Vol. 37, No. 1 (2015) 117-127

Tribology in Industry

www.tribology.fink.rs

RESEARCH

Effects of Viscosity Variation and Surface Roughness on the Couple stress Squeeze Film Characteristics of Short Journal Bearings

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Keywords:

Short Journal bearing
Squeeze film
Couple stress
Viscosity variation
Randomized roughness

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A B S T R A C T

The theoretical analysis of the combined effects of surface roughness and viscosity variation on the couple stress squeeze film characteristics of short journal bearings is presented. The modified stochastic Reynold's equation accounting for the viscosity variation of couple stresses fluid and randomized surface roughness structure on bearing surface is mathematically derived using the Christensen stochastic theory. It is observed that, the transverse roughness pattern improves the squeeze film characteristics whereas the bearing performance is affected due to the presence of one dimensional longitudinal surface roughness. Further, it is observed that, the effect of viscosity variation is to reduce the load carrying capacity and squeeze film time as compared to the case of constant viscosity.

1. INTRODUCTION

The technology of squeeze film are widely applied in many engineering applications, such as gears, disk clutches, machine tools, dampers, aircraft engines and human joints. The squeeze film behaviour arises from the phenomenon of two lubricated surfaces approaching each other with a normal velocity. Newtonian lubricants are conventionally used in the squeeze film bearings [1-3], with the development of modern machine equipments, the increasing use of fluids containing microstructures, such as additives suspensions, granular matter has received great interest. These kinds of lubricants exhibit the rheological behaviours of non-Newtonian fluids. A number of micro continuum theories have been developed to explain the peculiar behaviour of fluids containing a structure such as polymeric fluids [4, 5]. The micro continuum theory derived by Stokes [6] is the simplest generalization of the classical theory of fluids, which allows for the polar effects such as the presence of couple stress and body couples. By applying this couple stress fluid model, a number of researches in various squeeze film problems have been presented. The typical studies are the squeeze film behaviour between finite plates of various shapes [7], the squeeze film configuration with reference to synovial joints [8, 9], the squeeze film partial journal bearings [10] and squeeze film performance between a sphere and flat plate [11]. Generally speaking, a higher film pressure and larger load carrying capacity as well as long response time are obtained for the squeeze films by the use of fluids with couple stress.

The effect of surface roughness on the hydrodynamic lubrication of bearings has been studied by several investigators. The random character of the surface roughness prompted many researchers to adopt a stochastic approach for the study of surface roughness [12-14]. The stochastic model developed by Christensen [12] for the study of hydrodynamic lubrication of rough surfaces formed the basis for several studies [15-17]. The effect of surface roughness on the performance of short porous journal bearings is studied by Naduvinamani *et.al.* [18]. Lin *et.al.* [19] studied the effect of surface roughness on the oscillating squeeze film behaviour of a long partial journal bearings. Since the effect of couple stress is significant and the roughness cannot be avoided, it is worth to investigate the combined effect of both on the bearing performance. Although the isotropic rough plates with non-Newtonian couple stress fluid in the squeeze film has been studied by Lin *et.al.* [20], the study of journal bearing system is absent. Earlier theories were based on the assumptions that the viscosity is constant, although it is a function of both pressure and temperature. The variation in viscosity with temperature is important in many practical applications, where lubricants are required to function over a wide range of temperature [21]. To study the effect of viscosity variation, one has to consider a typical viscosity film thickness relation with thermodynamic problems [22-24]. A generalised form of Reynold's equation for stochastic lubrication applicable to rough bearings was derived by considering the viscosity variation and surface roughness in short journal bearings and slider bearings by Kumar and Sachidanand [25] and Kumar and Shukla [26] respectively. The effect of viscosity variation on the squeeze film performance of narrow journal bearing with couple stress fluid is studied by Reddy *et.al.* [27] by assuming bearing surfaces are smooth.

In this paper an attempt has been made to study the combined effect of viscosity variation and surface roughness on the couple stress squeeze film lubrication of narrow journal bearing.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Figure 1 shows the physical configuration of a squeeze film journal bearing. The shaft of radius R approaches the bearing surface with velocity $\left(\frac{dH}{dt}\right)$. The lubricant in the system is taken to be Stokes couple stress fluid.

To represent the surface roughness the mathematical expression for the film thickness is considered to be made of two parts:

$$H = h + h_s(\theta, z, \xi) \tag{1}$$

Where $h = c + e\cos\theta$ denote the nominal smooth part of the film geometry with c being the radial clearance and e is the eccentricity, while $h_s(\theta, z, \xi)$ is the part due to the surface asperities measured from the nominal level and is a randomly varying quantity of zero mean, ξ is an index determining definite roughness arrangements. Further, $\theta = x/R$ with R being the radius of the journal.

The basic equations derived by Stokes [6] for the motion of an incompressible couple stress fluid, in the absence of body forces and body couples are:

$$\nabla.\vec{V} = 0 \tag{2}$$

$$0 = -\nabla p + \mu\nabla^2\vec{V} - \eta\nabla^4\vec{V} \tag{3}$$

Where the vector $\vec{V}$ represents the velocity vector, p is the pressure, μ is the shear viscosity and η is the new material constant responsible for the couple stress fluid property.

