t)} + e^{a(T)}h_\infty,; \qquad \sigma_2(t,T) = e^{b(T)}h_2e^{-k(T-t)}$$

and $W_1(t)$ and $W_2(t)$ are independent Wiener processes (Karatzas, Shreve, 1998; Miltersen, 2003; Pham, 2010).

Note that a and b differ from a_u and b_u in their definition.

To derive the spot price process, we use lemma 2 in (Andersen, 2008), and get the following.

$$\begin{aligned}dG(t) = {}& \frac{\partial F(0,t)}{\partial t}dt + \left(\frac{1}{2}(h_1^2+h_2^2)e^{2a(t)+2d(t)} + \frac{1}{2}h_\infty^2e^{2a(t)}\right)dt\\ &-\Bigg(e^{2a(t)+2d(t)}\big(a'(t)+d'(t)\big)\frac{h_1^2+h_2^2}{2k}\big(1-e^{-2kt}\big) + \frac{h_1^2+h_2^2}{2}e^{2a(t)+2d(t)}\\ &+ e^{2a(t)+d(t)}\big(2a'(t)+d'(t)\big)\frac{h_1h_\infty}{2k}\big(1-e^{-kt}\big) + e^{2a(t)+d(t)}\frac{h_1h_\infty}{2}e^{-kt}\Bigg)dt\\ &+ J_t(a'(t)+d'(t)-k)dt + h_1e^{a(t)+d(t)}dW_1(t) + h_2e^{a(t)+d(t)}dW_2(t)\\ &- \left(\frac{1}{2}e^{2a(t)+d(t)}\big(2a'(t)+d'(t)\big)\frac{h_1h_\infty}{k}\big(1-e^{-kt}\big) + \frac{1}{2}e^{2a(t)+d(t)}h_1h_\infty e^{-kt}\right)dt\\ &+ I_ta'(t)dt + e^{a(t)}h_\infty dW_1(t),\end{aligned}$$

where $d(t) = a(t)+b(t)$, J_t and I_t have the following form:

$$J_t = e^{a(t)+d(t)}e^{-kt}\left(\int_0^t h_1e^{ku}dW_1(u) + \int_0^t h_2e^{ku}dW_2(u)\right); \quad I_t = e^{a(t)}h_\infty W_1(t),$$

or in differential form

$$dJ_t = J_t(a'(t)+b'(t)-k)dt + h_1dW_1(t) + h_2dW_2(t)$$

$$dI_t = I_ta'(t)dt + e^{a(t)}h_\infty dW_1(t)$$

Next we bypass the Monte Carlo method of Carmona, Ludkovski, 2006 and use numerical methods for deterministic PDE rather than SDE.

We Like to use HJB equation in our case (see Kechejian, Ohanyan, 2015), however to do it we should have an SDE for spot prices in the form,

$$dG(t) = \mu\big(t,G(t)\big)dt + \sigma\big(t,G(t)\big)dW(t).$$

Where $\sigma(t,S(t))$ is in matrix form.

However we note that in order to bring our SDE to the desirable form, we are to be able to express $J_t = f\big(t,G(t)\big)$, and $I_t = h\big(t,G(t)\big)$.

Using Ito formula for function $f(t,x)$ we have

$$dJ_t = y(t)\cdot dt + \big(h_1e^{a(t)+d(t)} + h_\infty e^{a(t)}\big)\frac{\partial f}{\partial x}dW_1(t) + h_2e^{a(t)+d(t)}\frac{\partial f}{\partial x}dW_2(t)$$

Equating the coefficients of above SDE with the ones in the original SDE of J_t, we get the following system

$$\begin{cases}\big(h_1e^{a(t)+d(t)} + h_\infty e^{a(t)}\big)\dfrac{\partial f}{\partial x} = h_1\\[2mm] h_2e^{a(t)+d(t)}\dfrac{\partial f}{\partial x} = h_2\end{cases}$$

This system has no solution, or in other words there is no function for which $J_t = f\big(t,S(t)\big)$.

The same can be stated for $I_t = h\big(t, S(t)\big)$, i.e. there is no such $h(t,x)$.

And at last there are no function $d(t,x)$, and $l(t,x)$, for which $J_t = d(t, I(t))$, or $I_t = l(t, J(t))$

So we come to an impasse while using two-dimensional HJB equation (for G and C). We should rather use four-dimensional HJB equations for $V(t,g,c,j,i,u)$.

