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RESEARCH

Effects of Surface Roughness in Squeeze Film Lubrication of Two Parallel Plates

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A B S T R A C T

A generalized form of Reynolds equation for two surfaces is taken by considering surface roughness at the bearing surfaces. This equation is applied to study the effects of surface roughness for the lubrication of squeeze film of two parallel plates. Expressions for the load capacity and squeezing time are obtained and studied theoretically for various parameters. The load capacity and squeeze time increases with an increase in the value of k which represent a ratio of the viscosity of the peripheral layer to the middle layer. In the case of transverse roughness the load capacity and squeezing time increases as the mean height of surface asperities increases and the load capacity and the squeezing time decreases as the mean height of surface asperities increases in the case of longitudinal roughness. Hence the effect of surface roughness is more pronounced in the case of transverse roughness.

1. INTRODUCTION

In general additives are added to the base lubricant to improve the bearing characteristics. Various theories have been proposed for this. These additives are generally long-chain organic compounds and they may form a high viscous layer near the surface. In most of the theoretical investigation of hydrodynamic lubrication, it has been assumed that the bearing surfaces are smooth. This is an unrealistic assumption for the bearing operating with small film thickness. In the recent years, a considerable amount of tribology research has been devoted to the study of the surface roughness on hydrodynamic lubrication. This is mainly because of the fact that all solid surfaces are rough to some extent and generally the height of roughness asperities is of the same order as the mean separation between lubricated contacts. Hence, the effect of surface roughness plays a significant role in the development of the science and technology of lubrication. In recent years the study of lubrication of rough surfaces has gained much attention, by deterministic and stochastic approaches [1-7]. In the deterministic approach, the effect of surface roughness is taken into account in the usual Reynolds equation by considering that the film thickness is a function of surface roughness which may represent by a series of sine and cosine waves. Dowson [1] has applied this procedure to study the bearing characteristics of rollers, spiral groove bearings etc. Shukla [2] gave a new deterministic theory to study the effects of surface roughness when the mean height of the asperities is same order of magnitude as the minimum film thickness.

The phenomenon of two lubricated surfaces approaching each other with a normal velocity is known as squeeze film lubrication. The thin film of lubricant present between the two surfaces acts as a cushion and it prevents the surfaces from making instantaneous contact. The time required to squeeze out the lubricant depends upon surface configuration, fluid properties and the load applied. In general, the relation between the load carrying capacity and the rate of approach is studied in the most squeeze film analysis. Much work is done in this line and the mathematical review on such process was done by various workers.

Shukla et al. [8] studied the effects of consistency variation of power law lubricants in squeeze films. Also R. Raghavendra Rao and K. Raja Sekhar [9,10] studied the effects of couple stresses and surface roughness under lightly and heavily loaded conditions on roller bearings. And after that R. Raghavendra Rao, K. Gowthami and myself [11,12] studied the effects of velocity-slip and viscosity variation in squeeze film lubrication of two circular plates and spherical bearings.

R.M. Patel et al. [13] studied the performance of a magnetic fluid based squeeze film between transversely rough triangular plates. Also M.E. Shimpi and G.M. Deheri [14] studied surface roughness and elastic deformation effects on the behavior of the magnetic fluid based squeeze film between rotating porous circular plates with concentric circular pockets and improved in 2012 to the rotating curved porous circular plates [15].

In this paper, a generalized Reynolds equation is derived for rough surfaces. It is applied to study the effects of surface roughness in squeeze film lubrication of two parallel plates.

2. BASIC EQUATIONS

Consider the flow of an incompressible lubricant between two rough surfaces rolling with velocity U as show in Fig. 1.

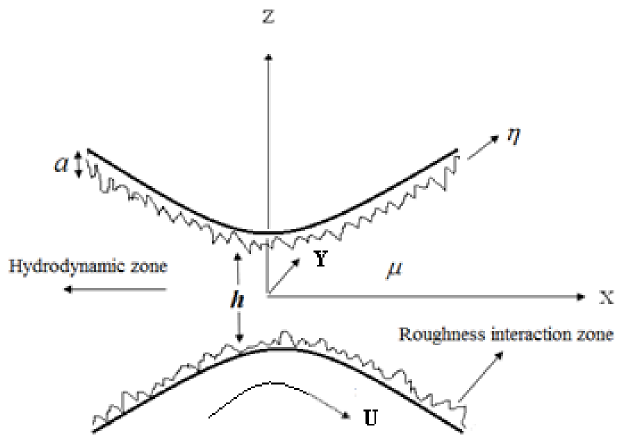

Fig.1. Lubrication between two rough surfaces.