With the usual assumptions of hydrodynamic lubrication applicable to thin films, equations of motion (2) and (3) take the form:

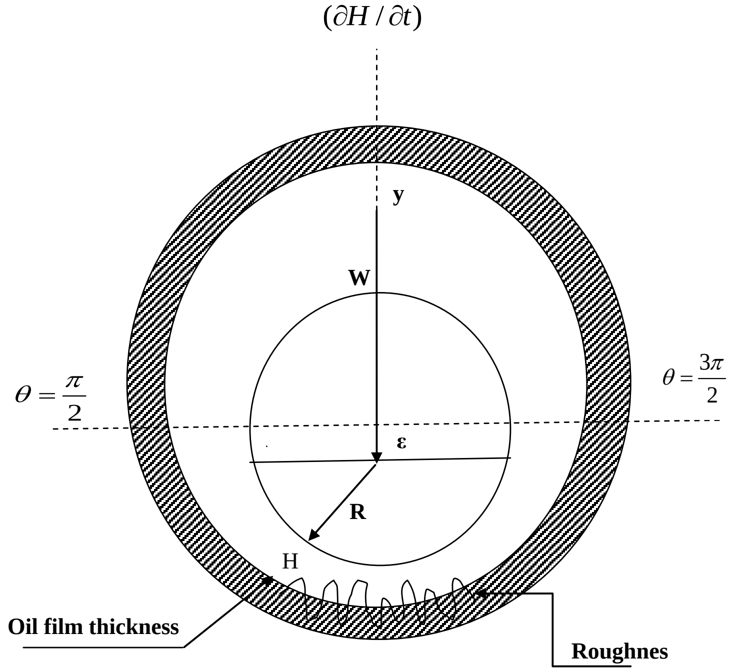

Fig. 1. Squeeze film geometry of a journal bearing.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad (4)$$

$$\frac{\partial p}{\partial x}=\mu\frac{\partial^2 u}{\partial y^2}-\eta\frac{\partial^4 u}{\partial y^4} \quad (5)$$

$$\frac{\partial p}{\partial y}=0 \quad (6)$$

$$\frac{\partial p}{\partial z}=\mu\frac{\partial^2 w}{\partial y^2}-\eta\frac{\partial^4 w}{\partial y^4} \quad (7)$$

where u, v and w denote the velocity components in the x, y, and z directions respectively.

The boundary conditions at the bearing surface are:

$$u(x,0,z)=v(x,0,z)=w(x,0,z)=0$$
$$\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=o}=\left(\frac{\partial^2 w}{\partial y^2}\right)_{y=o}=0 \quad (8)$$

and at the journal surface are:

$$u(x,H,z)=w(x,H,z)=0,\ v(x,H,z)=\frac{dH}{dt}$$
$$\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=H}=\left(\frac{\partial^2 w}{\partial y^2}\right)_{y=H}=0 \quad (9)$$

The solution of equations (5) and (7) subjected to the relevant boundary conditions given in equations (8) and (9) is obtained in the form

$$u=\frac{1}{2\mu}\frac{\partial p}{\partial x}\left\{y(y-H)+2l^2\left[1-\frac{\cosh((2y-H)/2l)}{\cosh(H/2l)}\right]\right\} \quad (10)$$

and

$$w=\frac{1}{2\mu}\frac{\partial p}{\partial z}\left\{y(y-H)+2l^2\left[1-\frac{\cosh((2y-H)/2l)}{\cosh(H/2l)}\right]\right\} \quad (11)$$

where: $l=\sqrt{\eta/\mu}$

Using the expressions (10) and (11) for velocity components u and w in the continuity equation (4) and integrating with respect to y and the use of boundary conditions (8) and (9), the modified Reynold's equation is obtained in the form:

$$\frac{\partial}{\partial x}\left\{\frac{g(H,l)}{\mu}\frac{\partial p}{\partial x}\right\}+\frac{\partial}{\partial z}\left\{\frac{g(H,l)}{\mu}\frac{\partial p}{\partial z}\right\}=12\frac{\partial H}{\partial t} \quad (12)$$

where:

$$g(H,l)=H^3-12l^2H-24l^3\tanh(H/2l) \quad (13)$$

It is noticed that, for Newtonian fluid, $\eta=0$ and $l=0$. As $l\to 0$, the function $g(H,l)$ defined in the above equation (13) approaches H^3 and modified Reynolds equation (12) reduces to the classical form of the Newtonian lubricant case [1].

Now it is assumed that, the Newtonian viscosity μ is varying along the fluid film thickness H according to the relation given below [22].

$$\mu = \mu_1 \left(\frac{H}{h_1} \right)^Q \tag{14}$$

where μ_1 is the inlet viscosity at:

$$H = h_1 = c(1+\varepsilon).$$

The parameter $Q(0 \le Q \le 1)$ depends on the particular lubricant used, for perfect Newtonian fluids Q=0, whereas for perfect gases Q=1. For mathematical simplicity, the couple stress parameter l is assumed to be independent of viscosity variation, this can be done by assuming that η is varying in the same way as μ.