Restating the four SDE-s in a matrix form we have the following

$$\begin{pmatrix} dG_t \\ dI_t \\ dJ_t \\ dC_t \end{pmatrix} = \begin{pmatrix} \mu(t,S,I,J) \\ I_t a'(t) \\ J_t(a'(t)+b'(t)-k) \\ a_u(t) \end{pmatrix} dt + \begin{pmatrix} h_1 e^{a(t)+d(t)} + h_\infty e^{a(t)} & h_2 e^{a(t)+d(t)} \\ e^{a(t)} h_\infty & 0 \\ h_1 & h_2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} dW_1(t) \\ dW_2(t) \end{pmatrix},$$

where

$$\begin{aligned}\mu(t,S,I,J) = {}& \frac{\partial F(0,t)}{\partial t} + \left(\frac{1}{2}(h_1^2+h_2^2)e^{2a(t)+2d(t)} + \frac{1}{2}h_\infty^2 e^{2a(t)}\right) \\ & - \Bigg(e^{2a(t)+2d(t)}\big(a'(t)+d'(t)\big)\frac{h_1^2+h_2^2}{2k}\big(1-e^{-2kt}\big) + \frac{h_1^2+h_2^2}{2}e^{2a(t)+2d(t)}e^{-2kt} \\ & + e^{2a(t)+d(t)}\big(2a'(t)+d'(t)\big)\frac{h_1 h_\infty}{2k}\big(1-e^{-kt}\big) + e^{2a(t)+d(t)}\frac{h_1 h_\infty}{2}e^{-kt}\Bigg) \\ & + J_t(a'(t)+d'(t)-k) \\ & - \left(\frac{1}{2}e^{2a(t)+d(t)}\big(2a'(t)+d'(t)\big)\frac{h_1 h_\infty}{k}\big(1-e^{-kt}\big) + \frac{1}{2}e^{2a(t)+d(t)}h_1 h_\infty e^{-kt}\right) + I_t a'(t)\end{aligned}$$

For simplicity we write $\mu(t,S,I,J)$ in the following form:

$$\begin{aligned}\mu(t,S,I,J) &= p(t,h_1,h_2,h_\infty,k,a(t),d(t),a'(t),d'(t)) + J_t(a'(t)+d'(t)-k) + I_t a'(t) \\ &= p + J_t(a'(t)+d'(t)-k) + I_t a'(t)\end{aligned}$$

Also we have used $dC(t) = a_u(t)dt$. We get the following form for HJB equation

$$\begin{aligned}\sup_{u\in U}\Bigg[& u(t)g(t)b_u - K_u\big(c(t)\big) + \frac{\partial V}{\partial t} + \frac{\partial V}{\partial s}\big(p(t) + j(t)(a'(t)+d'(t)-k) + i(t)a'(t)\big) + \frac{\partial V}{\partial i}i(t)a'(t) \\ & + \frac{\partial V}{\partial j}j(t)(a'(t)+d'(t)-k) + \frac{\partial V}{\partial c}a_u(t) \\ & + \frac{1}{2}\Bigg(\frac{\partial^2 V}{\partial s^2}\Big(\big(h_1 e^{a(t)+d(t)} + h_\infty e^{a(t)}\big)^2 + h_2^2 e^{2a(t)+2d(t)}\Big) + \frac{\partial^2 V}{\partial i^2}e^{2a(t)}h_\infty^2 + \frac{\partial^2 V}{\partial i^2}(h_1^2+h_2^2) \\ & + 2\frac{\partial^2 V}{\partial s\partial i}h_1 h_\infty e^{2a(t)+d(t)} + 2\frac{\partial^2 V}{\partial s\partial j}\big(e^{a(t)+d(t)}(h_1^2+h_2^2) + h_\infty e^{a(t)}\big) + 2\frac{\partial^2 V}{\partial j\partial i}h_1 h_\infty e^{a(t)}\Bigg) \\ & - rV\Bigg] = 0,\end{aligned}$$

where we have used

$$\psi(u,t,c,s) = u(t)g(t)b_u - K_u\big(c(t)\big)$$

Here we note, that the main differential equation also contains term $a_u(t)$.

As the profit is linear to injection/withdrawal rates it is always optimal to inject/withdraw at maximum allowable rate.