The lubricant zone is divided into a purely hydrodynamic zone in the middle, and a roughness interaction zone near the surfaces. Let 'a' be the average heights of the roughness interaction zones along the lower and upper surfaces respectively and 'h' be the film thickness in the purely hydrodynamic zone.

Under the usual assumptions of lubrication theory, the equations governing two dimensional flow of the lubricant in the purely hydrodynamic zone can be written as

$$\mu \frac{\partial^2 u_2}{\partial z^2} = \frac{\partial P}{\partial x} \quad in \quad 0 \le z \le \frac{h}{2} \tag{1}$$

where u_2 is the velocity and μ the viscosity.

Considering the interaction of the moving surface with the lubricant in contact, the equations governing the flow of the lubricant in the roughness interaction zone [16] can be written as:

$$\eta \frac{\partial^2 u_1}{\partial z^2} - k(u_1 - U) = \frac{\partial P}{\partial x} \quad in \quad \frac{h}{2} \le z \le \frac{h}{2} + a \tag{2}$$

where $k = \frac{\mu}{\phi}$, u_1 be the velocity, parameter ϕ corresponds to permeability of porous matrix at the surfaces and η be the effective viscosity.

The pressure P is assumed to be constant across the film. The second term $k(u_1 - U)$ on the left hand side of equation (2) corresponds to be resistance offered by the roughness asperities to the flowing fluid. It is assumed to be proportional to the relative velocity.

Keeping in view of the matching of velocities and shear stresses near the interface, the boundary conditions [16] for equations (1) and (2) can be written as:

$$\frac{\partial u_2}{\partial z} = 0 \quad at \quad z = 0 \tag{3}$$

$$u_1 = U \quad at \quad z = \frac{h}{2} + a \tag{4}$$

$$u_1 = u_2 = U_i \quad at \quad z = \frac{h}{2} \tag{5}$$

$$\eta \frac{\partial u_1}{\partial z} = \mu \frac{\partial u_2}{\partial z} \quad at \quad z = \frac{h}{2} \tag{6}$$

Solving the equations (1) and (2) and using the boundary equations (3) to (5), the velocities in the different regions are determined as:

$$u_1 = A\left[\sinh\left[\alpha\left(\frac{h}{2}+a-z\right)\right]+\sinh\left[\alpha\left(z-\frac{h}{2}\right)\right]-\sinh[\alpha a]\right] + \frac{U\sinh\left[\alpha\left(z-\frac{h}{2}\right)\right]}{\sinh[\alpha a]} + \frac{U_i\sinh\left[\alpha\left(\frac{h}{2}+a-z\right)\right]}{\sinh[\alpha a]} \tag{7}$$

$$u_2 = \frac{1}{2\mu}\frac{\partial P}{\partial x}\left[z^2 - \frac{h^2}{4}\right] + U_i \tag{8}$$

where:

$$A = \frac{1}{k\sinh[\alpha a]}\left[\frac{\partial P}{\partial x} - kU\right], \quad \alpha = \sqrt{\frac{k}{\eta}}$$

Using the condition (6), the interface velocity U_i is determined as:

$$U_i = \frac{1}{k}\frac{\partial P}{\partial x} Tanh(2M)\left[Tanh(M) + \frac{hk}{2\eta a}\right] + U \tag{9}$$

The lubricant flux Q, per unit width can be defined as:

$$Q = \int_0^{\frac{h}{2}+a} U dz = \int_0^{\frac{h}{2}} u_2\, dz + \int_{\frac{h}{2}}^{\frac{h}{2}+a} u_1\, dz \tag{10}$$

Using the expressions for velocities u_1 and u_2, lubricant flux, Q can be determined as:

$$Q = \frac{ga^3}{4}\left[\alpha^2 U - \frac{1}{\eta}\frac{\partial P}{\partial x}\right] - \frac{h^3}{24\mu}\frac{\partial P}{\partial x} + U\frac{Tanh(M)}{\alpha} + U\left(\frac{h}{2}\right) + \frac{U_i}{\alpha}Tanh(M) + U_i\left(\frac{h}{2}\right) \tag{11}$$

where:

$$g = \frac{\left(\frac{\alpha a}{2}\right) - Tanh\left(\frac{\alpha a}{2}\right)}{\left(\frac{\alpha a}{2}\right)^3}, \quad \alpha = \sqrt{\frac{k}{\eta}}$$