2.1 Stochastic Reynolds equation

Let $f(h_s)$ be the probability density function of the stochastic film thickness h_s. Taking the stochastic average of equation (12) with respect to $f(h_s)$, the stochastic Reynolds equation is obtained:

$$\frac{\partial}{\partial x}\left\{ \frac{E[g(H,l)]}{\mu} \frac{\partial E(p)}{\partial z} \right\} + \frac{\partial}{\partial z}\left\{ \frac{E[g(H,l)]}{\mu} \frac{\partial E(p)}{\partial z} \right\} = 12 \frac{\partial E(H)}{\partial t} \tag{15}$$

where:

$$E(.) = \int_{-\infty}^{\infty} (.) f(h_s) dh_s \tag{16}$$

In accordance with Christensen [12], it is assumed that:

$$f(h_s) = \begin{cases} \frac{35}{32C^7}(C^2 - h_s^2)^3, & -C < h_s < C \\ 0 & \text{elsewhere} \end{cases} \tag{17}$$

where $\sigma = C/3$ is the standard deviation.

Longitudinal roughness pattern

For the one dimensional longitudinal roughness pattern, the roughness striations are in the form of ridges and valleys in the x-direction, in this case film thickness assumes the form:

$$H = h(z) + h_s(z,\xi) \tag{18}$$

and the stochastic modified Reynold's equation (15) takes the form

$$\frac{\partial}{\partial x}\left\{ \frac{1}{E((g(H,l))^{-1})\mu} \frac{\partial E(p)}{\partial z} \right\} + \frac{\partial}{\partial z}\left\{ \frac{E(g(H,l))}{\mu} \frac{\partial E(p)}{\partial z} \right\} = 12 \frac{\partial E(H)}{\partial t} \tag{19}$$

Transverse roughness pattern

For one dimensional transverse roughness striations are in the form of ridges and valleys in the y-direction in this case the film thickness assumes the form:

$$H = h(\theta) + h_s(\theta,\xi) \tag{20}$$

The modified Reynold's type equation (15) takes the form:

$$\frac{\partial}{\partial x}\left\{ \frac{E(g(H,l))}{\mu} \frac{\partial E(p)}{\partial z} \right\} + \frac{\partial}{\partial z}\left\{ \frac{1}{E((g(H,l))^{-1})\mu} \frac{\partial E(p)}{\partial z} \right\} = 12 \frac{\partial E(H)}{\partial t} \tag{21}$$

Equations (19) and (21) together can be written as:

$$\frac{\partial}{\partial x}\left\{ \frac{G(H,l,c)}{\mu} \frac{\partial E(p)}{\partial z} \right\} + \frac{\partial}{\partial z}\left\{ \frac{G(H,l,c)}{\mu} \frac{\partial E(p)}{\partial z} \right\} = 12 \frac{\partial E(H)}{\partial t} \tag{22}$$

$$\text{where } G(H,l,C) = \begin{cases} E(g(H,l)) & \text{for logitudinal roughness} \\ \frac{1}{E((g(H,l))^{-1})} & \text{for transeverse roughness} \end{cases}$$

2.2 Short bearing approximation

In order to simplify the problem and to obtain a closed form solution for the fluid pressure, a narrow bearing approximation is assumed. That is the circumferential variation of pressure can be neglected as compared to the axial variation, then the modified stochastic Reynold's equation (22) reduces to:

$$\frac{\partial}{\partial z}\left\{ \frac{G(H,l,C)}{\mu} \frac{\partial E(p)}{\partial z} \right\} = 12 \frac{\partial E(H)}{\partial t} \tag{23}$$

Substituting $\mu = \mu_1 \left(\frac{H}{h_1} \right)^Q$ in the above equation, to obtain:

$$\frac{\partial}{\partial z}\left\{\frac{G(H,l,c)}{\mu_1}\left(\frac{h_1}{H}\right)^Q\frac{\partial E(p)}{\partial z}\right\}=12\frac{\partial E(H)}{\partial t} \quad (24)$$

Integrating twice with respect to z and applying the following boundary conditions:

$$E(p)=0,\text{at } z=\pm\frac{L}{2}\text{ and }\frac{dE(p)}{dz}=0,\text{at } z=0 \quad (25)$$

The fluid film pressure is given by:

$$E(p)=\frac{6\mu_1}{G(H,l,c)}\left(\frac{H}{h_1}\right)^Q\left(\frac{dH}{dt}\right)\left(z^2-\frac{L^2}{4}\right) \quad (26)$$

Introducing the non-dimensional variables:

$$\lambda=\frac{L}{2R},\ \bar{z}=\frac{z}{L},\ \bar{l}=\frac{l}{c},\ \frac{dH}{dt}=c\cos\theta\frac{d\varepsilon}{dt},\ \bar{c}=\frac{C}{c}$$
$$\bar{H}=\frac{H}{c}=1+\varepsilon\cos\theta,\ \bar{p}=\frac{E(p)c^2}{\mu_1R^2\left(\frac{d\varepsilon}{dt}\right)} \quad (27)$$

