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Study on Bistable Behavior of Antisymmetric Laminated Cylindrical Shells with Different Ply Numbers

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Abstract. The antisymmetrical laminated cylindrical shell is a multilayer composite structure composed of single-layer fiber reinforced composites bonded by fiber-opposing layer, and it is a kind of advanced composite structure with the characteristics of bistable which is not possessed by general shell structure. In this paper, based on the two-point loading method in the experimental study, taking T700/TDE-85 antisymmetrical laminated cylindrical shells as an example, the steady-state transition model of antisymmetrical laminated cylindrical shells is established by using ABAQUS finite element software and the numerical calculation is carried out. The bistable transformation process of antisymmetric laminated cylindrical shells is simulated. The influence of ply number on the bistable behavior of the laminated structure is studied by changing the number of layers of the opposing called laminated cylindrical shells. The results show that the critical load of steady-state transition and the second steady-state residual stress increase with the increase of layer number of laminated shells, but the second steady-state crimp radius is almost unchanged.

Keywords: Antisymmetric laminated cylindrical shell; bistable; ply number; Finite Element method

1 Introduction

Some composite structures have two different stable state properties under certain conditions, which are called bistable characteristics. The first to study the structure of bistable shells is the University of Cambridge. Daton-Lovett [1] invented the bistable composite shell in 1996. It is a new type of deployable structure, which can keep stable state during stretching and winding, that is, it has two stable states. As a new deformable structure, bistable composite structure can be converted into two stable states under the driving of external force, and can maintain a stable state without external force. As a new composite structure, it has the characteristics of bistable, which can be expanded or rolled up, which makes it have excellent mechanical properties and higher space utilization. Advanced composite structures, such as bistable structures, deployable composite structures and lattice structures, are widely used in aerospace, energy exploitation and other comprehensive fields. The bistable structure has attracted the attention of researchers from all over the world.

The antisymmetrical laminated cylindrical shell can be kept stable in both states. As shown in Figure 1, the first stable state of the cylindrical shell is on the left side, and the strain energy of the cylindrical shell has a minimum value in this state; the second stable state of the cylindrical shell is on the right side, and the strain energy of the cylindrical shell is minimal in this state. It is said that the bistable structure of antisymmetrical laminated cylindrical shells has two minimum values of strain energy during its deformation process.

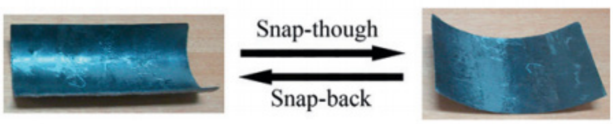


Figure 1. Two stable states of bistable laminated cylindrical shells.

The Pellegrino[2] group first proposed a linear elastic model without considering the coupling effect of tension and bending, and successfully predicted the curl radius of bistable composite structures[1]. After this, galletly and guest[3] simplified the laminated cylindrical shell into a beam model, considering the effect of torsion on the bistability of laminated cylindrical shells, and developed the study of the University of Cambridge, which successfully distinguished the bistable of the symmetric laminated cylindrical shell and the opposing symmetric laminated cylindrical shell. Then, they predicted the second steady-state radius of the antisymmetrical laminated cylindrical shells based on the "beam model" [4]. Pellegrino and Guest[5] then proposed that the bistable model previously obtained did not take into account the key factors affecting the bistable characteristics. So they put forward a "two-parametric model"-for each possible stable shape of a multi-stationary shell can be determined by two parameters, that is, the corresponding transverse radius of the cylindrical shell and the direction of the cylindrical shell. However, the model assumes that the structure has no initial stress, the neutral plane has no tension and the structure produces uniform deformation. Through the calculation and analysis of this model, the calculated results are in good agreement with the complicated theoretical analysis. Lei Yiming of Tsinghua University studied the mechanical mechanism of the folding and unfolding process of bistable structure, and gave the governing equation of the bistable structure. Based on the analysis of the strain energy of the shell structure, a simple relationship between the two steady states of antisymmetrical laminated cylindrical shells is obtained. By solving the governing equations, it is found that the bistability of antisymmetric laminated cylindrical shells is premised that the initial center angle must be greater than a certain critical value [6]. Nie Guohua and Gu Xin of Tongji University studied the mechanical properties of antisymmetrical laminated cylindrical shells and established a bistable structural mechanical model considering the coupling effect of tension and bending. At the same time, the expression of strain energy for the transition of antisymmetrical laminated cylindrical shells between two steady states is obtained[7,8]. In recent years, the University of Cambridge College of Engineering has carried out a series of studies on the bistability of laminated shells with prestressing isotropic materials and polystable composite shells with surface folds.