Substituting the expression for U_i in (11) the final expression for fluid flux is obtained as:

$$Q = \frac{\partial P}{\partial x}\left[\frac{g a^3}{4\eta} + \frac{h^3}{24\mu} + \frac{h^2}{4k\mu\alpha} + \frac{h}{k}Tanh(M) + \frac{1}{k}Tanh^2(M) Tanh(2M)\right] + U\left[\frac{\alpha^2 g a^3}{4} + \frac{h}{2}\right] \tag{12}$$

where:

$$g = \frac{\left(\frac{\alpha a}{2}\right) - Tanh\left(\frac{\alpha a}{2}\right)}{\left(\frac{\alpha a}{2}\right)^3}, \quad \alpha = \sqrt{\frac{k}{\eta}}$$

The continuity equation in the lubricant zone is given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{13}$$

Integrating (13) across the film thickness from 0 to h we get,

$$\frac{\partial Q}{\partial x} + [V]_0^h = 0$$

Using (12) in the above equation, the generalized Reynolds equation for the lubrication between two rough surfaces with squeezing velocity V is obtained as:

$$\frac{\partial}{\partial x}\left[F_1\frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial x}\left[U\left(a + \frac{h}{2}\right)\right] = -V \tag{14}$$

where:

$$F_1 = \frac{(M - Tanh(M))}{M^3}\frac{a^3}{2k\mu} + \frac{1}{4}\left(h + a\frac{Tanh(M)}{M}\right)^2 2a\frac{Tanh(2M)}{k\mu(2M)} + \frac{h^3}{12\mu} \tag{15}$$

$$M = \frac{\alpha a}{2}, \quad \alpha = \sqrt{\frac{1}{k\phi}}$$

In Reynolds equation, the effective viscosity, η near the rough surfaces is taken as $\eta = k\mu$, where k is constant and μ is the fluid viscosity in the middle.

2.1 Various Cases

Case (i). When $U = 0$

If there is no rolling motion of the surfaces, we get squeeze film lubrication for rough surfaces.

The equation (14) becomes:

$$\frac{\mathrm{d}}{\mathrm{dx}}\left[F_1 \frac{dP}{dx}\right] = -V \qquad (16)$$

where F_1 is given by (15).

Case (ii). When $\phi \to \infty$ *then* $M \to 0$.

In the case of high permeability of roughness interaction zone, i.e. the fluid flows more freely in the roughness interaction zones.

The equation (14) becomes:

$$\frac{d}{d\,x}\left[F_1 \frac{d\,P}{d\,x}\right] + \frac{d}{d\,x}\left[U\left(a + \frac{h}{2}\right)\right] = -V \qquad (17)$$

Where:

$$F_1 = \frac{1}{k\mu}\left[\frac{a^3}{6} + \frac{h^2}{2} + h\,a^2 + \frac{a^3}{2}\right] + \frac{h^3}{12\,\mu}$$

If $k = 1$ then

$$F_1 = \frac{(h + 2a)^3}{12\,\mu}$$

In this case we get the lubrication between two surfaces with the lubricant zone divided into two layers with different viscosities.

Case (iii). When $\phi \to 0$ *then* $M \to \infty$.

When the permeability parameter of the roughness interaction zones tends to zero, the concentration of roughness asperities increases and there is a little flow at the roughness interaction zones. In this case we get ordinary lubrication between two surfaces.

The equation (14) becomes:

$$\frac{d}{d\,x}\left[F_1 \frac{d\,P}{d\,x}\right] + \frac{d}{d\,x}\left[U\left(a + \frac{h}{2}\right)\right] = -V \qquad (18)$$

where:

$$F_1 = \frac{h^3}{12\,\mu}$$

Case (iv). When $a = 0$

If the mean height of roughness asperities tends to zero we get the ordinary lubrication between two surfaces.

The equation (14) becomes:

$$\frac{d}{d\,x}\left[F_1 \frac{d\,P}{d\,x}\right] + \frac{d}{d\,x}\left[U\left(a + \frac{h}{2}\right)\right] = -V \qquad (19)$$

where:

$$F_1 = \frac{h^3}{12\,\mu}$$

Case (v). When $h = 0$

When the film thickness of purely hydrodynamic zone is zero, we get mixed lubrication between two rough surfaces.

