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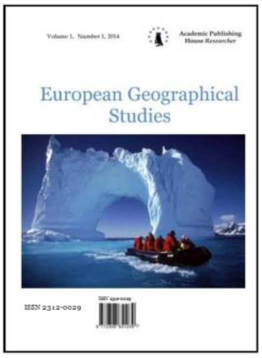

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Towards the Analytical Approximation of Weathering Forms based on Fitting of the Geomorphological Structures by the "Tafeln Höherer Funktionen" Profile Database

Brief communication

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Abstract

A novel approach for the analytical approximation of weathering forms is proposed in this brief communication. This novel technique based on different isosurfaces and profiles of special functions (such as Euler–Riemann zeta function, Jacoby function, Mathieu function, Weierstrass function, Hankel function, Auger function, modular elliptic functions, etc.) is a very effective method for prognostic geomorphology and physical geography, including simulations of the landscape relief instabilities, nonspecific weathering forms and geomorphogenesis.

Keywords: weathering forms, geomorphology, approximation, special functions, Euler–Riemann zeta function, Jacoby function, Mathieu function, Weierstrass function, Hankel function.

1. Introduction

It is well known that the weathering activity and morphogenesis of the Earth surfaces can be qualitatively correlated and spatially colocalized (Govindarajan, Murthy, 1969; Modenesi, 1983; Pavich, 1985; Tokuyama, 1986; Le Pera, Sorriso-Valvo, 2000; Rochette Cordeiro, 2014). The form of the surface peculiarities can be analyzed and approximated using multifactor analysis, including the influence of some geophysical and geochemical factors such as temperature and mineralization of the geographical environment and also surface reactions on the solid state rock interfaces and within the pores (Velbel, 1990). A number of eastern and Asian scientific organizations (*Institute of Geology and Geophysics of Chinese Academy of Sciences [Beijing]; Laboratory of Marine Geology, MOE, Tongji University [Shanghai]; College of Geology Engineering and Geomatics, Chang'an University [Xi'an]; Aero Geophysical Survey & Remote Sensing Center for Land and Resources [Beijing]* etc.) support the projects with the elements of approximation of geophysical and geomorphological structures and processes such as potential fields and seismic migration (Sheng-Chang et al., 2001; Shengchang et al., 2001; Zhang et al., 2005; Liu et al., 2007). The main problem of the concepts addressing the possibility of approximation of the weathering forms is misunderstanding of the full multiphysical complexes of different (and qualitatively inequivalent – such as chemical and physical weathering forces) factors and counter-directional forces of a morphogenesis process in different weathering and environmental conditions and for different weathered substances. For example, multiphysical correlations for soils (Prokofiev and Dunec.,