With the further study of the bistable deformation of laminated cylindrical shells and the combination of optical, electrical and automatic control, the bistable composites will be applied in more fields in the future [9-11]. Generally speaking, the asymmetric laminated composite structure has the bistable characteristics, the laminate shows the cylindrical bistable only when the laminated angle is $\left[0_n^o / 90_n^o\right]$. Most of the previous work focused on laminated laminates, but this has been widely studied for nearly 30 years. However, the cylindrical composite structure is bistable but not necessarily asymmetric laminated [12,13]. Of course, the antisymmetric static cylindrical shell is made of a cylindrical steel mold which is pre-loaded in its upper part. This is different from asymmetric laminated laminates, where asymmetric laminates are solidified in a press or a heater without support and then cooled to room temperature. This means that the initial dimension and curvature of antisymmetrical laminated bistable structures can be designed freely according to practical engineering needs. Therefore, it provides greater design flexibility and wider application for antisymmetric cylindrical shells. In this paper, based on the two-point loading method in the experimental study, taking T700/TDE-85 antisymmetrical laminated cylindrical shells as an example, the ABAQUS finite element software is used to establish the steady-state transition model of the anti-laminated cylindrical shell and the numerical calculation is carried out to simulate the double steady-state transition process of the opposing called laminated cylindrical shell.

2 Theoretical Analysis

The bistable composite structure studied in this paper is an antisymmetrical composite shell made of one-way T700 carbon fiber laminated from TDE-85 epoxy matrix. It is a kind of deployable space structure with good elongation, and has the mechanical behavior of small strain and large deformation. The one-way T700 carbon fiber has high strength, high elongation and good mechanical properties. The bistability of cylindrical shells formed by antisymmetrically laminated layers is achieved by using its superior performance[14].

The geometrical parameters of the composite shells studied in this paper are as follows: longitudinal length $L = 100mm$, initial transverse radius of neutral layer $R_1 = 25mm$, angle of holding $\beta = 170^o$, of layers = 5(when n is odd, ply angle of middle layer is 0) and ply angle $\alpha = \left[45^o / -45^o / 0 / 45^o / -45^o\right]$.

As shown in Figure 2, K_x and K_y are longitudinal and transverse curvatures, M_x and M_y represent the transverse and longitudinal moments of the shell edges, respectively. The X and Y axes are defined as consistent with the longitudinal and transverse directions of antisymmetric shells. The relationship between the strain and curvature of the neutral layer and the corresponding force and moment is given by the following formula:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ K \end{bmatrix} \tag{1}$$

The N, M are external loads and moments, A, B and D are laminated plate tensile stiffness matrix, coupling stiffness matrix and bending stiffness matrix, respectively. All matrices are symmetrical. According to formula (1), the A, B and D matrices of antisymmetric composite shells are given as follows:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & B_{66} \\ A_{21} & A_{22} & 0 & 0 & 0 & B_{26} \\ 0 & 0 & A_{66} & B_{61} & B_{62} & 0 \\ 0 & 0 & B_{16} & D_{11} & D_{12} & 0 \\ 0 & 0 & B_{26} & D_{21} & D_{22} & 0 \\ B_{61} & B_{62} & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ K_x \\ K_y \\ K_{xy} \end{bmatrix} \tag{2}$$

Note: $D_{16} = D_{26} = 0$, this indicates that there is no coupling between the curved item and the torsion item. So suppose torsional curvature $K_{xy} = 0$. It is also noted that the terms of tension and bending of the composite shell are coupled.

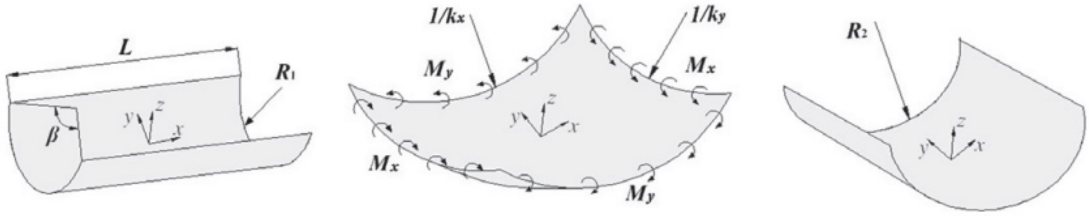


Figure 2. Geometric parameters and deformation of cylindrical shells.