The equation (14) becomes:

$$\frac{d}{d\,x}\left[F_1 \frac{d\,P}{d\,x}\right] + \frac{d}{d\,x}\left[U\left(a + \frac{h}{2}\right)\right] = -V \qquad (20)$$

where:

$$F_1 = \frac{(M - Tanh\,(M))}{M^3}\,\frac{a^3}{2k\mu} + \frac{1}{4}\left(a\frac{Tanh(M)}{M}\right)^2 2a\frac{Tanh\,(2M)}{k\mu\,(2M)}$$

$$M = \frac{\alpha\,a}{2}\,,\ \alpha = \sqrt{\frac{1}{k\phi}}$$

It may be noted that the above mentioned equation (14) involve various parameters characterizing roughness. Now, the film thickness h is written in terms of nominal film thickness h_n and mean height of roughness asperities h_s corresponding to the surfaces as shown in the Fig. 2.

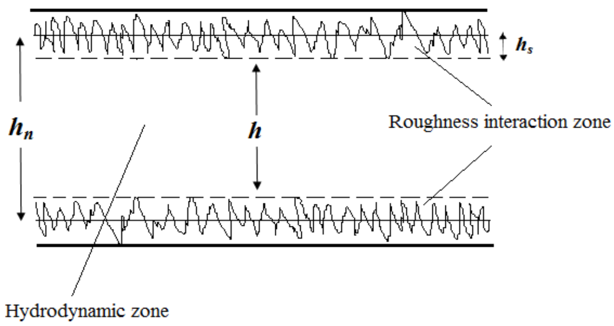

Fig. 2. Lubrication of two rough surfaces considering nominal film thickness.

Thus $h = h_n - 2h_s$, $a = 2h_s$.

Then the equation (14) can be written as:

$$\frac{d}{dx}\left[F_2 \frac{dP}{dx}\right] + \frac{d}{dx}\left[U\left(a + \frac{h}{2}\right)\right] = -V \quad (21)$$

where:

$$F_2 = \frac{(M - Tanh(M))}{M^3}\frac{(2h_s)^3}{2k\mu} + \frac{1}{4}\left((h_n - 2h_s) + 2h_s \frac{Tanh\,(M)}{M}\right)^2 4h_s \frac{Tanh\,(2M)}{k\mu\;(2M)} + \frac{(h_n - 2h_s)^3}{12\mu}$$

$$M = \alpha\, h_s\,, \quad \alpha = \sqrt{\frac{1}{k\phi}} \quad (22)$$

Where h_n is the nominal film thickness and h_s is the height of roughness asperities.

3. SQUEEZE FILM LUBRICATION OF TWO PARALLEL PLATES

Consider squeeze film lubrication between two symmetrical rough parallel plates with thickness h as shown in the Fig. 3.

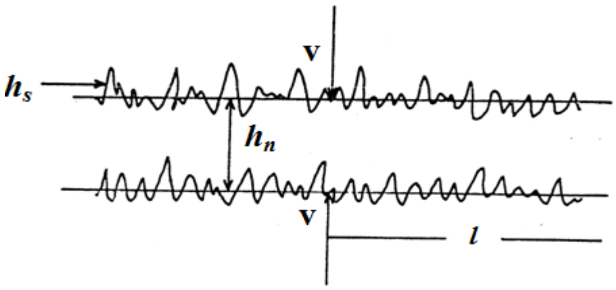

Fig. 3. Squeeze film between two Parallel Plates.

Let the two plates approach each other with a velocity, V symmetrically. The governing equation of flow of the lubricant in the case of squeeze film lubrication is given by equation:

$$\frac{d}{dx}\left(F_2 \frac{dP}{dx}\right) = -V \quad (23)$$

where:

$$F_2 = \frac{(M - Tanh(M))}{M^3}\frac{(2h_s)^3}{2k\mu} + \frac{1}{4}\left((h_n - 2h_s) + 2h_s \frac{Tanh\,(M)}{M}\right)^2 4h_s \frac{Tanh\,(2M)}{k\mu\;(2M)} + \frac{(h_n - 2h_s)^3}{12\mu}$$

$$M = \alpha\, h_s\,, \; \alpha = \sqrt{\frac{1}{k\phi}}$$

where k is the ratio of viscosities near the surfaces to the purely hydrodynamic zone, ϕ is the roughness parameter.

