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RESEARCH

# Analysis of HD Journal Bearings Considering Elastic Deformation and Non-Newtonian Rabinowitsch Fluid Model

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A B S T R A C T

*The purpose of this paper is to study the performance of a finite length journal bearing, taking into account effects of non-Newtonian Rabinowitsch flow rheology and elastic deformations of the bearing liner. According to the Rabinowitsch fluid model, the cubic-stress constitutive equation is used to account for the non-Newtonian effects of pseudoplastic and dilatant lubricants. Integrating the continuity equation across the film, the nonlinear non-Newtonian Reynolds-type equation is derived. The elasticity part of the problem is solved on the base of Vlassov model of an elastic foundation. The numerical solution of the modified Reynolds equation is carried out by using FDM with over-relaxation technique. The results for steady state bearing performance characteristics have been calculated for various values of nonlinear factor and elasticity parameters. It was concluded that in comparison with the Newtonian lubricants, higher values of film pressure and load carrying capacity have been obtained for dilatant lubricants, while the case was reversed for pseudoplastic lubricants.*



## 1. INTRODUCTION

Most of studies on hydrodynamic (HD) journal bearings rely on the assumption that both the journal and bearing are rigid bodies. It is well known that under heavy loading conditions of the bearings in some applications and/or when using bearings with layers on the contact surfaces, distortion of these contact surfaces is significant and cannot be ignored.

Elastic deformation of the bearing liner under hydrodynamic pressure changes the fluid film profile, modifies the pressure distribution and therefore changes the performance characteristics of journal bearings. Thus, operational performance become significantly different from those computed in classical HD theory where the bearing liner is assumed to be rigid.

During the last decades, the interest to the elastohydrodynamic (EHD) analysis of HD journal bearings became really extensive. Typical contributions can be found in the works by Monmousseau et al. [1,2], ArreguI and Vazquez [3], Osman [4], Elsharkawy [5], Javorova et al. [6,7], Prabhakaran et al. [8], Ma [9], Attia et al. [10].

Furthermore, it is well known that the non-Newtonian lubricants are widely encountered in various lubrication processes. Many non-Newtonian fluid models like micropolar (Guha [11]), power law (Nessil et al. [12]), couple stress (Ma [13]), Bingham (Jang and Khonsari [14]), etc. have been investigated at rendering into account the effects of the lubricants additives on the performance characteristics of thin film bearings. However, many of these models either work for a limited range of strain rate or lack of experimental data for verification. On the other hand, to describe the nonlinear relationship between shear stress and shear strain rate for the non-Newtonian lubricants, the Rabinowitsch fluid (cubic equation) model has also been introduced. In this model the following empirical stress-strain relationship holds:

$$\tau + k\tau^3 = \eta \frac{du}{dy}, \tag{1}$$

where $\eta$ is the zero shear rate viscosity (or so-called initial viscosity and it is equivalent to the viscosity of Newtonian fluids) and $k$ represents a coefficient of pseudoplasticity (nonlinear factor responsible for the non-Newtonian effects of the fluid).

The flow characteristics of these kind lubricants are shown in Fig. 1. If values of $\eta$ do not vary, the nonlinearity of the flow curve increases with the value of the coefficient of pseudoplasticity $k$ (see Fig. 1 - on the right).

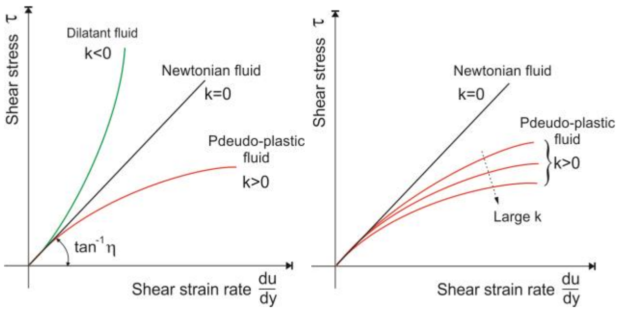


**Fig. 1.** Flow curves for non-Newtonian Rabinowitsch fluid.

Bearing in mind a relationship (1), Rabinowitsch fluid model can be applied to the case of Newtonian lubricants for $k=0$, dilatant lubricants for $k<0$, pseudo-plastic lubricants for $k>0$. One important advantage of this model is that the theoretical analysis for it is verified with the experimental justification by Wada and Hayashi [15]. By using the Rabinowitsch fluid model, many researchers have studied theoretically the effects of non-Newtonian lubricants on the performance characteristics of lubricating contacts, as: slider bearings by Lin [16], Singh et al. [17] and Singh [18]; circular-disk squeeze films by Hashimoto [19]; slot-entry bearings by Sharma et al. [20]; hydrostatic bearings by Sinhasan and Sha [21]; journal bearings by Bourging and Gay [22] and Javorova et al. [23]; parallel rectangular plates by Hung [24]; parallel annular disks by Lin [25].