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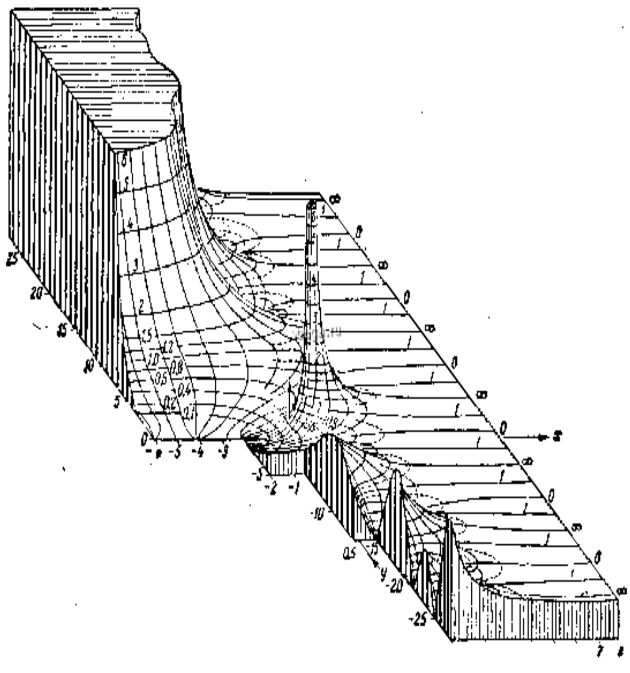
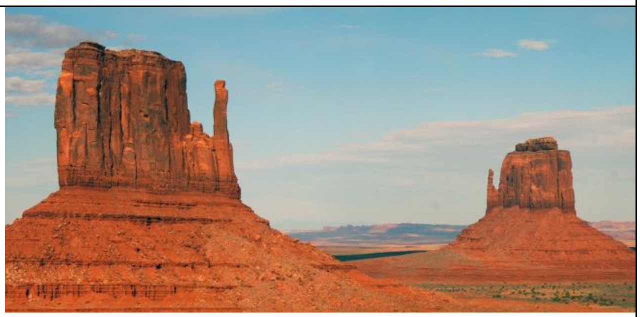
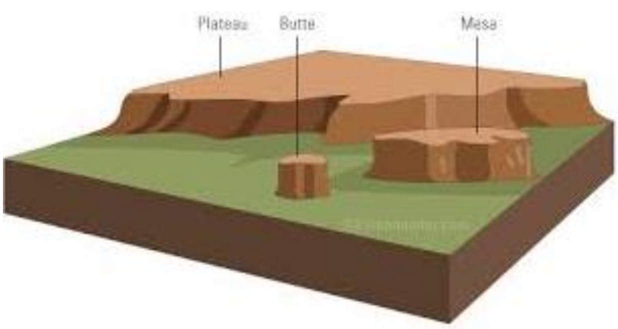
2005; Hemmati et al., 2010; Hayes et al., 2010; Navarro et al., 2014), rocks (EUROCK 2006; Guangmiao, 2006, 2007; Sheng et al., 2007; Selvadurai et al., 2011; Rutqvist and Tsang, 2012; Škarydová and Hokr, 2015; Kraishan et al., 2016), clays (Supian, 2011; Gerasch et al., 2014) and crystals (Roters et al., 2010; Yankin et al., 2013; Shizawa, 2014; Yoon et al., 2016) are different by the most key parameters / constants and variables. Consequently, approximation of the multiphysics of such different structures will be equivalent to the simulation (*ab initio* studies) of such processes, but will not be equivalent to the analytical approximations of the resulting forms using mathematical or statistical functions. But it is obvious that the computation process of the multivariable functions is very time-consuming. It seems acceptable that the resulting form of the weathering process must be interpreted as a consequent of the weathering process force approximation, and thereof, it is adequate to and correlated with the geo-environment force diagrams which can also be approximated by the analytical functions. According to the above listed considerations, we provide an attempt of su-field analytical approximation of the weathering process results using special functions for different forms of geostructures and fields (for example – colocalized morphology and geo-electric field in multiphysical modeling (Clément, Moreau, 2012)). The plausibility of our concept can be verified by the isomorphism (or other similar morphisms) between the morphological weathering process results and graphical representation or visualization of the certain special functions within the frame of comparative geomorphology.

2. Materials and methods

Methodological bases of this study include special function tables (by Jahnke E., Emde F. and Lösch F.), also known as a "Tafeln Höherer Funktionen" in original language editions (Jahnke, Emde und Lösch, 1960 [etc.]), initially provided by "B.G. Teubner Verlagsgesellsch" AFT, Leipzig. The study is based on the comparative analysis between the approximations (by some special functions from this book) and different images of the weathering structures, provided by the artificial intelligence or machine learning-assisted Web searching using "weathering"-like keywords. Some notations of such illustrations are introduced into the article body for clarification of the theoretical principles proposed and visual recognizing of similar objects in different figures.

3. Results

The results of the comparative studies are presented in Table 1.

<table>
<tr><th>Special functions</th><th>Geostructures</th></tr>
<tr><td></td><td>