On substituting equation (27) into above equation (26) the non-dimensional fluid film pressure is given in a closed form is obtained as

$$\bar{p}=\frac{24\lambda^2\cos\theta\bar{H}^Q}{\bar{G}(\bar{H},\bar{l},\bar{c})(1+\varepsilon)^Q}\left(\bar{z}^2-\frac{1}{4}\right) \quad (28)$$

$$\text{where } \bar{G}(\bar{H},\bar{l},\bar{c})=\begin{cases}E\left(\bar{g}(\bar{H},\bar{l})\right) & \text{for logitudinal roughness}\\ \dfrac{1}{\mathrm{E}\left((\bar{g}(\bar{H},\bar{l}))^{-1}\right)} & \text{for transeverse roughness}\end{cases}$$

$$\bar{g}(\bar{H},\bar{l})=\bar{\mathrm{H}}^3-12\bar{l}^{\,2}\bar{H}+24\bar{l}^{\,3}\tanh\left(\frac{\bar{H}}{2\bar{l}}\right)$$

2.3 Load carrying capacity

The load carrying capacity is evaluated by integrating the squeeze film pressure acting on the journal shaft is given by:

$$E(w)=-2R\int_{z=0}^{z=L/2}\int_{\theta=\pi/2}^{\theta=3\pi/2}E(p)\cos\theta\,d\theta\,dz \quad (29)$$

Introducing the non-dimensional quantity:

$$\bar{w}=\frac{E(w)c^2}{\mu_1R^2L(d\varepsilon/dt)} \quad (30)$$

The load carrying capacity can be expressed in non-dimensional form as:

$$\bar{w}=\frac{4\lambda^2}{(1+\varepsilon)^Q}\int_{\theta=\pi/2}^{\theta=3\pi/2}\frac{(1+\varepsilon\cos\theta)^Q}{\bar{G}(\bar{H},\bar{l},\bar{c})}\cos^2\theta\,d\theta \quad (31)$$

The non-dimensional load carrying capacity $\bar{w}$ in the above equation (31) cannot be obtained by direct integration. It can be numerically evaluated by the method of Gaussian quadrature.

2.4 Squeeze time eccentricity ratio relationship

For constant load *E(w)*, the time taken by the journal to move from ε=0 to ε= ε_1 can be obtained by integrating equation (30) with respect to time. Introducing the non-dimensional response time:

$$\bar{t}=\frac{E(w)c^2}{\mu_1R^3L}t \quad (32)$$

We have the time-height relationship expressed as:

$$\frac{d\varepsilon}{d\bar{t}}=\frac{1}{\bar{w}} \quad (33)$$

with initial condition of $\varepsilon=0$ at $\bar{t}=0$. In the limiting case of $\bar{c}\to 0$ equations (28), (31) and (33) reduce to that of smooth case studied by Reddy [27].

3. RESULTS AND DISCUSSIONS

This paper predicts the combined effects of surface roughness and viscosity variation on the couple stress squeeze film characteristics of short journal bearings. These effects are analyzed on the basis of various dimensionless parameters such as the viscosity variation parameter *Q*, roughness parameter $\bar{c}$, couple stress parameter $\bar{l}$ and the eccentricity ratio parameter ε.

In the present analysis, we choose the parameters, $\lambda=0.5$ (length to diameter ratio), since in practice the eccentricity ratio ranges from 0.4 to 0.6. Couple stress fluid is characterized by the non-dimensional parameter $\bar{l}$, the value of this couple stress parameter depends upon the characteristic material length of the polar suspensions *l* and the radial clearance *c*. Hence the values of $\bar{l}$ are chosen as 0.0, 0.2, 0.4. Viscosity variation parameter *Q* lies between 0 and 1. Numerical values of 0, 0.25, 0.5, 0.75 and 1 are assumed for *Q* in order to discuss the effect of viscosity variation in the present analysis and the roughness parameter $\bar{c}=0, 0.1, 0.2$ and 0.3 are chosen for the discussion.

3.1 Squeeze film pressure

Figure 2 shows the variation of non-dimensional pressure $\overline{p}$ as a function of circumferential coordinate θ (in degrees) on the mid-plane $\overline{z}=0$ at the eccentricity ratio ε =0.6, couple stress parameter $\overline{l}$ =0.4 and roughness parameter $\overline{c}$ =0.2 for both longitudinal and transverse roughness patterns. It is observed that, the effect of viscosity variation parameter is to decreases the squeeze film pressure for both longitudinal as well as transverse roughness patterns. Figure 3 shows the variation non-dimensional pressure $\overline{p}$ as a function of circumferential coordinate θ (in degrees) on the mid-plane $\overline{z}=0$ at the eccentricity ratio ε =0.6 and Q=1 for different values of $\overline{l}$ for both longitudinal and transverse roughness pattern. It is observed that, the pressure $\overline{p}$ increases for increasing values of $\overline{l}$ for both longitudinal as well as transverse roughness patterns. The presence of couple stress parameter provides an increase in squeeze film pressure. As the viscosity variation factor increases the squeeze film pressure decreases rapidly for couple stress fluid than Newtonian fluid. Thus the viscosity variation effect is significantly for couple stress fluid. The effect of $\overline{c}$ on the variation of $\overline{p}$ with θ is depicted in the Fig. 4. It is observed that, the pressure $\overline{p}$ decreases with increase in $\overline{c}$ for longitudinal roughness pattern, whereas $\overline{p}$ increases with increase in the values of $\overline{c}$ for transverse roughness pattern.