Although Rabinowitsch fluid model has been investigated on journal bearings, the analysis of HD lubrication of these bearings under conditions of elasticity contacts is still lacking and further investigations are motivated.

The study presented in this paper is related to the journal bearings lubrication aspect analysis using non-Newtonian fluids which are described by a Rabinowitsch fluid model and by taking into account the elastohydrodynamic aspect of the problem. The influence of the various values of the non-Newtonian nonlinear factor $\psi$ on the HD pressure, lubricant film thickness, bearing liner elastic deformations and some of the journal bearing characteristics are also analysed by using the Reynolds equation in its modified form.

## 2. MATHEMATICAL ANALYSIS

The geometrical configuration of the journal bearing is shown on a Fig. 2. It is assumed that the journal and bearing are circular and their surfaces are perfectly smooth, the load is applied in vertical direction, the groove is filled with a lubricant of a constant pressure, and the journal rotates with a constant angular velocity $\omega$ about its axis. The radius of bearing is approximately equal to the journal radius because of which it is possible to neglect the curve shape of the film. A bearings' liner with elastic properties $\mu$ and $E$ is press-fitted on a bush, the layers material is homogenous and isotropic, as the journal is rigid. It is assumed that the lubricant thickness $h$ and liner thickness $d$ are very thin and they are with the same order of magnitude.

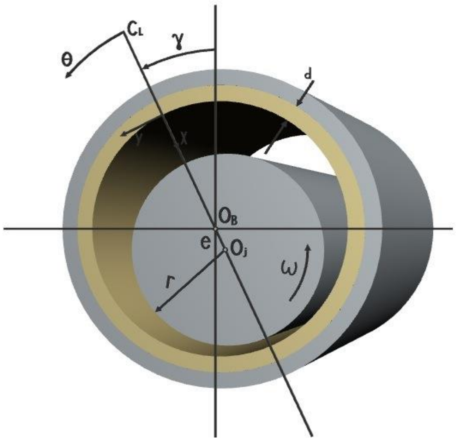


**Fig. 2.** Schematic representation of a journal bearing with soft layer on the bush.

The non-Newtonian lubricant is taken to be a Rabinowitsch fluid. In the present analysis the following assumptions are also considered: the flow is isothermal, incompressible and laminar, and the lubricant inertia effect is negligible.

### 2.1 Modified Reynolds equation

According to the thin-film theory of hydrodynamic lubrication as considered by Cameron [26] and used by Wada et al. [27], equations of motion and of continuity in Cartesian coordinates reduce to:

$$\frac{\partial p}{\partial x}=\frac{\partial \tau_x}{\partial y}; \qquad (2)$$

$$\frac{\partial p}{\partial y}=0; \qquad (3)$$

$$\frac{\partial p}{\partial z}=\frac{\partial \tau_z}{\partial y}; \qquad (4)$$

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=0. \qquad (5)$$

The constitutive relationship between the shear stress component and the shear strain rate for a Rabinowitsch fluid obeys the cubic equation (1). The boundary conditions for the components of velocities are:

at $y=0$:

$$u=0,\ v=0,\ w=0; \qquad (6)$$

at $y=h$:

$$u=u_h=\omega r;\ v=v_h=\omega r\frac{\partial h}{\partial x};\ w=w_h=0. \qquad (7)$$

Integration of Eqns. (2) and (4) with respect to $y$ yields:

$$\tau_x=\frac{\partial p}{\partial x}y+\tau_1; \qquad (8)$$

$$\tau_z=\frac{\partial p}{\partial z}y+\tau_2, \qquad (9)$$

where $\tau_1$ and $\tau_2$ are integration constants.

Substituting equations (8) and (9) into the constitutive equation (1) leads to the velocity gradients in following forms:

$$\frac{du}{dy}=\frac{1}{\eta}\left[\left(\frac{\partial p}{\partial x}y+\tau_1\right)+k\left(\frac{\partial p}{\partial x}y+\tau_1\right)^3\right]; \qquad (10)$$

$$\frac{dw}{dy}=\frac{1}{\eta}\left[\left(\frac{\partial p}{\partial z}y+\tau_2\right)+k\left(\frac{\partial p}{\partial z}y+\tau_2\right)^3\right]. \qquad (11)$$