Algorithmic approach

Here we attempt to explicitly solve the stated problem in several steps (layers). To accomplish the latter, we propose the following algorithm. First divide the whole decision space into subclasses with no switching, 1 switching , 2 switching etc. strategies.

$$V = E\left(\int_0^T e^{-rs}\psi_{u_s}\big(G_s, \bar{C}_s(u)\big)ds\right)$$

$u \in \{-1, 0, 1\}$,

$$\psi_{u_s}\big(G_s, \bar{C}_s(u)\big) = uG_s b_u(\overline{C_s}) - K_u(\bar{C}_s)$$

<u>No switching strategy</u> The only possible case with no change is of form (0,0)

$$V = E\left(\int_0^T e^{-rs}\psi_{u_s}\big(G_s,\bar{C}_s(u)\big)ds\right) = E\left(-\int_0^T e^{-rs}K_0(\bar{C}_s)ds\right) = -\int_0^T e^{-rs}K_0(\bar{C}_s)ds$$

One switch strategy The only feasible strategy is (-1,1). Here -1 means we are buying gas (injection), 1 means withdrawal.

For each such strategy the maximal V is obtained when we change our u at the most profitable time.

Let's denote it by t^*. V then becomes

$$V = E\left(\int_0^{t^*} e^{-rs}\left(-G_s b_{in}(\bar{C}_s) - K_{-1}(\bar{C}_s)\right)ds + \int_{t^*}^{T} e^{-rs}\left(G_s b_{out}(\bar{C}_s) - K_1(\bar{C}_s)\right)ds\right)$$

To obtain t^*, we should solve the following equation

$$\frac{\partial V}{\partial t}|_{t=t^*} = 0$$

which becomes

$$E\left(e^{-rt^*}\left(-G_{t^*} b_{in}(\bar{C}_{t^*}) - K_{-1}(\bar{C}_{t^*})\right)\right) - E\left(e^{-rt^*}\left(G_{t^*} b_{out}(\bar{C}_{t^*}) - K_1(\bar{C}_{t^*})\right)\right) = 0$$

$$K_1(\bar{C}_{t^*}) - K_{-1}(\bar{C}_{t^*}) = \left(b_{out}(\bar{C}_{t^*}) - b_{in}(\bar{C}_{t^*})\right)E(G_{t^*})$$

From the above we get

$$E(G_{t^*}) = \frac{K_1(\bar{C}_{t^*}) - K_{-1}(\bar{C}_{t^*})}{b_{out}(\bar{C}_{t^*}) - b_{in}(\bar{C}_{t^*})} \qquad 3.1)$$

We also have a boundary conditions, for $\overline{C}_t$:

$$\overline{C}_0 = \overline{C}_T = 0,$$

since

$$\overline{C}_t = \int_0^t u(s) a_u(\overline{C}_s)\, ds.$$

We can rewrite the condition in the following manner

$$\overline{C}_T = \int_0^T u(s) a_u(\overline{C}_s)\, ds = \int_0^{t^*} a_{in}(\overline{C}_s)\, ds - \int_{t^*}^T a_{out}(\overline{C}_s)\, ds = 0$$

Then the boundary conditions can be written as

$$\int_0^{t^*} a_{in}(\overline{C}_s)\, ds = \int_{t^*}^T a_{out}(\overline{C}_s)\, ds \qquad 3.2)$$

If there is such t^*, then we have found the optimal switching time.

The existence of a solution is shown in Carmona, Ludkovski, 2006, for k switching options where ≥ 1 . So in general for 1 switching the existence of such solution may not be assured.

However, if we impose some additional conditions on the function K(C(t)), C(t), bu and au, we can guarantee its existence.

Moreover, if the system (3.1) and (3.2) are incompatible, then this will imply that our maximum is on the boundary, which contradicts (3.2). So this implicitly imply that (0,0) is better than (-1,1).

The uniqueness is also obvious as $a_u(C) > 0$, whenever $C > 0$, regardless of u. So in (3.2) the left part is increasing, and the right part is decreasing in t^*.

$E(G_{t^*})$ is constant, as G_t is martingale (see Andersen, 2008), hence if $\frac{K_1(\bar{C}_{t^*})-K_{-1}(\bar{C}_{t^*})}{b_{out}(\bar{C}_{t^*})-b_{in}(\bar{C}_{t^*})}$ is decreasing in t^*, (3.1) will also have unique solution.

What remains to show is the coincidence of unique solutions of (3.1) and (3.2).

The latter can't be shown without an explicit form of K, and relation of $b_u(\cdot)$ and $a_u(\cdot)$.

In the case of two switching all possible variants are (0,-1,1) ; (-1,0,1) and (-1,1,0).