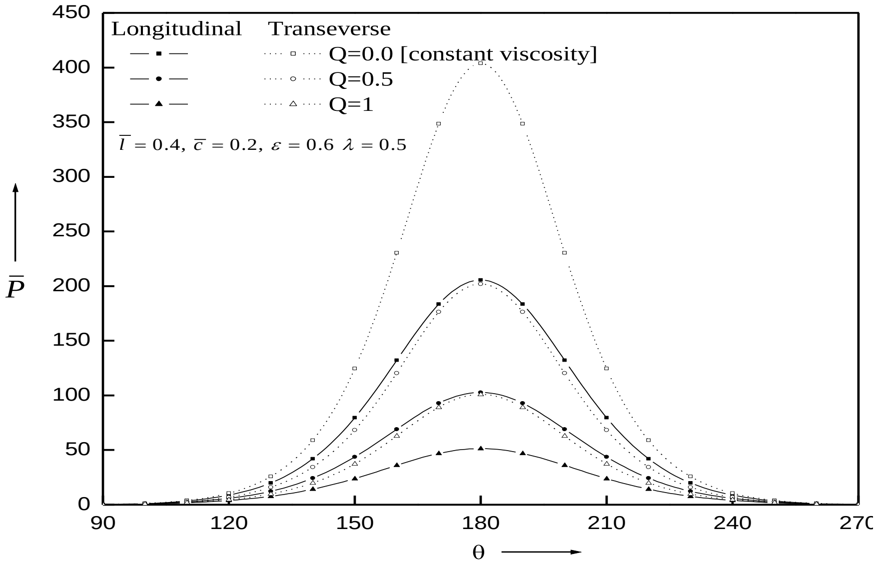

Fig. 2. Dimensionless pressure verses bearing circumferential angle for different viscosity variation parameters.

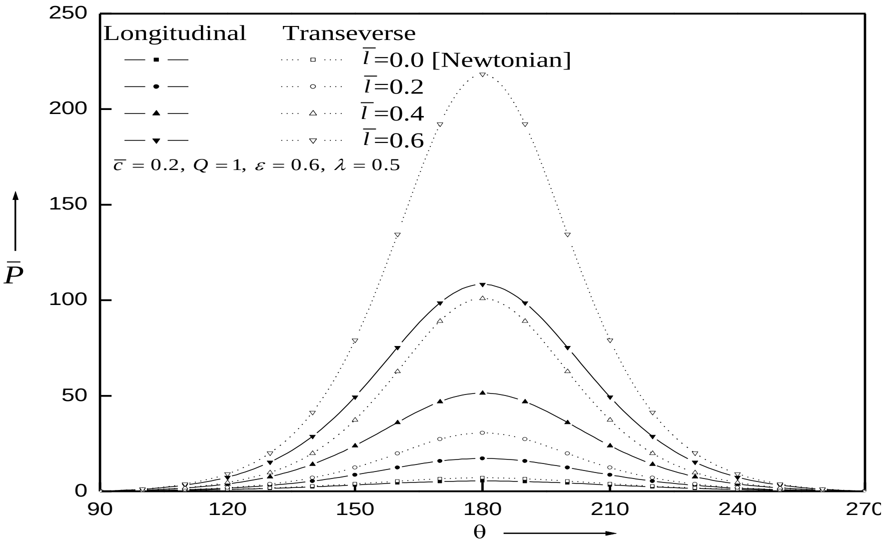

Fig. 3. Dimensionless pressure verses bearing circumferential angle for different couple stress parameters.

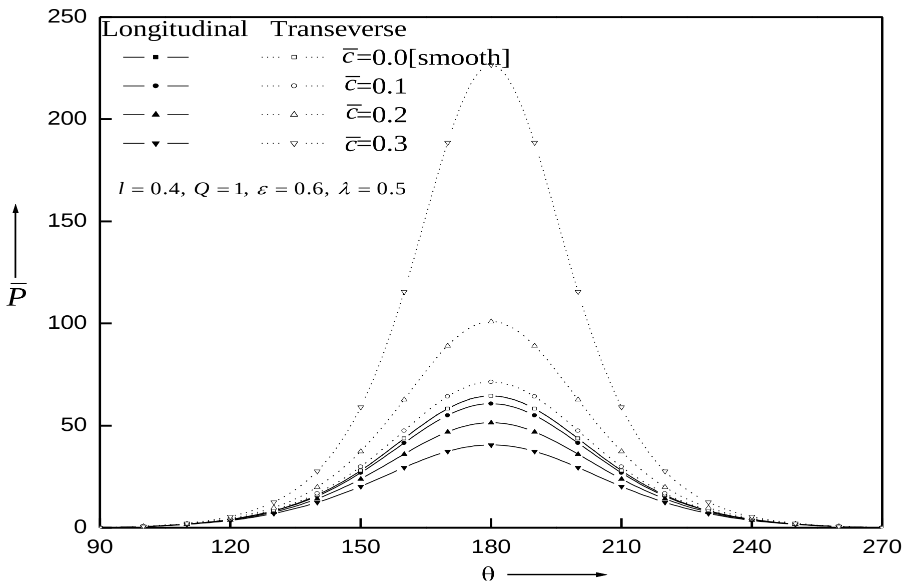

Fig. 4. Dimensionless pressure verses bearing circumferential angle for different roughness parameters.