Integrating equations (10) and (11) with respect to $y$, under the boundary conditions (6) and (7), the velocity components $u$ and $w$ are obtained the same as by Wada et al. [27] in the form:

$$u=\frac{1}{\eta}\left[\frac{1}{2}\frac{\partial p}{\partial x}y(y-h)+k\left(\frac{\partial p}{\partial x}\right)^3F_1\right]+$$

$$+u_h\left[1-\frac{y+k\left(\frac{\partial p}{\partial x}\right)^2F_2}{h\left(1+\frac{1}{4}k\left(\frac{\partial p}{\partial x}\right)^2h^2\right)}\right]; \qquad (12)$$

$$w=\frac{1}{\eta}\left[\frac{1}{2}\frac{\partial p}{\partial z}y(y-h)+k\left(\frac{\partial p}{\partial z}\right)^3F_1\right], \qquad (13)$$

where

$$F_1=\frac{1}{4}y^4-\frac{1}{2}hy^3+\frac{3}{8}h^2y^2-\frac{1}{8}h^3y; \qquad (14a)$$

$$F_2=y^3-\frac{3}{2}hy^2+\frac{3}{4}h^2y. \qquad (14b)$$

In the considered case it is assumed a linear distribution of the radial velocity $v(x,y,z)$ using the formulation by Kelzon et al. [28]; as the final form for this velocity component is given by:

$$v = (v_h / h)\, y. \tag{15}$$

Integrating the equation of continuity (5) with respect to $y$, under the boundary conditions (6) and (7), yields:

$$\frac{\partial}{\partial x}\int_0^{h(x,z)} u dy - u_h \frac{\partial h}{\partial x} + v_h + \frac{\partial}{\partial z}\int_0^{h(x,z)} w dy = 0. \tag{16}$$

Then the modified Reynolds equation for the finite length journal bearing lubricated with non-Newtonian Rabinowitsch fluids under steady-state conditions is represent as :

$$\frac{\partial}{\partial x}\left\{\frac{1}{\eta}\left[\frac{h^3}{12}\frac{\partial p}{\partial x} + k\frac{h^5}{80}\left(\frac{\partial p}{\partial x}\right)^3\right]\right\} +$$

$$+\frac{\partial}{\partial z}\left\{\frac{1}{\eta}\left[\frac{h^3}{12}\frac{\partial p}{\partial z} + k\frac{h^5}{80}\left(\frac{\partial p}{\partial z}\right)^3\right]\right\} = \frac{\omega r}{2}\frac{\partial h}{\partial x}. \tag{17}$$

It is known, that for Newtonian fluids $k = 0$ and from Eqn (17) can be obtained the classical Reynolds equation. The above equation is more general, as it includes the case of Newtonian fluids.

The term of the right-hand side of Eqn (17) indicates the action of so called "wedge" effect. In hydrodynamically lubricated bearings using non-Newtonian Rabinowitsch fluids, the film pressure is developed by the same three terms (wedge, stretch or squeeze) as are used for Newtonian fluids.

By applying the above mentioned modified Reynolds equation to journal bearings, the bearing performance can be obtained. Expressing in dimensionless form, the Reynolds type equation is depicted by:

$$\frac{\partial}{\partial\theta}\left[\frac{H^3}{12}\frac{\partial\Pi}{\partial\theta} + \psi\frac{36H^5}{80}\left(\frac{\partial\Pi}{\partial\theta}\right)^3\right] + \tag{18}$$

$$+\alpha^2\frac{\partial}{\partial z_1}\left[\frac{H^3}{12}\frac{\partial\Pi}{\partial z_1} + \alpha^2\psi\frac{36H^5}{80}\left(\frac{\partial\Pi}{\partial z_1}\right)^3\right] = \frac{1}{12}\frac{\partial H}{\partial\theta},$$

where $\psi$ means the "nonlinear factor" accounting for non-Newtonian Rabinowitsch fluids effects; such it is given as:

$$\psi = k\left(\frac{\eta u}{c}\right)^2 = \bar{k}\left(\frac{\bar{\eta}\omega r}{c}\right)^2. \tag{19}$$

As the value of $\psi$ is set equal to zero, the non-dimensional classical Reynolds equation for a journal bearing with Newtonian fluids is recovered. The value of $\psi$ depends on the radial clearance, journal velocity, initial lubricant viscosity and the coefficient of the pseudo-plasticity of the fluid. Because of the linear relation the value of nonlinear factor increases with the coefficient of plasticity $k$. For Newtonian fluids $\psi = 0$ since $k = 0$. In that means, it is considered that the lubricant' nonlinear factor is a dimensionless quantity which indicates the non-Newtonian characteristics.

## 2.2 Oil film thickness and elasticity equations

The pressure generated in the film is assumed to act normally on the bearing surfaces. All tangential forces to these surfaces are neglected. The present analysis will be made allowances for the elastic deflection of the bush liner. The other components of the bearing and the journal will be treated as rigid.