The Flexural strain per unit area of a laminate's neutral layer is given by the following formula:

$$U_b = \frac{1}{2}\left[M_x K_x + M_y K_y + M_{xy} K_{xy} \right] \tag{3}$$

Notice: $K_{xy} = 0, D_{16} = D_{26} = 0$ Substituting formula (2) into formula (3) can be obtained:

$$U_b = \frac{1}{2}\left\{\left[B_{16}\gamma_{xy}^0 + D_{11}K_x + D_{12}\left(K_y - \frac{1}{R_1}\right)\right]K_x + \left[B_{26}\gamma_{xy}^0 + D_{12}K_x + D_{22}\left(K_y - \frac{1}{R_1}\right)\right]\left(K_y - \frac{1}{R_1}\right)\right\} \tag{4}$$

According to the semi-inverse form of equation (2), the shear strain of NTO neutral layer can be expressed as:

$$\gamma_{xy}^0 = A_{16}^{'} N_x + A_{26}^{'} N_y + A_{66}^{'} N_{xy} + B_{61}^{'} K_x + B_{62}^{'} K_y + B_{66}^{'} K_{xy} \tag{5}$$

including $B^{'} = -A^{-1}B$. Notice $N_x = N_y = N_{xy} = 0$, the equation (5) can be rewritten as:

$$\gamma_{xy}^0 = B_{61}^{'} K_x + B_{62}^{'} K_y \tag{6}$$

Suppose that K_x and K_y are homogeneous in the entire opposing called laminated cylindrical shell, therefore, U_b is uniform. Therefore, the expression of the longitudinal length bending strain energy of a unit of a cylindrical shell is:

$$\mathrm{U_b} = \frac{1}{2} \quad \mathrm{R_1}\left\{\left[\mathrm{B_{16}} \;{}^{0}_{\mathrm{xy}} + \mathrm{D_{11}K_x} + \mathrm{D_{12}}\left(\mathrm{K_y} - \frac{1}{\mathrm{R_1}}\right)\right]\mathrm{K_x} + \left[\mathrm{B_{26}} \;{}^{0}_{\mathrm{xy}} + \mathrm{D_{12}K_x} + \mathrm{D_{22}}\left(\mathrm{K_y} - \frac{1}{\mathrm{R_1}}\right)\right]\left(\mathrm{K_y} - \frac{1}{\mathrm{R_1}}\right)\right\} \tag{7}$$

Substituting formula (6) into formula (7) can be obtained:

$$U_b = \frac{1}{2}\beta R_1 \left[\left(D_{11} + B_{16}B_{61}^{'} \right) K_x^2 + \left(2D_{12} + B_{16}B_{62}^{'} + B_{26}B_{61}^{'} \right) K_x \left(K_y - \frac{1}{R_1} \right) + \left(D_{22} + B_{26}B_{62}^{'} \right) \left(K_y - \frac{1}{R_1} \right)^2 \right] \tag{8}$$

In Figure 2, the tensile strain energy per unit area of the neutral layer of an antisymmetric laminated cylindrical shell is given by the following formula:

$$U_s = \frac{1}{2}\left[N_x \varepsilon_x^0 + N_y \varepsilon_y^0 + N_{xy} \gamma_{xy}^0 \right] \tag{9}$$

The tensile strain energy of antisymmetric laminated cylindrical shells is given by the following formula:

$$U_s = \frac{1}{2} A_{11} \left[\left(\frac{\beta R_1}{2} \frac{K_x^2}{K_y^2} + \frac{\sin\left(\beta R_1 K_y\right)}{2} \frac{K_x^2}{K_y^3} - \frac{4\sin^2\left(\beta R_1 K_y / 2\right)}{\beta R_1} \frac{K_x^2}{K_y^4} \right) \right] \tag{10}$$

The total strain energy of per unit length antisymmetric laminated cylindrical shell can be expressed as:

$$U = U_b + U_s \tag{11}$$

For an antisymmetric laminated cylindrical shell, the bistable solution can be obtained from the minimum total strain energy. When an equilibrium solution is stable, the first derivative of U is zero, and the second derivative of U must be positive. That is

$$\frac{dU}{dK_x} = 0 \tag{12}$$

Due to the transverse curvature in the second stable state $K_y \approx 0$, the longitudinal curvature of the second stable shape K_x can be obtained by substituting $K_y \approx 0$ into the formula (12) as:

$$K_x = \frac{2D_{12} + B_{16}B_{62}^{'} + B_{26}B_{61}^{'}}{2R_1(D_{11} + B_{16}B_{61}^{'})} \tag{13}$$

The crimp radius of the second stable shape R_2 is given by the following formula:

$$R_2 = \frac{1}{K_x} \tag{14}$$

In this way, the second steady-state radius of the antisymmetric laminated cylindrical shell was obtained.