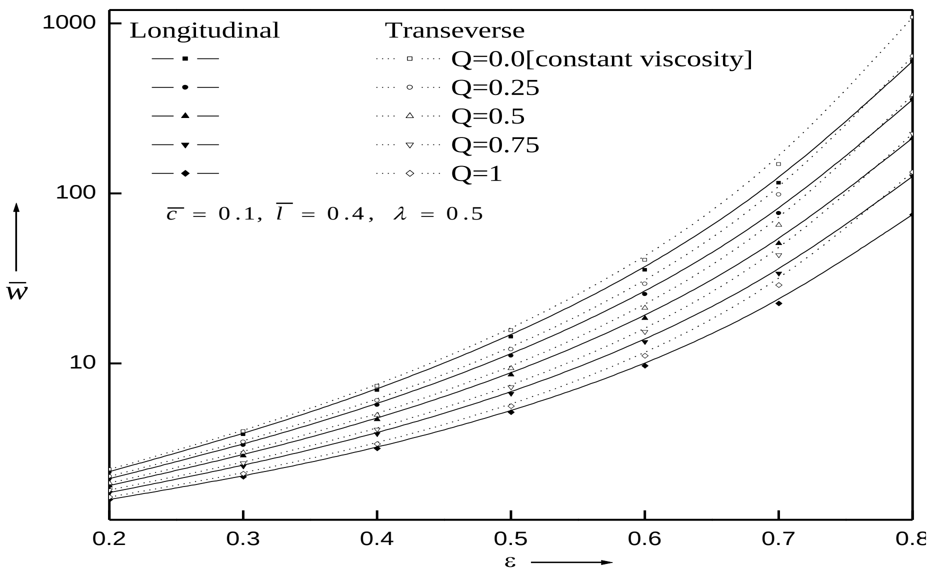

Fig. 5. Dimensionless load verses eccentricity ratio with different viscosity variation parameters.

3.2 Load carrying capacity

The variation of dimensionless load carrying capacity $\bar{w}$ with the eccentricity ratio ε for different values of viscosity variation factor *Q* for both longitudinal and transverse pattern is depicted in Fig.5. It is observed that, the load carrying capacity decreases for increasing values of viscosity variation parameter for both the types of roughness patterns. The variation of dimensionless load carrying capacity $\bar{w}$ with eccentricity ratio ε for different values of $\bar{l}$ is shown in Fig.6. It is observed that the load carrying capacity increases for the increasing values of $\bar{l}$. The curve corresponding to $\bar{l} = 0$ represents the Newtonian case. Figure 7 shows the variation of dimensionless load $\bar{w}$ with eccentricity ratio ε for different values of the roughness parameter, $\bar{c}$ for both the types of roughness patterns. It is observed that, $\bar{w}$ increases (decreases) for increasing values of $\bar{c}$ for the transverse (longitudinal) roughness case.

3.3 Squeeze time eccentricity ratio relationship

Figure 8 shows the variation of dimensionless squeeze film time $\bar{t}$ with eccentricity ratio, ε, for different values of the viscosity variation parameter *Q*, for both longitudinal and transverse roughness patterns.

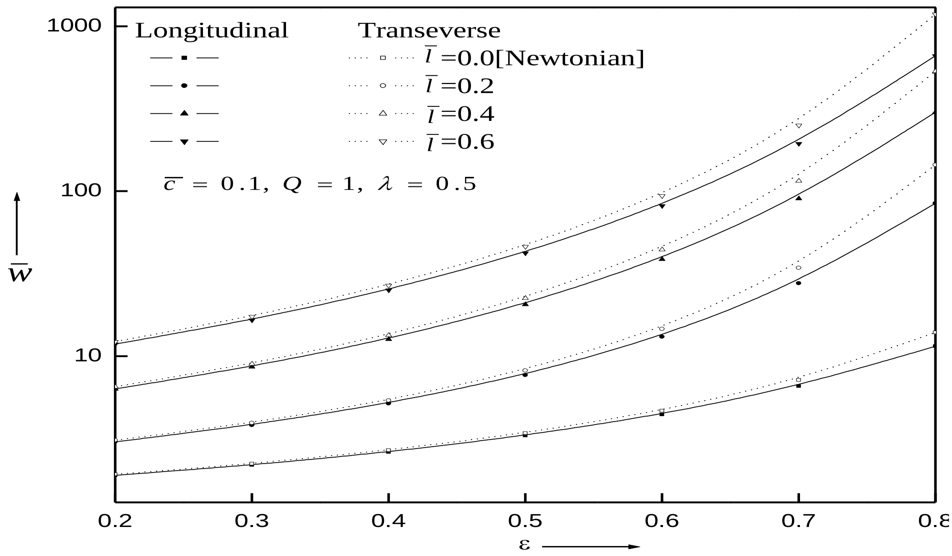

Fig. 6. Dimensionless load verses eccentricity ratio with different coupe stress parameters.