The used in a current study approach aims to superimpose the deformation of the layer on the bush, caused by hydrodynamic pressure generated onto the oil film thickness. The gap thickness is then modified to account for the estimated elastic deformation (represented by the last term of equation) as follows:

$$h(x,z) = c + e\cos\theta + \delta_y. \tag{20}$$

By introducing the non-dimensional variables the above equation is modified for the numerical solution to:

$$H = 1 + \varepsilon\cos\theta + \bar{\delta}. \tag{21}$$

Determination of the radial displacements of the liner's surface points is carried out in accordance with the Vlassov three-dimensional model of elastic foundation for the case of thin layer [7]:

$$\bar{\delta} = \frac{6\eta\omega r^2}{c^3}\frac{(1-2\mu)(1-\mu^2)d}{E(1-\mu)^2}\Pi. \tag{22}$$

## 2.3 Steady-state characteristics

Integrating the steady film pressure over the film region gives the steady load-carrying capacity of the bearing, calculated by:

$$\bar{W} = \sqrt{\bar{W}_1^2 + \bar{W}_2^2} = \frac{\beta^2}{6\eta\omega r L} W \,. \tag{23}$$

Here $\bar{W}_1$ and $\bar{W}_2$ represent the components along and perpendicular to the line of centres and they are given respectively as:

$$\bar{W}_1 = -\int_{-1}^{1}\int_{0}^{2\pi} \Pi \cos\theta \, d\theta dz_1 \,;$$

$$\bar{W}_2 = \int_{-1}^{1}\int_{0}^{2\pi} \Pi \sin\theta \, d\theta dz_1 \,. \tag{24}$$

Consequently, the attitude angle is calculated by

$$\gamma = \tan^{-1}\left(\bar{W}_2 / \bar{W}_1\right) \tag{25}$$

and the Sommerfeld number may be defined as [6]:

$$S = \frac{W\beta^2}{\eta\omega r L} = 6\bar{W} \,. \tag{26}$$

## 3. SOLUTION PROCEDURE

EHD problem presupposes simultaneous solution of the modified Reynolds equation for non-Newtonian Rabinowitsch fluids (18), film thickness equation (21) and elasticity equation (22).

The dimensionless modified Reynolds equation is solved numerically using the finite difference method; as the iterative scheme with application of a successive over-relaxation procedure in order to improve the convergence rate is employed.

For pressure field the Reynolds boundary conditions were used: - pressure at the journal bearing edges is equal to zero; - pressure is zero in the areas where the pressure gradient in the circumferential direction becomes zero. The application of these conditions imposes the use of the Christopherson assumption negative pressure is set to zero in each iteration.

For EHD analysis, an iterative process is repeated until the required convergence is achieved. The converged nodal pressures are then used to calculate the nodal displacements. The film thickness is modified by considering the radial component of the nodal displacements to get the solution of the nodal pressures. Iterations are also required to obtain performance characteristics for a wide range of values of the parameters included in Eq. (22) to takes into account the flexibility of the bearing liner.

The film domain (see Fig. 3) is divided by the orthogonal grid spacing, as the used mesh has 67 nodes along the circumferential direction and 17 nodes in the axial direction. This size gives a rapid rate of convergence. The convergence criterion adopted for pressure is:

$$\left|\left(1 - \sum \Pi_{old} / \sum \Pi_{new}\right)\right| \leq 1.10^{-7} \,.$$

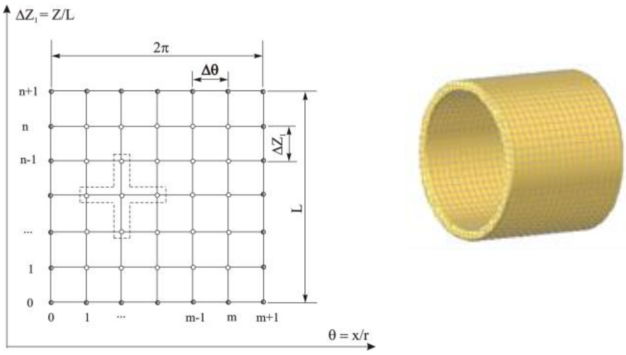


**Fig. 3.** Discretization of the domain of study.

## 4. RESULTS AND DISCUSSIONS

The present analysis showed that the effect of non-Newtonian properties of Rabinowitsch lubricant can be presented by nonlinear factor $\psi$ while from another side the effect of deformability of the bearing' layer - by parameters $\mu$ and $E$. Then, considering the mathematical model, the governing parameters are eccentricity ratio $\varepsilon$, diameter to length ratio $\alpha$, nonlinear factor $\psi$ and elastic layer parameters $\mu$, $E$.

In the computations are used the following main operating conditions, which represent the investigated effects:

- related to values of nonlinear factor are considered three kinds of fluids:

Newtonian fluid - $\psi = 0$;

pseudo-plastic fluid - $\psi = 0{,}01$, $\psi = 0{,}1$;

dilatant fluid - $\psi = -0{,}01$, $\psi = -0{,}1$;

- related to values of elasticity parameters are considered four separate cases:

rigid case - $E = 2.10^{11}$ [Pa], $\mu = 0{,}25$;

soft case 1 - $E = 1{,}63.10^8$ [Pa], $\mu = 0{,}38$;
soft case 2 - $E = 7{,}33.10^7$ [Pa]; $\mu = 0{,}4$;
soft case 3 - $E = 4{,}07.10^7$ [Pa]; $\mu = 0{,}41$.