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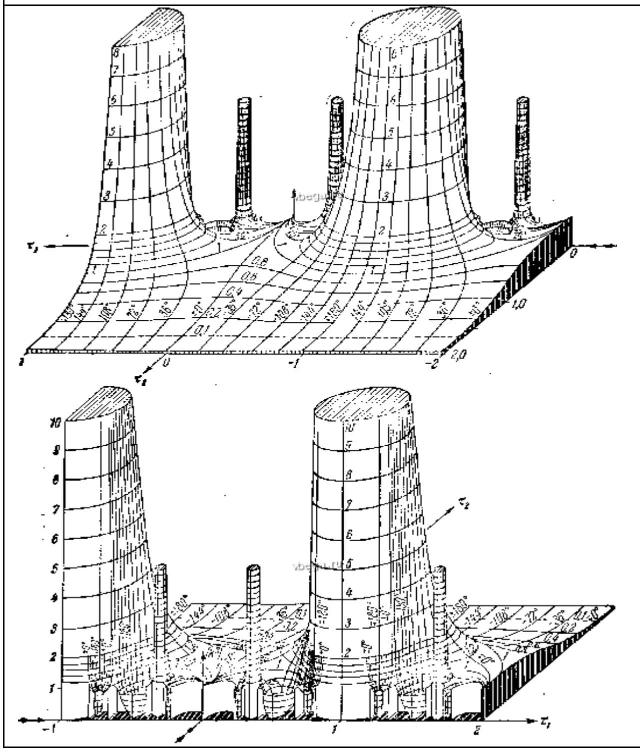
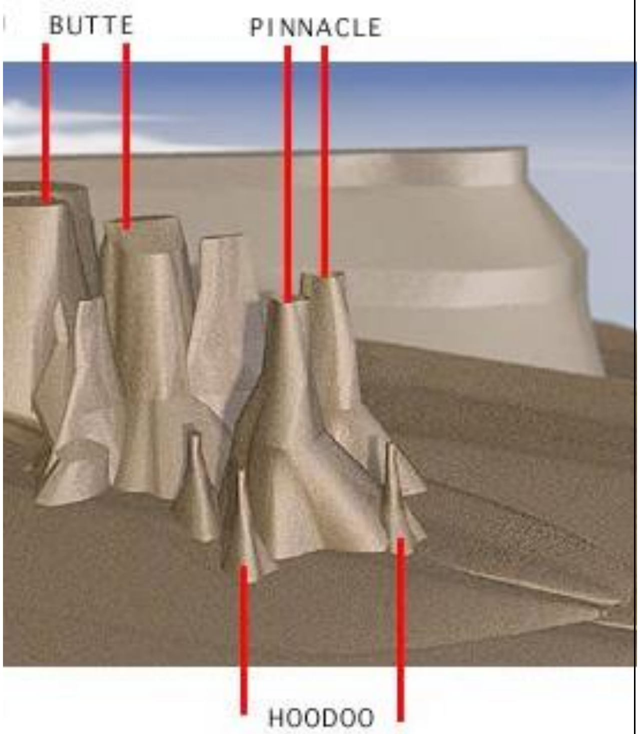
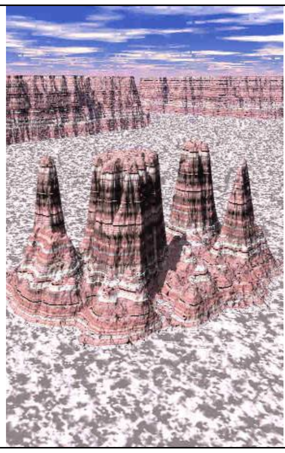
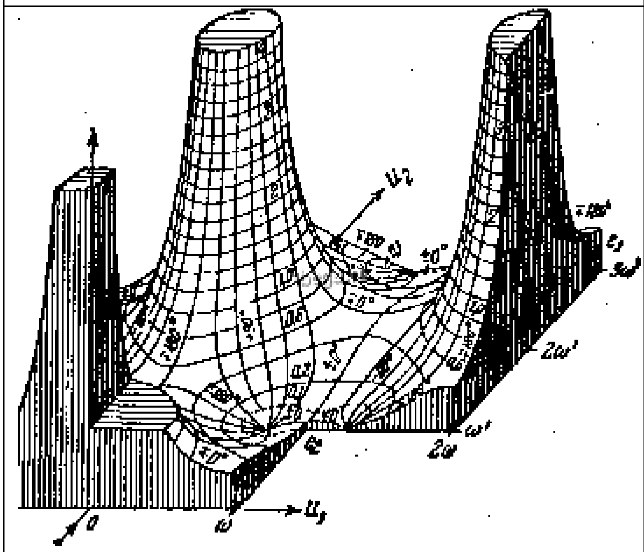
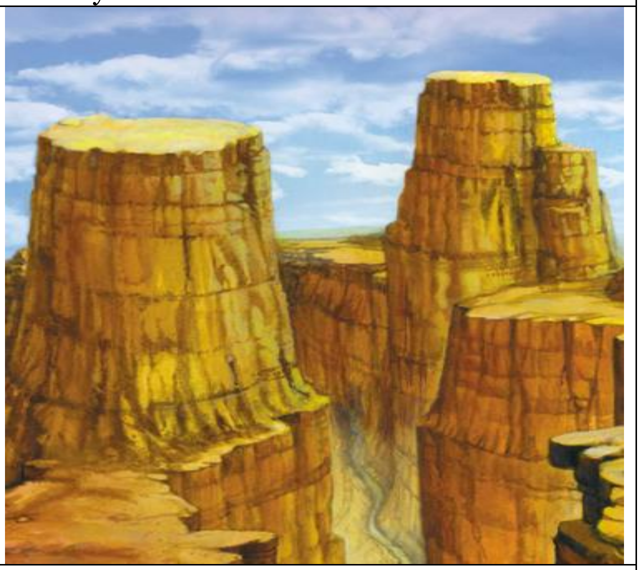
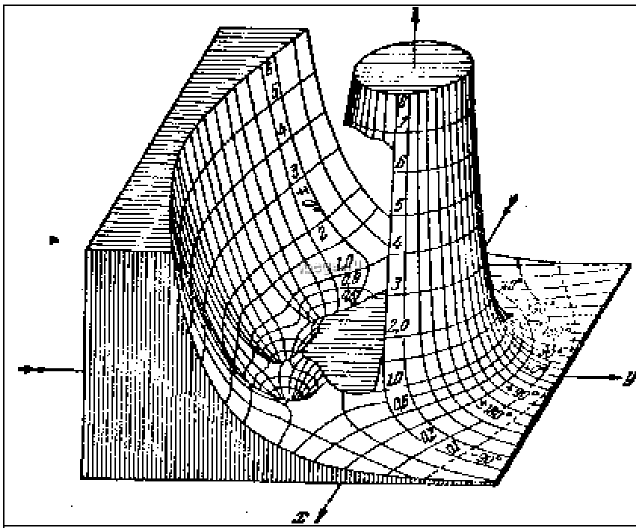
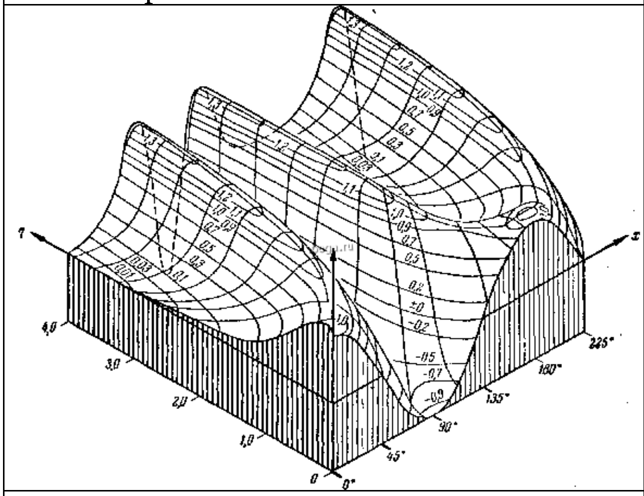
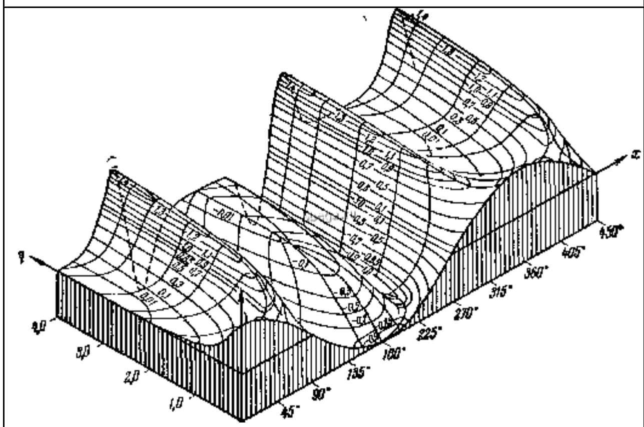
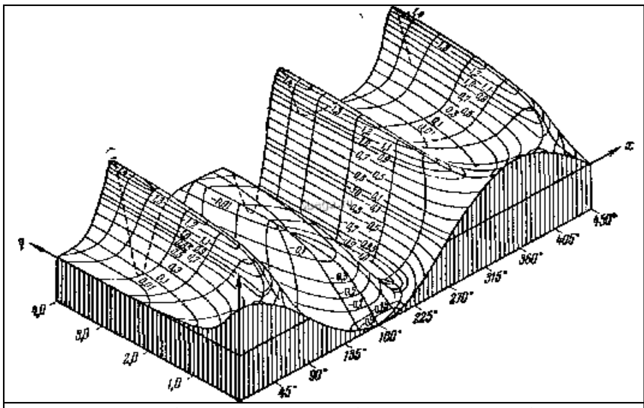
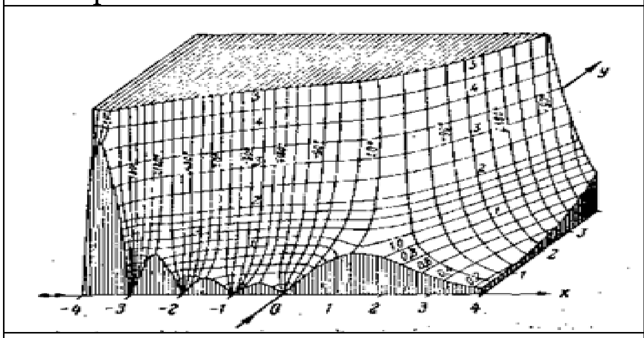
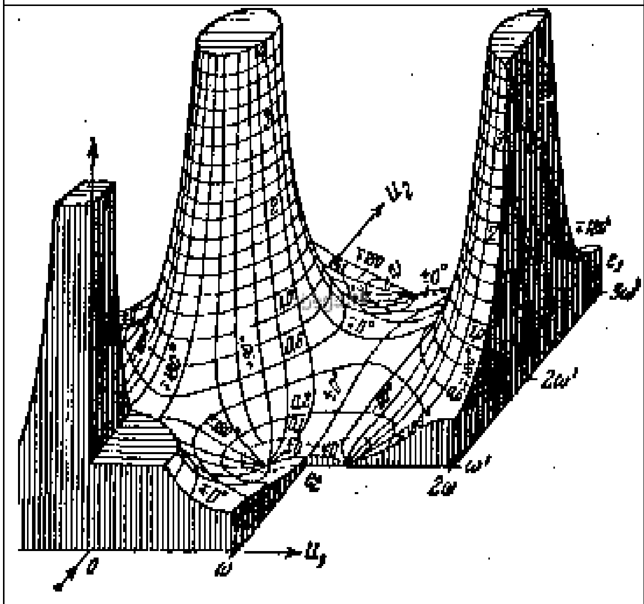
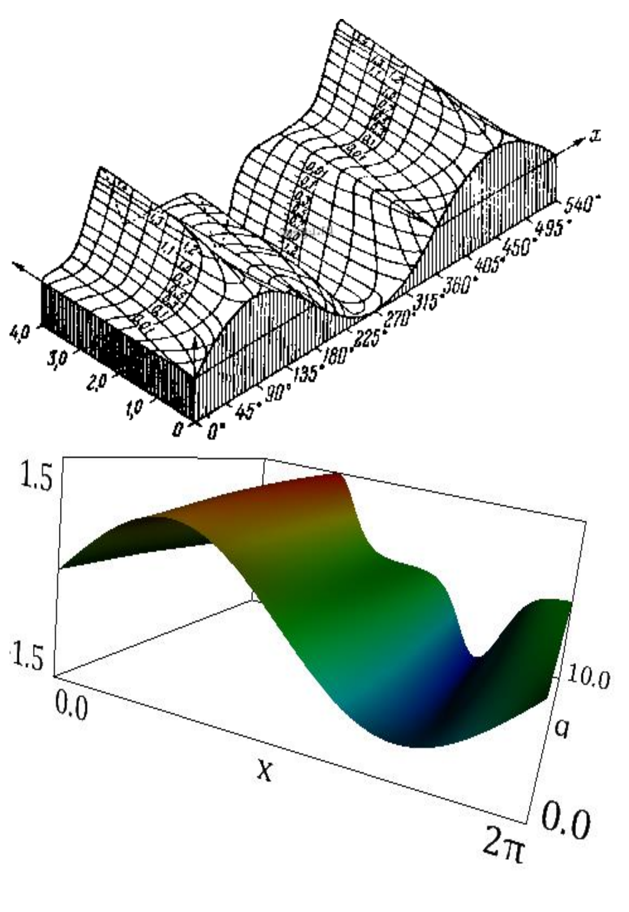
Riemann zeta function or Euler–Riemann zeta function ζ(s) is a function of a complex variable s that analytically continues the sum of the Dirichlet series: $$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^{s}}$$ when the real part of s is not greater than 1. More general representations of ζ(s) for all s are given below. The values of the Riemann zeta function at even positive integers were computed by Euler. The first of them, ζ(2), provides a solution to the Basel problem. The values at negative integer points, also found by Euler, are rational numbers and play an important role in the theory of modular forms. Many generalizations of the Riemann zeta function (Dirichlet series, Dirichlet L-functions) are known	"The Mittens" are a pair of buttes in Monument Valley, Utah. Each of these formations includes a thick tower of rock with a thin spire alongside it, making the two buttes look like a giant pair of mittens. Buttes are tall, flat-topped, steep-sided towers of rock. Buttes were created through the process of erosion, the gradual wearing away of earth by water, wind, and ice. Buttes were once part of flat, elevated areas of land known as mesas or plateaus. In fact, the only difference between a mesa and a butte is its size. Buttes are created as streams slowly cut through a mesa or plateau. The caprock, resist weathering and erosion. As a result, the formations stay about the same height as the plateau or mesa. Weathering and erosion, most often by wind and rainwater, slowly erode the softer rock surrounding the caprock. Caprock protects the more vulnerable rock beneath it
	