The boundary conditions for equation (23) are:

$$\frac{dP}{dx} = 0 \quad at \quad x = 0 \quad (24)$$

$$P = 0 \quad at \quad x = l \quad (25)$$

Using the conditions (24) and (25) and integrating twice equation (23), then we get the expression for pressure, P as:

$$P = \frac{V}{2F_2}\left(l^2 - x^2\right) \quad (26)$$

The load capacity per unit width is given by:

$$W = \int_0^l P\,dx \quad (27)$$

Substituting the expression for P in (27), we obtain:

$$W = \frac{V}{3F_2}\,l^3 \quad (28)$$

The squeezing time T, is obtained by taking $V = \left(-\frac{dh}{dt}\right)$ in (28) and integrating the resultant expression form initial film thickness h_i to final film thickness h_f

$$T = \frac{l^3}{3W}\int_{h_f}^{h_i} \frac{dh}{F_2} \quad (29)$$

where F_2 is given in equation (22).

Now equations (28) and (29) are non-dimensionalised in the following manner:

$$\overline{h_s} = \frac{h_s}{l},\; \overline{h_n} = \frac{h_n}{l},\; \overline{h_f} = \frac{h_f}{l},\; \overline{h_i} = \frac{h_i}{l},$$

$$\overline{F_2} = \frac{F_2}{\left(l^3 / \mu\right)}\ ,\ \overline{\phi} = \frac{\phi}{l^2}\ ,\ \overline{V} = \frac{V\mu}{P_0 l}$$

then:

$$\overline{W} = \frac{W}{P_0\, l} = \frac{1}{3}\frac{\overline{V}}{\overline{\overline{F_2}}} \tag{30}$$

and:

$$\overline{T} = \frac{T}{\left(\frac{\mu l}{3W}\right)\overline{h_f}} = \int_{h_f}^{h_i} \frac{d\overline{h_n}}{\overline{\overline{F_2}}} \tag{31}$$

where:

$$\overline{F_2} = \frac{(M - Tanh(M))}{M^3}\frac{\left(2\overline{h_s}\right)^3}{2k} + \frac{1}{4}\left(\left(\overline{h_n} - 2\overline{h_s}\right) + 2\overline{h_s}\frac{Tanh(M)}{M}\right)^2 4\overline{h_s}\frac{Tanh(2M)}{k(2M)} + \frac{\left(\overline{h_n} - 2\overline{h_s}\right)^3}{12} \tag{32}$$

$$M = \alpha\,\overline{h_s}\ ,\quad \alpha = \sqrt{\frac{1}{k\overline{\phi}}}$$

The equations (30) and (31) are analyzed numerically and graphs have been plotted.

4. RESULTS AND DISCUSSIONS

In Figs. 4 and 5, $\overline{W}$ and $\overline{T}$ are plotted with k for various $\overline{h}_s$ and in Fig. 6, $\overline{W}$ is plotted with $\overline{h}_n$ for various k.

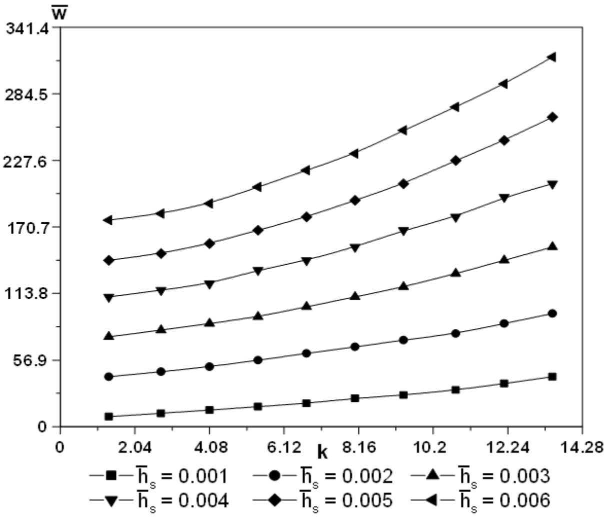

Fig. 4. Variation of $\overline{W}$ with k for various $\overline{h_s}$ and $\overline{\phi}$ = 0.000001.

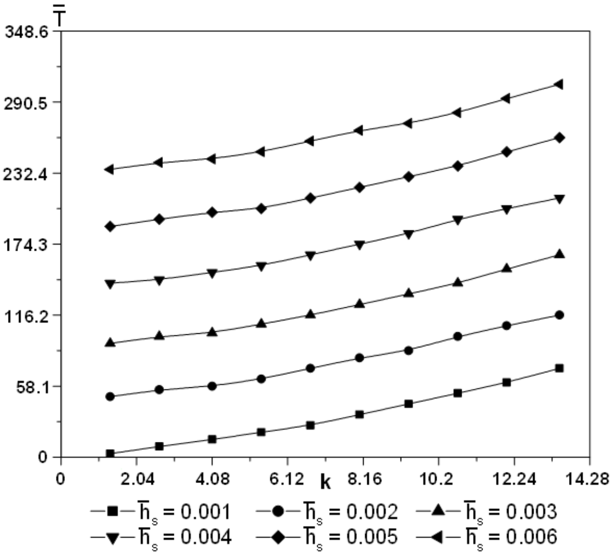

Fig. 5. Variation of $\overline{T}$ with k for various $\overline{h_s}$ and $\overline{\phi}$ = 0.000001.