The results were obtained for diameter to length ratio $\alpha$ equal to 1,0 whereas the eccentricity ratio $\varepsilon$ was varied from 0,1 to 0,9.

The first group of the presented results are related to the effect of elastic deformations, while the second group show the influence of the nonlinear factor accounting for non-Newtonian lubricants effects as well as the combination of above effects.

On Figs. 4-6 is presented pressure distribution along both angular and axial coordinates; such the results correspond to rigid case, soft case 1 and soft case 3 respectively.

As it was expected, for the soft cases a visible reduction of HD pressure profile is obtained. This effect is a more visible for the more soft materials of the bearing elastic liner.

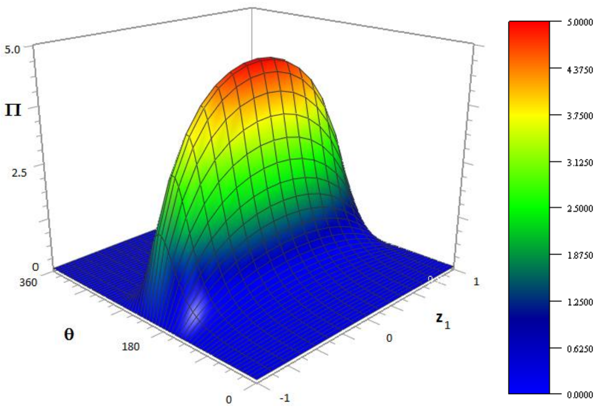


**Fig. 4.** HD pressure distribution – rigid case.

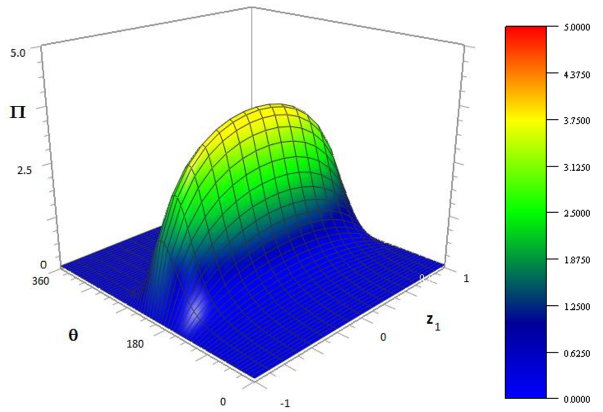


**Fig. 5.** HD pressure distribution – soft case 1.

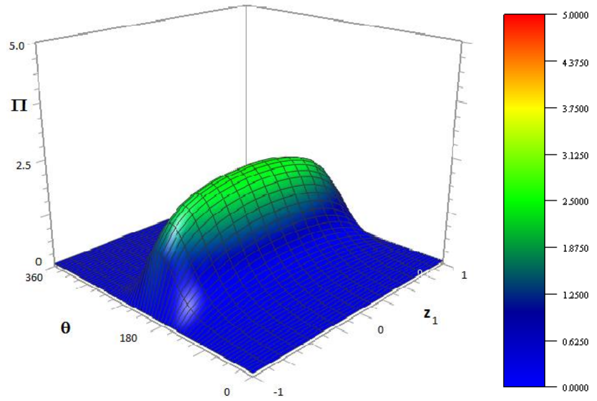


**Fig. 6.** HD pressure distribution – soft case 3.

Variation of the oil film thickness is shown in Figs. 7 and 8 for typical HD theory (rigid case) and for EHD theory when the bearing liner is deformed (soft case 2 as example), respectively; as the profile in the circumferential direction has a sinusoidal shape with a minimum located around $\theta = 180^o$.