A modular form is a complex analytic function on the upper half-plane satisfying a certain kind of functional equation with respect to the group action of the modular group, and also satisfying a growth condition. The theory of modular forms therefore belongs to complex analysis but the main importance of the theory has traditionally been in its connections with the number theory. A modular form can equivalently be defined as a function F from the set of lattices in C to the set of complex numbers which satisfies certain conditions: I) If we consider the lattice Λ = Zα + Zz	

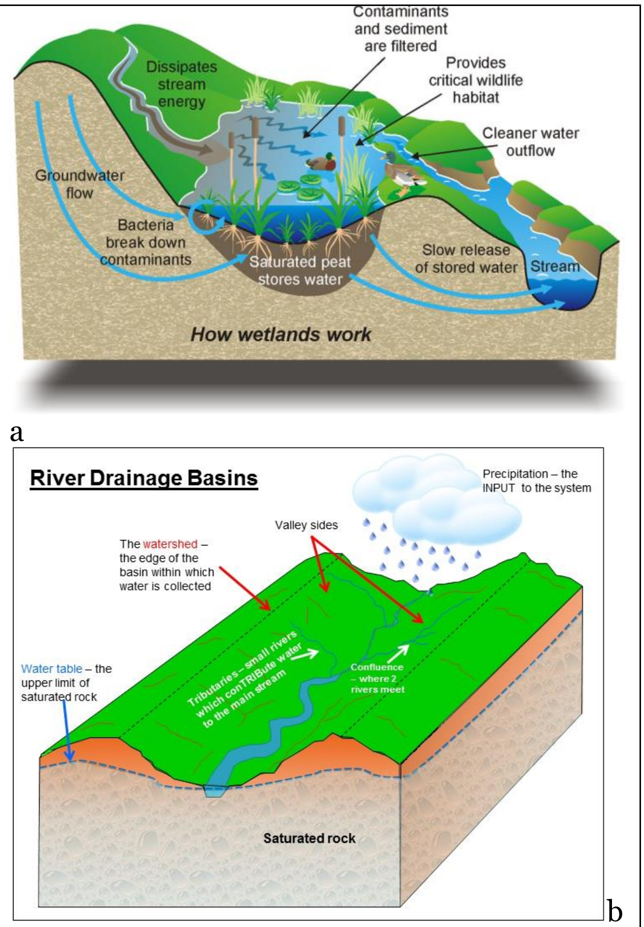
generated by a constant α and a variable z, then F(Λ) is an analytic function of z. II) If α is a non-zero complex number and αΛ is the lattice obtained by multiplying each element of Λ by α, then F(αΛ) = α−kF(Λ) where k is a constant (typically a positive integer) called the weight of the form. III) The absolute value of F(Λ) remains bounded above as long as the absolute value of the smallest non-zero element in Λ is bounded away from 0	 Monument Valley is a Arizona-Utah border tribal park famous for its towering butte sandstones. The first images below were constructed using Metatools Bryce3D (both the Windows and Mac versions), a virtual landscape generator. These and many similar images, along with numerous color photographs, will be incorporated in some books, including the bestseller "Geologic Scenery of the Southwest"
	