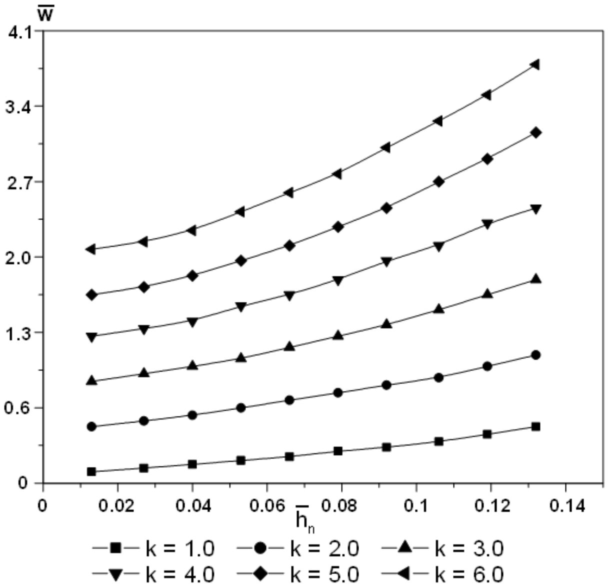

Fig.6. Variation of $\overline{W}$ with $\overline{h_n}$ for various k and $\overline{\phi}$ = 0.000001.

It observed that, the load capacity and squeezing time increases with an increase in the value of k which represents a ratio of the viscosity of the peripheral layer to the middle layer. Hence it can be seen that due to high viscosity in the peripheral layer the load capacity and squeezing time increases which is observed in many experiments [17,18].

In Figs. 7 and 8, $\overline{W}$ and $\overline{T}$ are plotted with $\overline{h_s}$ for various $\overline{\phi}$. It is seen that, the load capacity and squeezing time increases for lower values of

$\bar{\phi}$ with an increase of $\overline{h_s}$ but decreases for higher values of $\bar{\phi}$ with an increase in $\overline{h_s}$. From the nature of $\bar{\phi}$, it may view that lower values of $\bar{\phi}$ may represents transverse roughness and higher values of $\bar{\phi}$ may represents longitudinal roughness. Hence for transverse roughness, the load capacity and squeezing time increases as the mean height of the roughness parameter, $\overline{h_s}$ increases and for longitudinal roughness the load capacity and squeezing time decreases as the mean height of the roughness parameter, $\overline{h_s}$ increases.

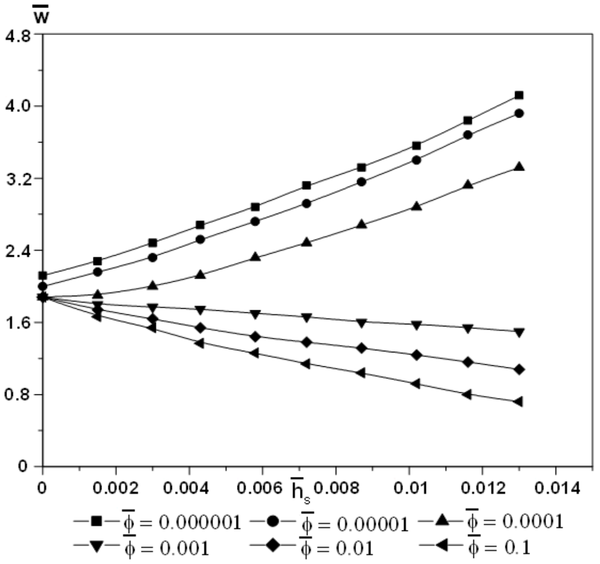

Fig. 7. Variation of $\overline{W}$ with $\overline{h_s}$ for various $\bar{\phi}$ and k =5.0.