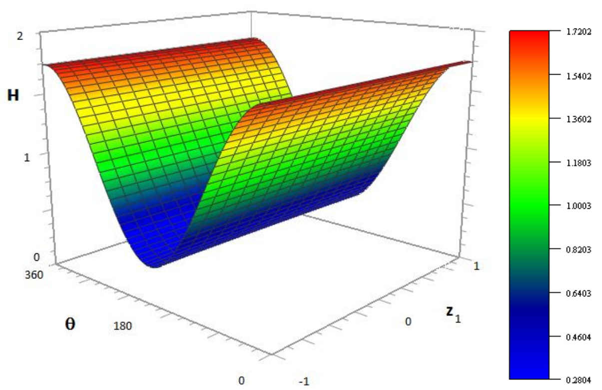


**Fig. 7.** Film thickness geometry – rigid case

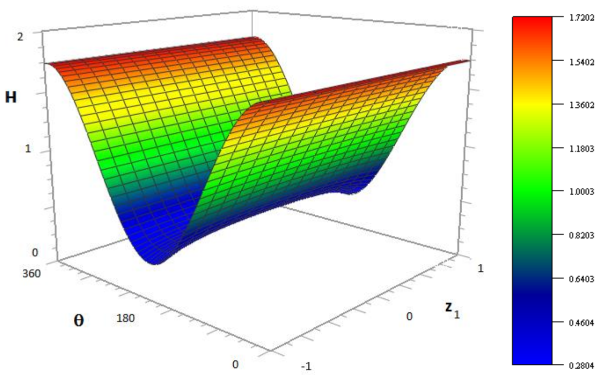


**Fig. 8.** Film thickness geometry – soft case 2

For the second case (Fig. 8) the film geometry is changed in the bottom; such the increase of film thickness in the middle section and the displacement of its minimum explain the drop of the maximum pressure and the spreading of the pressure field compared to our rigid case (pure HD theory).

Fig. 9 shows the area of radial displacements of inner surface points of the bearing liner for the soft case 2. The displacements distribution is the same as a type of the pressure profile but the maximal displacements values are very lower compared with maximum HD pressure.

The presented numerical results about pressure distribution, film thickness geometry and deformation rise area are in a close agreement with the results by Osman [4], Elsharkawy [5], Attia et al. [10] for EHD lubrication of cylindrical journal bearings.

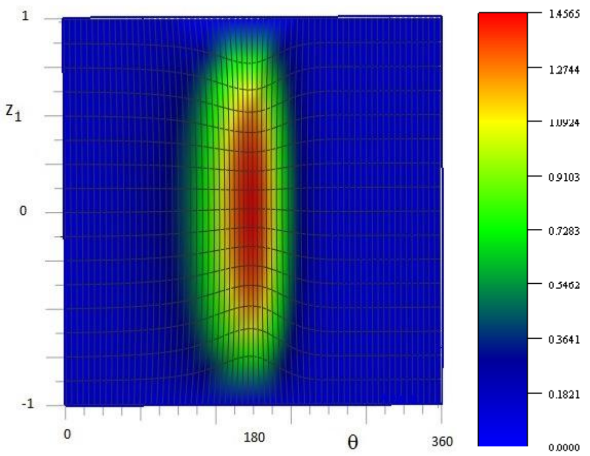


**Fig. 9.** Area of the deformation rise.

The pressure distribution according to the circumferential coordinate for dilatant, pseudo-plastic and Newtonian fluids is given in Fig. 10. The results correspond to values of nonlinear factor between -0,1 till +0,1 under elastic parameters for rigid case. It is evident that the non-Newtonian effect of cubic stress flow can produce different results to steady film pressure. For pseudo-plastic lubricant the values of pressure decrease, while for dilatant fluids the pressure increases, compared with Newtonian lubricants; such it agrees with the results by Wada and Hayashi [15], Lin [16], Hung [24], Singh [17]. It was observed that the above effects are most significant for the areas of maximum pressure.

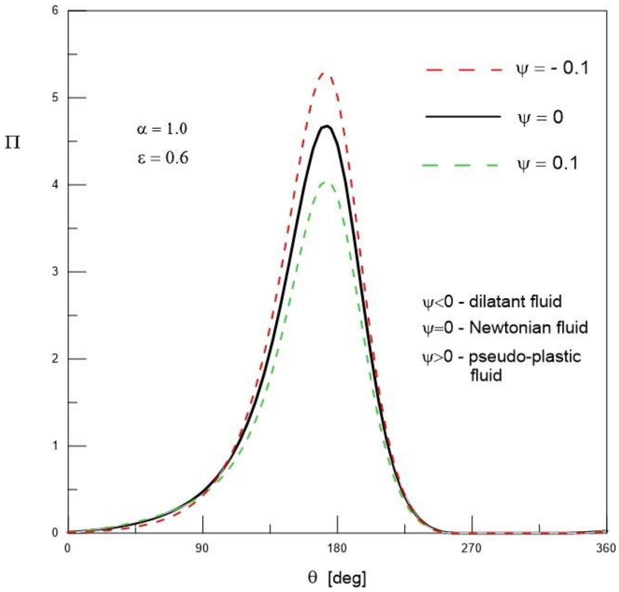


**Fig. 10.** Variation of HD pressure with circumferential coordinate for different values of $\psi$.

More detailed view of the same effect on the maximum pressure is shown on a Fig. 11 where are chosen more values in the considered limits of the nonlinear factor $\psi$. The observed tendency in journal bearings for influence of the lubricants pseudo-plasticity is in a good agreement with the results for slider bearings [16,17].