The example of the Jacoby function relief	The Grand Canyon landscape elements. Daniel Colon illustration from the J. O'Connor book about Grand Canyon

	
The example of the Hankel function relief	The Grand Canyon landscape elements
	
The example of the Mathieu function relief	Buttes, mesas and canyons are created after coal companies strip mine land in
	

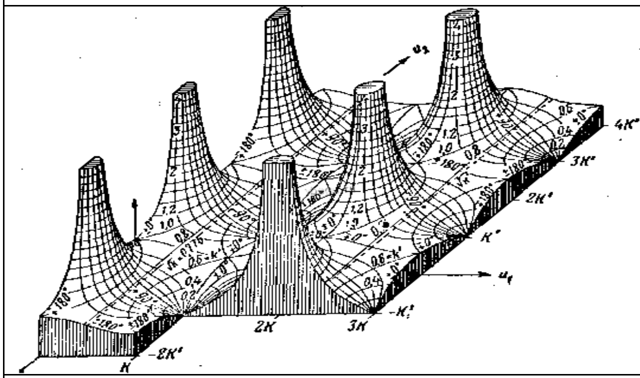
	
Examples of the Mathieu function relief	Cappadocian weathering forms
	
The map of the inversed values of the gamma function	Himalayas. Streaming weathering forms
	
The example of the Jacoby function relief	Cappadocian weathering forms



Examples of the Mathieu function relief (including NIST resource visualization https://dlmf.nist.gov/28.3)