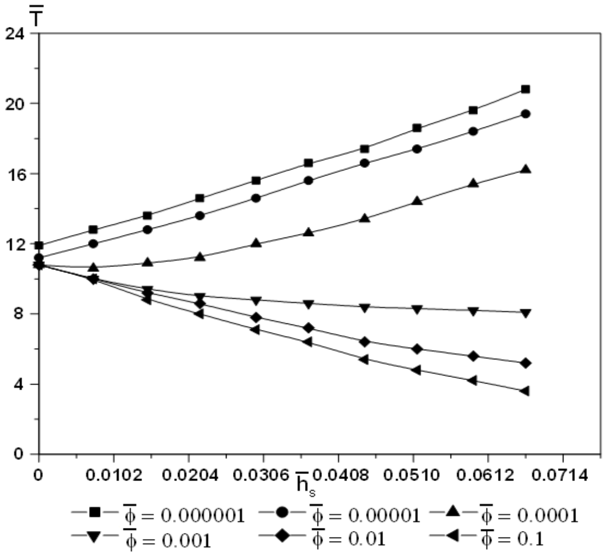

Fig. 8. Variation of $\overline{T}$ with $\overline{h_s}$ for various $\bar{\phi}$ and k = 5.0.

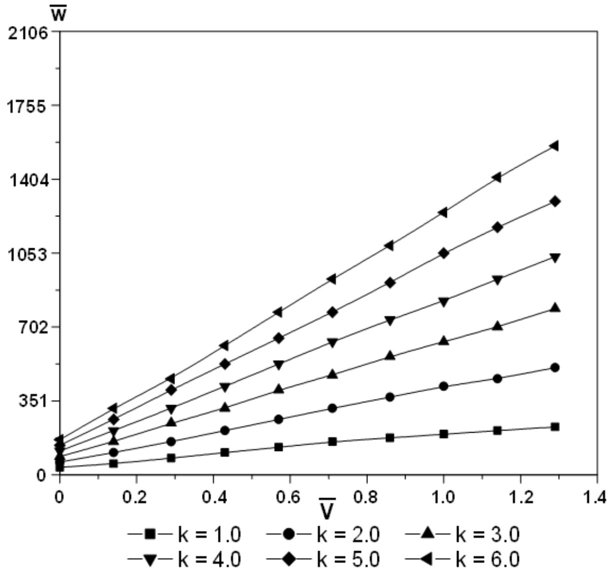

Fig. 9. Variation of $\overline{W}$ with $\overline{V}$ for various k and $\bar{\phi}$ = 0.000001.

From these figures, it is also observed that the effect of roughness is more pronounced in the case of transversal roughness. The results are consistent with stochastic theory [19,20].

In Fig. 9, the load capacity $\overline{W}$ is plotted with $\overline{V}$, the squeeze velocity for various k. The load capacity increases with an increase of squeeze velocity.

5. CONCLUSION

In this paper, the Reynolds equation derived for rough surfaces is applied to squeeze film between two parallel plates. It is found that, the load capacity and squeezing time increase with an increase of peripheral viscosity of the lubricant. Similarly the load capacity and squeezing time increases with an increase in surface roughness asperities for lower values of permeability of roughness interaction zone and decreases for higher values of permeability of roughness interaction zone. The lower values of $\bar{\phi}$ may represent transverse roughness and higher values of $\bar{\phi}$ may represent longitudinal roughness. Also load capacity increases with an increase in squeeze velocity.

The above results are consistent with various experimental observations performed by various workers [21-23].

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Nomenclature

- a Average height of roughness interaction zone
- h Film thickness of hydrodynamic zone
- h_f Final film thickness
- h_i Initial film thickness
- h_n Nominal film thickness
- h_s Mean height of roughness asperities
- k Ratio of viscosities
- l Length of the bearing
- P Hydrodynamic pressure
- t Squeeze time
- V Squeeze velocity
- W Load capacity for rough surfaces
- ϕ Permeability of porous matrix at the surfaces
- μ Viscosity of the purely hydrodynamic zone
- η Effective viscosity of the lubricant in peripheral layer

REFERENCES

[1] D. Dowson and T.L. Whomes, 'The effects of surface roughness upon the lubrications of rigid cylindrical rollers', *Wear*, vol. 18, no. 2, pp. 129-140, 1971.

[2] J.B. Shukla, 'A New theory of lubrication of rough surfaces', *Wear*, vol. 49, no. 1, pp. 33-42, 1978.

[3] P. Sinha, C. Singh and K.R. Prasasd, 'Non-Newtonian lubrication theory for rough surfaces, Application to rigid and Elastic Rollers', *Journal of Mechanical Engineering Science*, vol. 24, pp. 147-154, 1982.