Let's look for example to the first one - (0,-1,1).

$$V = E\left(\int_0^{t_1^*} e^{-rs}\left(-K_0(\bar{C}_s)\right)ds + \int_{t_1^*}^{t_1^*+t_2^*} e^{-rs}\left(-G_s b_{in}(\bar{C}_s) - K_{-1}(\bar{C}_s)\right)ds + \int_{t_1^*+t_2^*}^{T} e^{-rs}\left(G_s b_{out}(\bar{C}_s) - K_1(\bar{C}_s)\right)ds\right)$$

From which we have

$$\frac{\partial V}{\partial t}|_{t=t_1^*}=0 \qquad \frac{\partial V}{\partial t}|_{t=t_2^*}=0$$

that is

$$-E\left(e^{-rt_1^*}K_0(\bar C_{t_1^*})\right)+E\left(e^{-r(t_1^*+t_2^*)}\left(-G_{t_1^*+t_2^*}b_{in}(\bar C_{t_1^*+t_2^*})-K_{-1}(\bar C_{t_1^*+t_2^*})\right)\right)$$
$$-E\left(e^{-rt_1^*}\left(-G_{t_1^*}b_{in}(\bar C_{t_1^*})-K_{-1}(\bar C_{t_1^*})\right)\right)$$
$$-E\left(e^{-r(t_1^*+t_2^*)}\left(G_{t_1^*+t_2^*}b_{out}(\bar C_{t_1^*+t_2^*})-K_1(\bar C_{t_1^*+t_2^*})\right)\right)=0$$

and

$$E\left(e^{-r(t_1^*+t_2^*)}\left(-G_{t_1^*+t_2^*}b_{in}(\bar C_{t_1^*+t_2^*})-K_{-1}(\bar C_{t_1^*+t_2^*})\right)\right)-E\left(e^{-r(t_1^*+t_2^*)}\left(G_{t_1^*+t_2^*}b_{out}(\bar C_{t_1^*+t_2^*})-K_1(\bar C_{t_1^*+t_2^*})\right)\right)=0$$

We can conclude that

$$-E\left(e^{-rt_1^*}K_0(\bar C_{t^*})\right)-E\left(e^{-rt_1^*}\left(-G_{t_1^*}b_{in}(\bar C_{t_1^*})-K_{-1}(\bar C_{t_1^*})\right)\right)=0$$

So, the final form will be

$$\begin{cases} E\left(K_0(\bar C_{t_1^*})\right)-E\left(G_{t_1^*}b_{in}(\bar C_{t_1^*})+K_{-1}(\bar C_{t_1^*})\right)=0 \\ \boldsymbol{E}\left(-G_{t_1^*+t_2^*}b_{in}(\bar C_{t_1^*+t_2^*})-K_{-1}(\bar C_{t_1^*+t_2^*})\right)-E\left(G_{t_1^*+t_2^*}b_{out}(\bar C_{t_1^*+t_2^*})-K_1(\bar C_{t_1^*+t_2^*})\right)=0 \end{cases} \qquad 3.3)$$

And the second equation will become

$$\int_0^{t_1^*}a_0(\overline{C}_s)\,ds+\int_{t_1^*}^{t_1^*+t_2^*}a_{in}(\overline{C}_s)\,ds-\int_{t_1^*+t_2^*}^{T}a_{out}(\overline{C}_s)\,ds=0$$

Comparing the second equation of (3.3) and (3.1) one can see that they constitute the same time (or times) for second switching.

General Case: Finally let's consider the general case.

For n switches we have the following formula for V

$$V=E\left(\sum_{i=0}^{n}\int_{\sum_{k=0}^{i}t_i}^{\sum_{k=0}^{i+1}t_i}\left(u_iG_sb_{u_i}(\overline{C}_s)-K_{u_i}(\overline{C}_s)\right)ds\right)=\sum_{i=0}^{n}\int_{\sum_{k=0}^{i}t_i}^{\sum_{k=0}^{i+1}t_i}E\left(u_iG_sb_{u_i}(\overline{C}_s)-K_{u_i}(\overline{C}_s)\right)ds$$
$$=\sum_{i=0}^{n}\int_{\sum_{k=0}^{i}t_i}^{\sum_{k=0}^{i+1}t_i}\left(u_ib_{u_i}(\overline{C}_s)E(G_s)-K_{u_i}(\overline{C}_s)\right)ds$$
$$=G_0\sum_{i=0}^{n}u_i\int_{\sum_{k=0}^{i}t_i}^{\sum_{k=0}^{i+1}t_i}b_{u_i}(\overline{C}_s)ds-\sum_{i=0}^{n}\int_{\sum_{k=0}^{i}t_i}^{\sum_{k=0}^{i+1}t_i}K_{u_i}(\overline{C}_s)ds,$$

where we used the martingale property of G_s (Ross, Pekoz Erol, 2007).

Here $\sum_{j=1}^{n}t_j=T$, $t_0=0$ and $u_i=u(t_i)$, which is in general non-differentiable function at exactly n points.