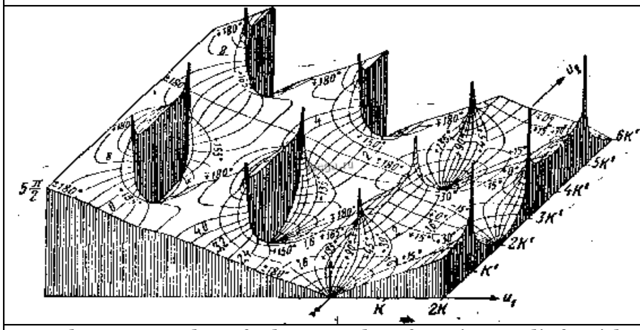


Wetlands purification diagram (a) and the water cycle as a series of processes by which water is evaporated from the sea and eventually condensed and precipitated over the land (b)



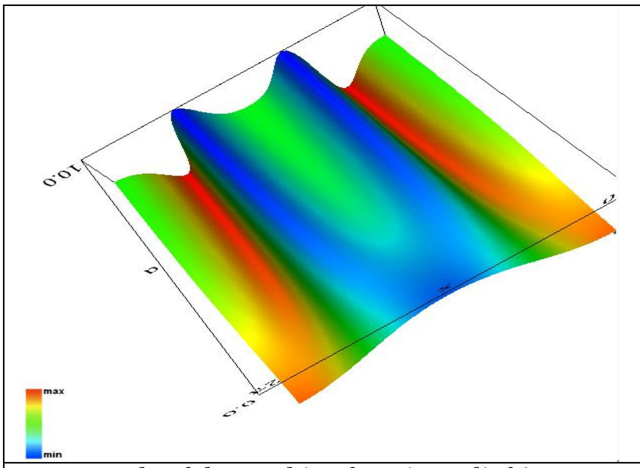
The relief of the Weierstrass function

Badlands are the examples of a naturally occurring erosion-shaped terrain



Another example of the Jacoby function relief with branching or bifurcation

Cappadocia-like landscape in the Himalayas (the weathering factor illustration)



Example of the Mathieu function relief (NIST resource graph https://dlmf.nist.gov/28.3)

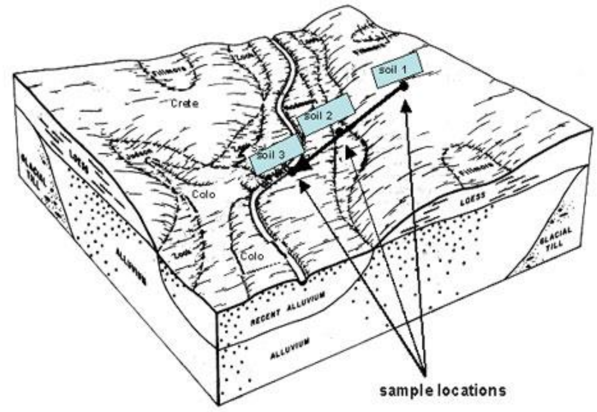
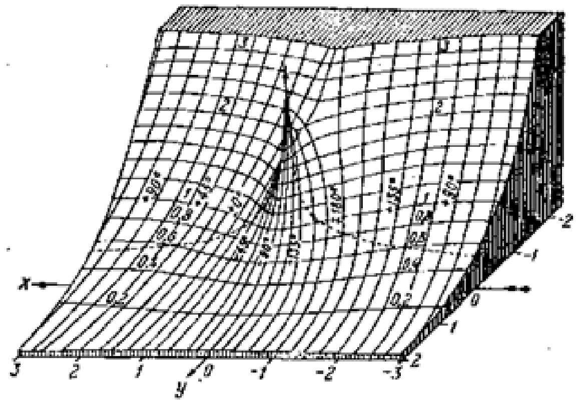
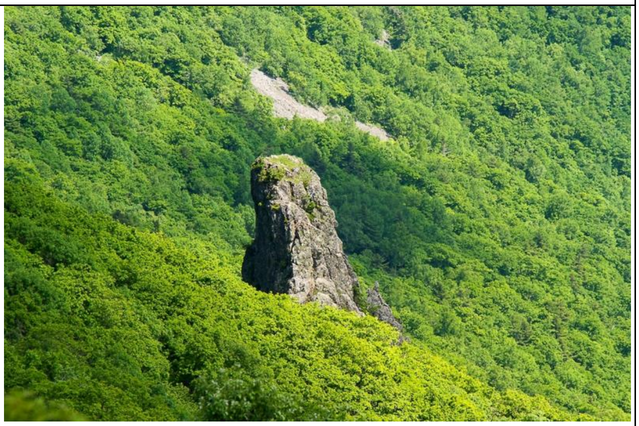