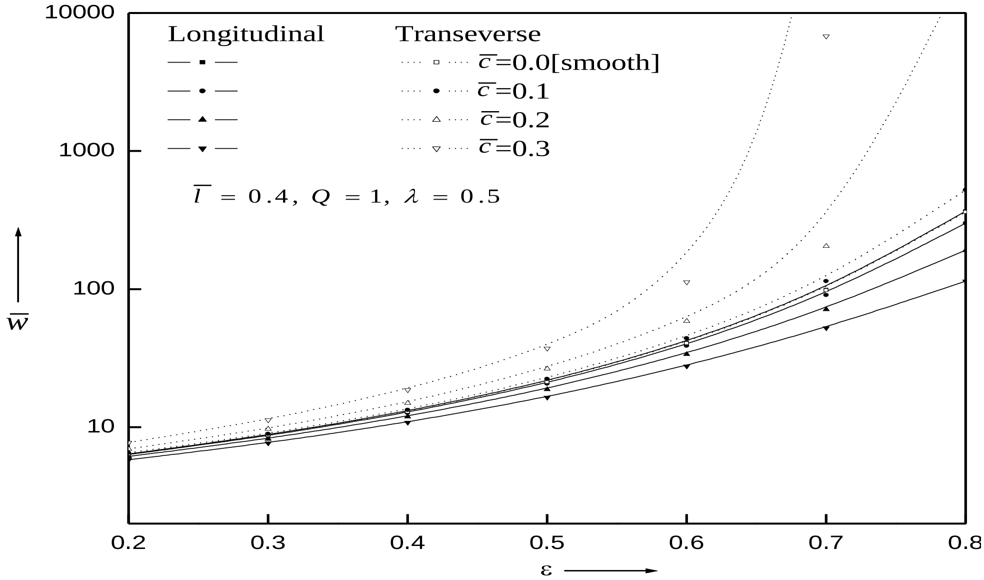

Fig. 7. Dimensionless load verses eccentricity ratio with different roughness parameters.

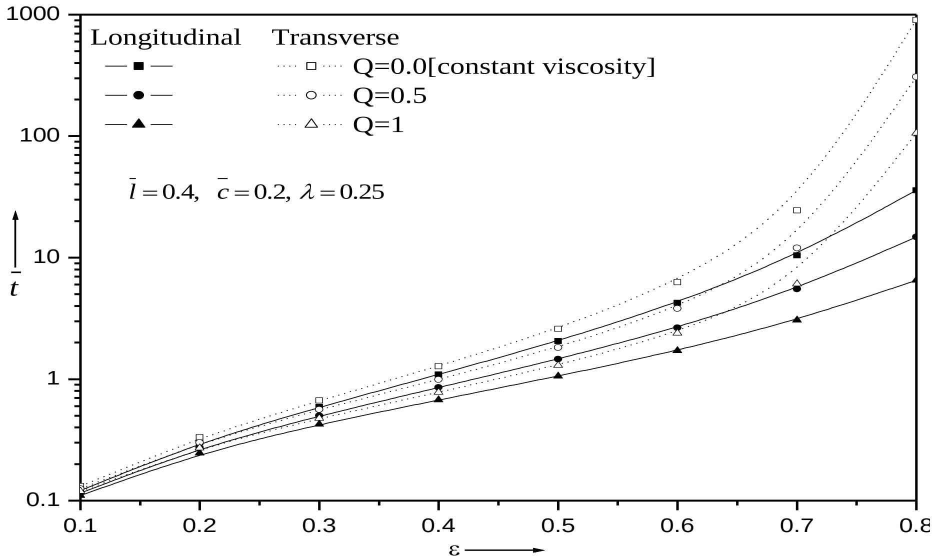

Fig. 8. Dimensionless time verses eccentricity ratio with different viscosity variation parameters.

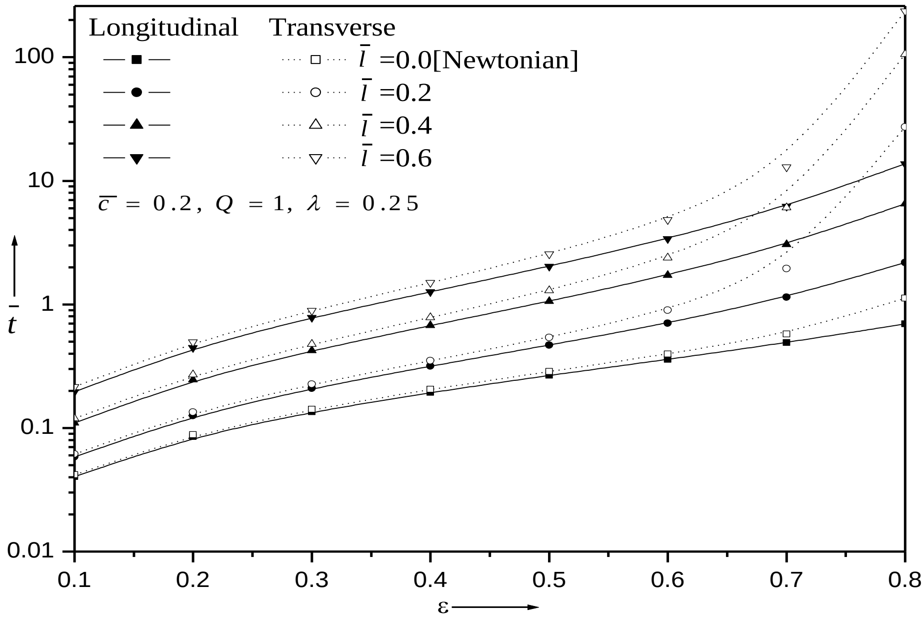

Fig. 9. Dimensionless time verses eccentricity ratio with different coupe stress parameters.