However, if we have specified the vector $(u_0,u_1,\dots,u_n)$, than we can easily differentiate V, to maximize with respect to $(t_1,t_2,\dots,t_n)$ under the conditions of positivity ($t_i>0$).

For last switching time the following necessary condition for optimality should be,

$$G_0\left(-u_nb_{u_n}\left(\overline{C}_{\sum_{k=0}^{n}t_k}\right)+u_{n-1}b_{u_{n-1}}\left(\overline{C}_{\sum_{k=0}^{n}t_k}\right)\right)+\left(K_{u_n}\left(\overline{C}_{\sum_{k=0}^{n}t_k}\right)-K_{u_{n-1}}\left(\overline{C}_{\sum_{k=0}^{n}t_k}\right)\right)=0$$

Let's look back for the optimality of t_{n-1}, for which we get the following necessary condition

$$G_0\left(-u_nb_{u_n}\left(\overline{C}_{\sum_{k=0}^{n}t_k}\right)+u_{n-1}b_{u_{n-1}}\left(\overline{C}_{\sum_{k=0}^{n-1}t_k}\right)-u_{n-1}b_{u_{n-1}}\left(\overline{C}_{\sum_{k=0}^{n-1}t_k}\right)+u_{n-2}b_{u_{n-2}}\left(\overline{C}_{\sum_{k=0}^{n-1}t_k}\right)\right)$$
$$+\left(K_{u_n}\left(\overline{C}_{\sum_{k=0}^{n}t_k}\right)-K_{u_{n-1}}\left(\overline{C}_{\sum_{k=0}^{n}t_k}\right)+K_{u_{n-1}}\left(\overline{C}_{\sum_{k=0}^{n-1}t_k}\right)-K_{u_{n-2}}\left(\overline{C}_{\sum_{k=0}^{n-1}t_k}\right)\right)=0$$

From which subtracting the previous one we get

$$G_0\left(-u_{n-1}b_{u_{n-1}}\left(\overline{C}_{\sum_{k=0}^{n-1}t_k}\right)+u_{n-2}b_{u_{n-2}}\left(\overline{C}_{\sum_{k=0}^{n-1}t_k}\right)\right)+\left(K_{u_{n-1}}\left(\overline{C}_{\sum_{k=0}^{n-1}t_k}\right)-K_{u_{n-2}}\left(\overline{C}_{\sum_{k=0}^{n-1}t_k}\right)\right)=0$$

Finally all necessary conditions can be described by the following system, $m = 1, \dots, n$

$$G_0\left(-u_m b_{u_m}\left(\overline{C}_{\sum_{k=0}^{m} t_k}\right) + u_{m-1} b_{u_{m-1}}\left(\overline{C}_{\sum_{k=0}^{m} t_k}\right)\right) + \left(K_{u_m}\left(\overline{C}_{\sum_{k=0}^{m} t_k}\right) - K_{u_{m-1}}\left(\overline{C}_{\sum_{k=0}^{m} t_k}\right)\right) = 0\,; \qquad 3.4)$$

If this system is incompatible, then one can infer that no n switching solution exists.

Also another condition should be satisfied is that $C_0 = C_T = 0$, or

$$\sum_{i=0}^{n} \int_{\sum_{k=0}^{i} t_i}^{\sum_{k=0}^{i+1} t_i} a_{u_i}(\overline{C}_s) ds = 0$$

In conclusion the algorithm should contain the following steps. First one should construct all feasible n-vectors for values of u. For each n-step the decision vector,

- must start either with 0, or with -1,
- must end either with 0 or with 1,
- no consecutive elements of the vector can be the same
- 1 cannot occur before the first occurrence of -1.
- Cannot have -1 after than last occurrence of 1.

After constructing all possible combinations satisfying above conditions, we can proceed to solving (3.4) for each of these cases. For the n step switching case we need also perform the procedure for case $k < n$, as there is no guarantee that the n –th step solution will result in more value than any k-th step, for $k < n$.

For implementation of this algorithm, we must have either functional form for functions $K_u(\cdot), a_u(\cdot)$ and $b_u(\cdot)$, or some estimated dependence upon u.

3. Conclusion

We propose a simple algorithmic approach to evaluate the optimal strategy and value of gas storage. Though we haven't yet analyzed properties of convergences of proposed algorithm, the convergence itself is obvious as a consequence of Bellman principle (Shevre, 2008). At last without some explicit form of functions used, one can't proceed to numerical analysis. So at least some explicit relation is necessary for K, a and b.

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