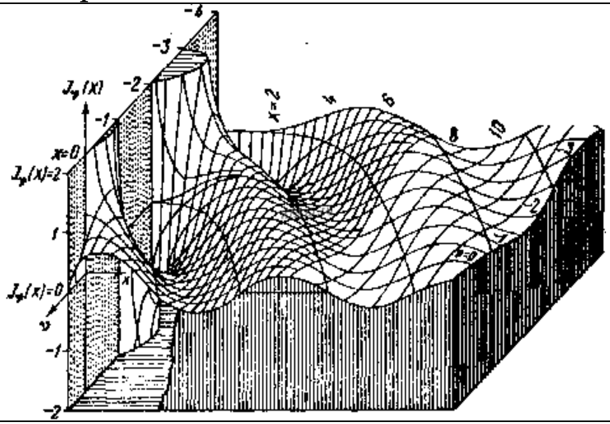
Diagram of soil and parent material distribution.
Image Court. of USDA-NRCS,
Ed. Court. of UNL



Example of the Hankel function relief

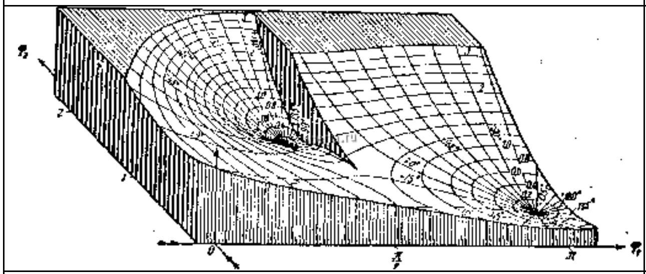


Single weathered crag on the hillside Chitinza



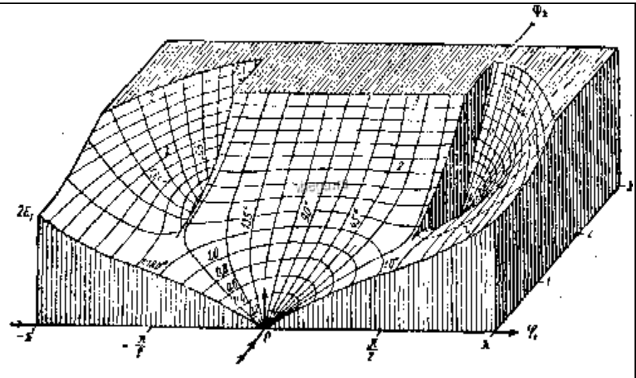
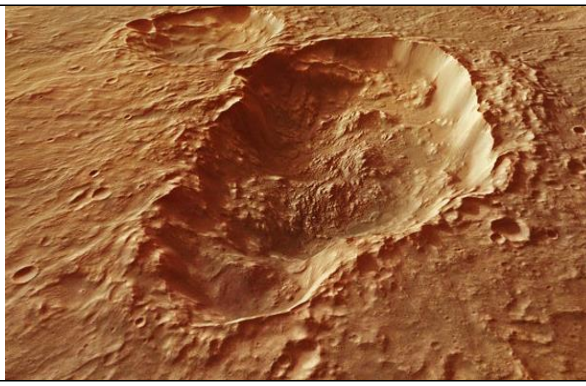
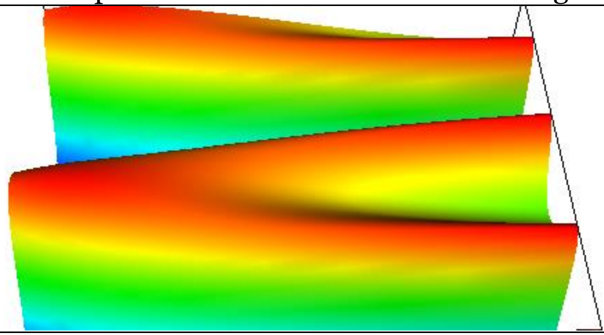
Example of the Bessel functions

Fold in the Green Pond Formation Location:
Rt 23 North Newfoundland
(NJ Fossil Web Site)



A special function with two extremums and one peculiarity ("saltation")

"Double-crater" and the two-level volcano lake with weathering water. Kelimutu, Indonesia

	
A special function with one global extremum and two peculiarities “saltations” with a “stage”	Space weathering (meteorite weathering)
	
Example of the Mathieu function relief (NIST graph https://dlmf.nist.gov/28.3.F13.webgl)	Sand dunes weathering and erosion

4. Conclusion

Some results of this computations and comparative geomorphological studies very good confirm our primary hypothesis about approximability of different types of weathering forms using different special functions, including Euler–Riemann zeta function, Jacoby function, Mathieu function, Weierstrass function, Hankel function.

5. Acknowledgements

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