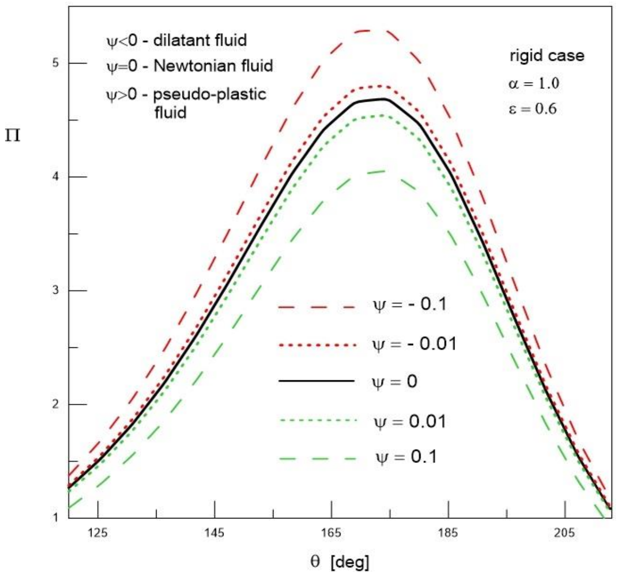


**Fig. 11.** Effect of nonlinear factor on maximum pressure values.

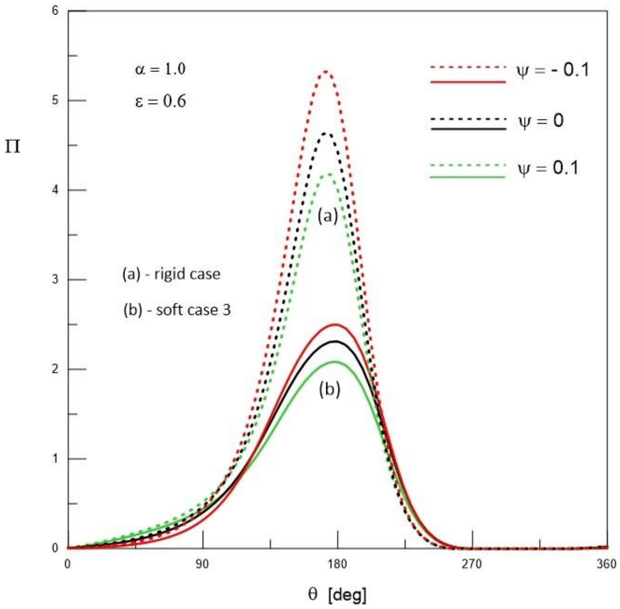


**Fig. 12.** Effect of nonlinear factor on pressure distribution for rigid and soft cases.

The same visible influence of the nonlinear factor of the non-Newtonian Rabinowitsch fluids is presented in Fig. 12, as here are plotted pressure distribution results for both rigid and soft case regarding elasticity parameters of the bearing liner. In the soft case the tendency is kept but the influence of pseudoplastic and dilatant behaviour is slightly lower.

The pressure distribution in the axial direction for the considered three lubricants types is shown in Fig. 13. Here in the abscissa axis $z=-1$ and $z=1$ indicate the sides of the bearing and $z=0$ is the middle section of the bearing. From this figure it is also understood that the film pressure in pseudo-plastic fluids is smaller than that of Newtonian fluids, and for dilatant lubricants the inverse effect is observed.

The numerical results from Fig. 14 describe the variation of the dimensionless load-carrying capacity with the eccentricity ratio. In comparison with Newtonian lubricants, at the increasing of the nonlinear factor, the load capacity of journal bearing using pseudo-plastic fluids decreases, while at the dilatant behaviour load capacity is with higher values.

