New Horizons in Mathematical Physics, Vol. 2, No. 4, December 2018
https://dx.doi.org/10.22606/nhmp.2018.24003

C in Complex PT-symmetry

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Abstract We propose a model PT - symmetry operator(H) in the form of (2x2) matrix and find corresponding charge operator (C) such that it stisfies all the necessary appropriate commutation relations [H,PT]=0;[H,C]=0,[C,PT]=0 and [H,CPT]=0 in order to justify a model in complex quantum system like that to preserve the necessary conditions in Hilbert space.

Keywords: Charge conjugation matrix, $\mathcal{PT}$-symmetry matrix,commutative $\mathcal{PT}$-symmetry matrices.

PACS no-11.30 Er,03.65-w.

1 Introduction

In order to extend the horizon of understanding quantum mechanics in complex space Bender, Brody and Jones [1] introduced that all the complex $\mathcal{PT}$ invariant operators must satisfy the relations

$$[H, PT] = 0 \tag{1}$$

and

$$[H, CPT] = 0 \tag{2}$$

so that one can easily understand the properties of vectors in Hilbert spaces similar to that of Hermitian operator. Accordingly authors [1] proposed a four parameter operator in the form of (2x2) matrix as

$$H = \begin{bmatrix} re^{i\theta} & s \\ t = s & re^{-i\theta} \end{bmatrix} \tag{3}$$

and its corresponding charge conjugation operator[1](which is valid for s=t only, see erratum[1]).

$$C = \begin{bmatrix} i\tan\alpha & \sec\alpha \\ \sec\alpha & -i\tan\alpha \end{bmatrix} \tag{4}$$

where α and θ are related as

$$\sin\alpha = \frac{r}{\sqrt{st}}\sin\theta \tag{5}$$

BBJ[1] claim that H and C are **always** commutative .

$$[H, C] = \circ \tag{6}$$

We believe that the purpose of previous work [1] is to suggest CPT analysis on any non-Hermitian operator, that can include their Hamiltonian [1]

$$H_{x,p} = p^2 + x^2(ix)^\nu (\nu = i, 3, 5, 7..) \tag{7}$$

However, a critical analysis reveals that the Hamiltonian in Eq(7) when converted in to matrix form one will notice the following

$$H_{x,p} = \begin{bmatrix} H_D & H_N & ... \\ H_N & H_D & .... \\ .. & .. & ... \end{bmatrix} \tag{8}$$

Explicitly using the second quantisation formalism one will find that[2,3]

$$H_D = \frac{(2a^{\dagger}a + 1)}{2} \tag{9}$$

and

$$H_N = \frac{i^{\nu}}{(\sqrt{2})^{\nu+2}} [a + a^{\dagger}]_N^{\nu+2} \tag{10}$$

where N stands for normal ordering and

$$[a, a^{\dagger}] = i \tag{11}$$

$$a|n >= \sqrt{n}|n - 1 > \tag{12}$$

$$a^{\dagger}|n >= \sqrt{n+1}|n + 1 > \tag{13}$$

$$[a, a^{\dagger}] = i \tag{14}$$

where a stands for anihilation operator and $a^{\dagger}$ stands for the creation operator. (for details see the second quantisation algebra[2,3]). One can easily verify that diagonal terms reflect unequal real values. However ignoring this authors [1] have suggested a new PT -symmetry model, where diagonal terms are complex and non-diagonal terms are real. Anyway we prposed a model non-Hermitian matrix model ,where diagonal terms are real and non-diagonal terms are imaginary. Here our aim is to calculate the charge conjugation operator satisfying the commutative behaviour[4].

2 Model PT-symmetry

Motivated by this work we present a simple model using an arbitrary matrix as

$$H = \begin{bmatrix} 9 & 3i \\ 3i & 1 \end{bmatrix} \tag{15}$$

and notice that above model will be suited to physical Hamiltonianon understanding requirements of Hilbert spaces.. In order to show that above model is a PT-symmetric we define P as [1]