[4] J.B. Shukla and K.R. Prasad, 'Roughness Effects in squeeze films with power law lubricants', in: *Proceedings of 15th National conference on F. N. EF.P.*, 1987.

[5] K.R. Prasad, 'Roughness effects in roller bearings under lightly loaded conditions', in: *Proceedings of Srinivasa Ramanujan Centenary Celebrations, International Conference on Mathematics*, Madras, 1987.

[6] J. White, 'Analytic Solution of a Finite-Width Rough Surface Hydrodynamic Bearing', *Journal of Applied Mechanics*, vol. 53, no. 2, p. 450, 1986.

[7] H. Christensen and K. Tonder, 'Tribology of rough surfaces: Parameters study and comparison of lubrication models', *SINTEF Report No. 22169*, pp. 1013.

[8] J. Shukla, K. Prasad and P. Chandra, 'Effects of consistency variation of power law lubricants in squeeze films', *Wear*, vol. 76, no. 3, pp. 299-319, 1982.

[9] R. Raghavendra Rao and K. Raja Sekhar, 'Effects of Couple Stresses and Surface Roughness on Roller Bearings under lightly loaded conditions', *Indian Journal of Tribology*, vol. 2, no. 1, pp. 11-25, 2007.

[10] R. Raghavendra Rao and K. Raja Sekhar, 'Effects of Couple Stresses and Surface Roughness on the film thickness of heavily loaded Rollers', *Industrial Lubrication and Tribology*, vol. 59, no. 6, pp. 285-296, 2007.

[11] R. Raghavendra Rao, K. Gowthami and J. Vijaya Kumar, 'Effects of velocity-slip and viscosity variation in squeeze film lubrication of two circular plates', *Tribology in Industry*, vol. 35, no. 1, pp. 51-60, 2013.

[12] R. Raghavendra Rao, K. Gowthami and J. Vijaya Kumar, 'Effects of velocity-slip and viscosity variation in squeeze film lubrication of Spherical bearing', in: *Lecture Notes in Mechanical Engineering (Proceedings of ICATES)*, Springer India, pp. 35-47, 2013.

[13] R.M. Patel, G.M. Deheri, P.A. Vadhar, 'Performance of a magnetic fluid based squeeze film between Transversely Rough Triangular plates', *Tribology in Industry*, vol. 32, no. 1, pp. 33-38, 2010.

[14] M.E. Shimpi, G.M. Deheri, 'Surface roughness and Elastic Deformation Effects on the behavior of the Magnetic fluid based squeeze film between rotating porous circular plates with concentric circular pockets', *Tribology in Industry*, vol. 32, no. 2, pp. 21-30, 2010.

[15] M.E. Shimpi, G.M. Deheri, 'Magnetic fluid-Based squeeze film performance in Rotating curved porous circular plates: The effects of Deformation and Surface roughness', *Tribology in Industry*, vol. 34, no. 2, pp. 57-67, 2012.

[16] H. Brinkman, 'A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles', *Applied Scientific Research*, vol. 1, no. 1, pp. 27-34, 1949.

[17] P. Sinha, C. Singh and K. Prasad, 'Viscosity variation considering cavitation in a journal bearing lubricant containing additives', *Wear*, vol. 86, no. 1, pp. 43-56, 1983.

[18] P. Sinha, C. Singh and K. Prasad, 'Lubrication of human joints — a microcontinuum approach', *Wear*, vol. 80, no. 2, pp. 159-181, 1982.

[19] J.B. Shukla, 'A generalized theory of stochastic lubrication: Effects of additives and surface interaction', in: *Symp. Math. Modeling and its applications*, I.I.T., Kanpur, India, 1978.

[20] H. Christensen, 'A theory of mixed lubrication', *Proc. Instn, Mech. Engrs.*, vol. 186, no. 41, pp. 421- 430, 1972.

[21] H. Christensen, J. Shukla and S. Kumar, 'Generalized reynolds equation for stochastic lubrication and its application', *ARCHIVE: Journal of Mechanical Engineering Science 1959-1982 (vols 1-23)*, vol. 17, no. 5, pp. 262-270, 1975.

[22] T. Askwith, A. Cameron and R. Crouch, 'Chain Length of Additives in Relation to Lubricants in Thin Film and Boundary Lubrication', *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 291, no. 1427, pp. 500-519, 1966.

[23] A. Cameron and R. Gohar, 'Theoretical and experimental studies of the oil film in lubricated point contacts', *Proc. Roy. Soc., S. A.*, vol. 291, pp. 520-536, 1966.