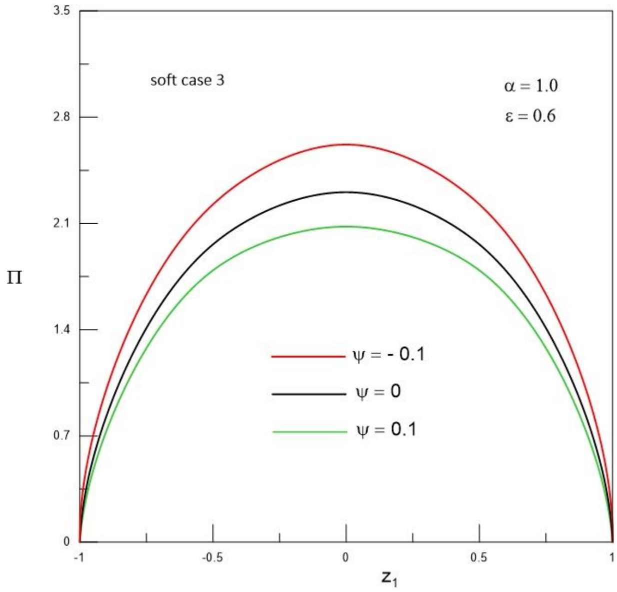


**Fig. 13.** Variation of HD pressure with axial coordinate for different values of $\psi$ .

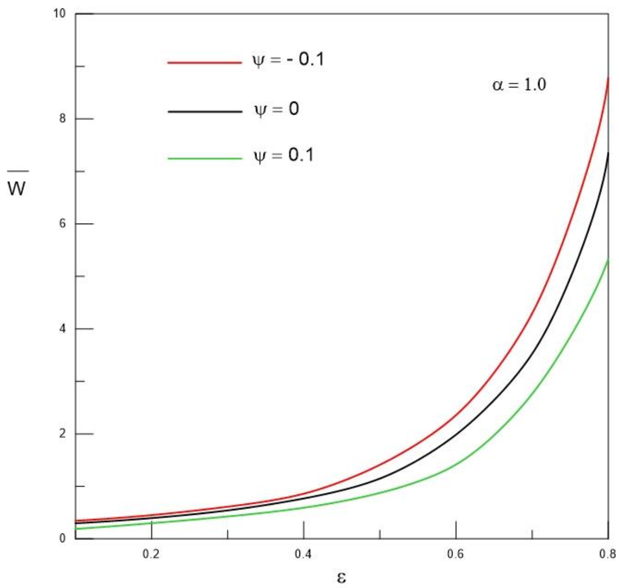


**Fig. 14.** Effect of nonlinear factor on load-carrying capacity.

For the bigger eccentricity ratios this tendency is better observed while for the small $\varepsilon$ these effects almost cannot be recognized. Obviously, for the load capacity coefficient Sommerfeld number must be waited the same effects because of the linear relation of $S$ with $\bar{W}$ in our study.

## 5. CONCLUSION

On the base on the Rabinowitsch fluid model, the non-Newtonian effects on the steady state pressure and operational characteristics in EHD journal bearing with finite length were investigated. Especially the influence of non-Newtonian properties of lubricant and elastic deformation effects of bush liner on the bearing performance were studied.

Based on the presented results and their analysis and discussions, the following conclusions can be drawn:

The effects of deformability of the bush liner result to a reduction of the peak pressure and respectively to increase in minimum film thickness. When the value of Young's modulus is smaller ( $\mu$ is large), this influence is visibly distinguished.

At the comparison with the Newtonian lubricants, the effects of pseudo-plastic behaviours provide a decrease in the steady state pressure, while the influence of non-Newtonian dilatant properties results in a reversed trend. Further, the quantitative effects of cubic stress Rabinowitsch fluids are more pronounced for larger absolute values of the nonlinear factor.

In the case of EHD lubrication (soft material of the bearing liner) the lubricants' pseudoplastic/dilatant behaviours show the same influences on the pressure values; but non-Newtonian effects are slightly lower for the soft material of bearing liner compared with these for rigid bearing liner.

The steady state performance parameters load-carrying capacity and Sommerfeld number of a bearing with pseudoplastic lubricants are lower than that for Newtonian ones; such for the dilatant lubricants these performance parameters have higher values regarding to Newtonian case.

The combined influence of the nonlinear non-Newtonian parameter (lubricants rheology) and elastic deformations on pressure distribution and operational performance of bearings is significant and cannot be overlooked.

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## NOMENCLATURE

| | |
|---|---|
| $c$ | Radial clearance, [m] |
| $d$ | Bearing liner thickness, [m] |
| $E$ | Young's modulus, [Pa] |
| $e$ | Eccentricity, [m] |
| $h$ | Film thickness, [m] |
| $H$ | Dimensionless film thickness, $H = h/c$ |
| $k$ | Coefficient of pseudo-plasticity |
| $L$ | Bearing axial length, [m] |
| $p$ | Hydrodynamic pressure, [Pa] |
| $r$ | Radius of the journal, [m] |
| $S$ | Sommerfeld number |
| $x, y, z$ | Cartesian rectangular coordinates |
| $z_1$ | Dimensionless axial coordinate, $z_1 = z/(L/2)$ |
| $u,v,w$ | Velocity components, [m.s$^{-1}$] |
| $W$ | Load-carrying capacity, [N] |
| $\bar{W}$ | Dimensionless load-carrying capacity, $\bar{W} = W(c/r)^2/6\eta\omega rL$ |
| $\alpha$ | Diameter to length ratio, $\alpha = 2r/L$ |
| $\beta$ | Clearance ratio, $\beta = c/r$ |
| $\gamma$ | Attitude angle, [rad] |
| $\varepsilon$ | Eccentricity ratio, $\varepsilon = e/c$ |
| $\theta$ | Circumferential coordinate, $\theta = x/r$ |
| $\psi$ | Nonlinear factor of lubricant |
| $\delta$ | Liner surface points radial displacement, [m] |
| $\eta$ | Initial viscosity of the lubricant, [Pa.s] |
| $\mu$ | Poisson's ratio |
| $\Pi$ | Dimensionless pressure, $\Pi = p.(c/r)^2/6\eta\omega$ |
| $\tau$ | Shear stress |
| $\omega$ | Shaft angular velocity, [s$^{-1}$] |

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