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{16}$$

and the corresponding time-reversal T can be expressed considering antilinear property as

$$T = \mathit{\Gamma} K_0 \tag{17}$$

where K_0 ia an anti-unitary and

$$\mathit{\Gamma} = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \tag{18}$$

is unitary. Further interested reader will see that $\mathit{\Gamma}^{\dagger} = \mathit{\Gamma}^{-1} = \mathit{\Gamma}^{\star}$. It is easy to verify that

$$[H, PT] = \circ \tag{19}$$

The eigenvalue relation for the above operator can be written as

$$H\mathit{\Phi} = \lambda_1 \mathit{\Phi} \tag{20}$$

$$H\mathit{\Psi} = \lambda_2 \mathit{\Psi} \tag{21}$$

Here eigenvalues of H are $\lambda_{1,2}$ and corresponding eigenfunctions are $(\mathit{\Phi}, \mathit{\Psi})$ as reflected above. This can be written as

$$H\mathit{\Phi} \to \frac{2072}{271} = \begin{bmatrix} \phi_1 = \frac{2542}{2789} \\ \phi_2 = \frac{518i}{1259} \end{bmatrix} \tag{22}$$

and

$$H\Psi \to \frac{1801}{765} = \begin{bmatrix} \psi_1 = \frac{-518i}{1259} \\ \psi_2 = \frac{2542}{2789} \end{bmatrix} \tag{23}$$

Further we notice that wave functions are normalised to unit but non-orthogonal in nature, i.e.

$$< \Phi||\Phi >= 1 \tag{24}$$

$$< \Psi||\Psi >= 1 \tag{25}$$

and

$$< \Psi|\Phi >\neq \circ; < \Phi|\Psi >\neq \circ \tag{26}$$

Hence the important relation

$$|\phi_1 >< \phi_1| + |\phi_2 >< \phi_2| \neq I \tag{27}$$

or

$$|\psi_1 >< \psi_1| + |\psi_2 >< \psi_2| \neq I \tag{28}$$

is no longer satisfied.

3 C Operator

For the above Hamiltonian (2x2 matrix) we find the charge operator (2x2 matrix) as[4]

$$C = \begin{bmatrix} \frac{765}{506} & \frac{2024i}{1785} \\ \frac{2024i}{1785} & -\frac{765}{506} \end{bmatrix} \tag{29}$$

Further we notice that

$$[H, C] = \circ \tag{30}$$

$$[C, PT] = \circ \tag{31}$$

and

$$[H, CPT] = \circ \tag{32}$$

More interestingly

$$C|\Phi >= +1|\Phi > \tag{33}$$

and

$$C|\Psi >= -1|\Psi > \tag{34}$$

4 Conclusion

Before we draw a conclusion in the present analysis, we would like to state that non-orthogonal character on wave functions in non-hermitian operator was reported earlier by Lee, Hsich, Flammia and Lee [5]. Present results are in conformity with the previous work [5]. Interestingly previous authors[5] did not say anything about complete set of eigenfunction[6], however we reflect it here. In this context we would like to state that in the case of Harmonic oscillator under simultaneous transformation[5,6] of co-ordinate and momentum wave functions are normalised, orthogonal and satisfy closure property[6,7] using perturbation theory. Hence we conclude that all $\mathcal{PT}$-invariant operators can not be member of Hilbert space preserving the behaviour of Hermiticity. However, a few of them deserve similar behaviour like that of Hermitian operator.

Acknowledgments. Author greatfully acknowledges the suggestions of the Referee.

References

1. C. M. Bender, D. C. Brody and H. F. Jones,"Complex Extension of Quantum Mechanics" Physical Review Letters. vol. 89, no. 27, pp.270401,2002 and Erratum ,vol. 92, no. 11, pp. 119902,2004.
2. B. Rath, "Energy Level Calculation through perturbation theory", Physical Review A, vol 42. no 5, pp 2520-2523, 1990.
3. B. Rath, "A correlation among orthogonality relation,variational principle and perturbation theory", Fortschritte der Physik, vol 39, no-7, pp 491-499, 1991.
4. B. Rath, "Commutative Matrices", International Journal of Mathematics, Trends and Technology, vol 60, no 3, pp 191-194, 2018.
5. Y. C. Lee, M. H. Hsich, S. T. Flammia and R. K. Lee, "Local $\mathcal{PT}$ -symmetry violates the No-Signalling Principle", Physical. Review. Letters, vol 112, pp 130414, 2014.
6. B. Rath and P. Mallick, "Energy and Wave function analysis on Harmonic Oscillator under simultaneous non-Hermitian transformation of co-ordinate and momentum", Open Physics, vol 14, pp 492-497, 2014.
7. P. Mohapatra, B. Rath and P. Mallick "Spectral analysis on Swanson's Hamiltonian", Acta Physica Polonica B, (to appear), 2018.