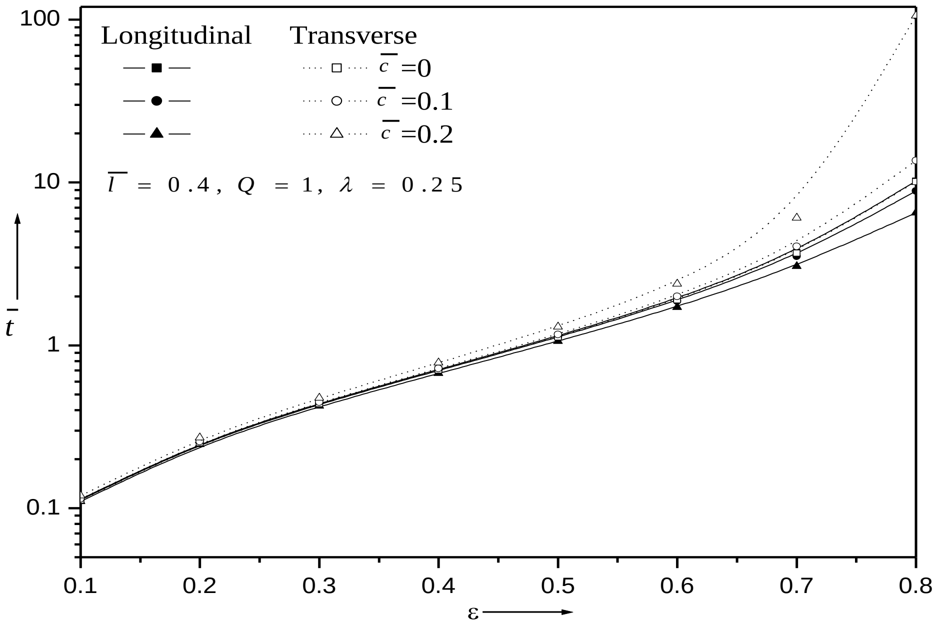

Fig. 10. Dimensionless time verses eccentricity ratio with different roughness parameters.

It is observed that the effect of variation of viscosity is to decrease the squeeze film time for both types of roughness patterns. The variation $\overline{t}$ with ε for different values of $\overline{l}$ for both the types of roughness pattern is shown in Fig. 9. It is observed that, the effect of couple stress fluids is to increase $\overline{t}$ as compared to the corresponding Newtonian case ($\overline{l} = 0$) for both types of roughness patterns. The variation $\overline{t}$ with ε for different roughness parameters $\overline{c}$ for both longitudinal and transverse roughness pattern is depicted in the Fig.10. It is observed that, $\overline{t}$ increase (decrease) for the increasing values of $\overline{c}$ for the transverse (longitudinal) roughness patterns.

4. Conclusions

The present investigation reveals the effect of viscosity variation and surface roughness on the couple stress squeeze film characteristics of short journal bearings. The modified stochastic Reynold's equation is solved for the squeeze film pressure and obtained the load carrying capacity and squeeze film time. According to the results presented in the above section the following conclusions can be drawn:

1. The effect of viscosity variation is to decreases the squeeze film pressure, load carrying capacity and squeeze film time for both the types of roughness patterns.
2. The effect of couple stresses is to increases load carrying capacity and to lengthen the squeeze film time as compared to the corresponding Newtonian case for both types of roughness patterns.
3. The effect of transverse (longitudinal) roughness pattern is to increase (decrease) the load carrying capacity and squeeze film time as compared to the corresponding smooth case.

NOMENCLATURE

c	radial clearance
C	roughness parameter
$\bar{c}$	dimensionless roughness parameter (C/c)
E	expectancy operator
h	film thickness
h_1	fluid film thickness at the inlet
h_s	random variable
H	film thickness, $h + h_s$
$\bar{H}$	dimensionless film thickness
l	couple stress parameter $\left(\frac{\eta}{\mu}\right)^{1/2}$
$\bar{l}$	dimensionless couple stress parameter (l/c)
L	Length of the bearing
p	film pressure
$\bar{p}$	dimensionless film pressure $\left(\dfrac{E(p)c^2}{\mu_1 R^2\left(\dfrac{d\varepsilon}{dt}\right)}\right)$
Q	viscosity variation parameter
R	radius of the journal
u,v,w	fluid velocity components in the x, y, z directions respectively
t	response time
$\bar{t}$	dimensionless response time $\left(\dfrac{E(w)c^2}{\mu_1 r^3 L}t\right)$
$\vec{V}$	velocity vector
w	load carrying capacity
$\bar{w}$	dimensionless load carrying capacity $\left(\dfrac{E(w)c^2}{\mu_1 R^2 L(d\varepsilon/dt)}\right)$
Δ	gradient operator
ε	eccentricity ratio (e/c)
η	material constant responsible for the couple stress property
μ_1	viscosity of the lubricant at the inlet

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