3 Basic Parameters and Modeling

3.1 Basic Parameters of the Model

A 0.185 mm thick unidirectional T700 carbon fiber antisymmetric composite shell was fabricated from TDE-85 epoxy matrix in this paper. The antisymmetric composite shells of these sizes are solidified and cooled in a cylindrical steel mold in sequence. Each layer is linear elastic and the material properties of the carbon fiber unidirectional plate are shown in Table 1. T700 carbon Fiber material parameters [11]:

Table 1. Material properties of carbon fiber unidirectional plates.

$E_{11}(GPa)$	$E_{22}(GPa)$	$G_{12}(GPa)$	$G_{13}(GPa)$	$G_{23}(GPa)$	ν_{12}	$t_{ply}(mm)$
132	10.3	6.5	6.5	3.91	0.25	0.185

The geometrical parameters of the composite shells studied in this paper are as follows: longitudinal length $L = 100mm$, initial transverse radius of neutral layer $R_1 = 25mm$, angle of holding $\beta = 170^o$, of layers = 5 (when n is odd, ply angle of middle layer is 0) and ply angle $\alpha = \left[45^o / -45^o / 0 / 45^o / -45^o \right]$.

For convenience, we can simply remember: L-R- β -nanti α _shell. For example: 100-25-170-5anti45_shell.

3.2 Establishing Finite Element Model

The finite element method is used to simulate the steady-state transition process of antisymmetric laminated shells, which involves contact and large geometric nonlinear deformation. Especially in the process from initial state deformation to extension state and then to the second different stable state, the shell produces large deformation, which leads to great changes in structural stiffness. The commercial finite element software ABAQUS has a strong ability to compute nonlinear problems and is helpful to obtain convergent finite element results. Therefore, ABAQUS is used to calculate the bistable transformation process of antisymmetric laminated shells. In order to obtain accurate results and significantly reduce the calculation cost, a four-node reduced integral shell element S4R is used for shell lamination element, and a rigid cylinder with a radius of 5 mm is used to replace the indenter for shell deformation, which is set as an analytical rigid body in the ABAQUS environment.

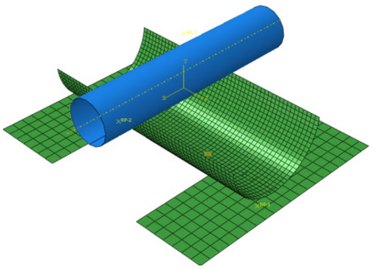


Figure 3. Finite element model for simulating the bistable behavior of antisymmetric laminated shells.

4 Result Analysis

4.1 Bistable Transformation Process

The model is imported into ABAQUS, and a 5 mm radius indenter model is created according to the two-point loading method. Constraints are applied to the supporting contact surface, and load and displacement boundary conditions are applied to the rigid cylinder. The process of the antisymmetrical laminated cylindrical shell changing from the first steady state to the second steady state is obtained. After unloading, it remains at second steady state. As shown in Figure 4.

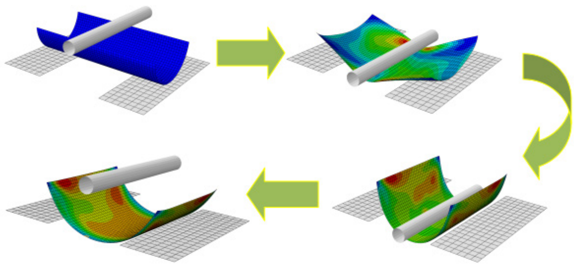
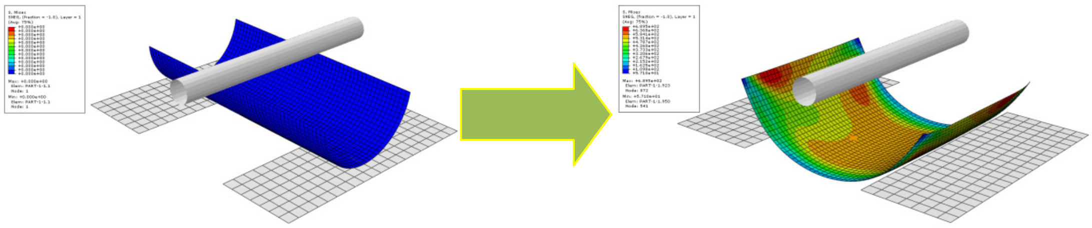


Figure 4. Steady state transformation process.

4.2 Stress Changes during Bistable Transformation

In ABAQUS, a step-by-step view of the bistable transition process is used. By observing the stress nephogram, it is easy to find that the residual stress is produced by coupling during the transition from the first steady state to the second steady state, as shown in Figure 5.



the first steady state the second steady state

Figure 5. The residual stress generated.

4.3 Influence of Layer Number on Bistable Characteristics

In order to investigate the effect of the number of layers on the bistable transition process, the load-displacement curves for different layers are given in Figure 6.

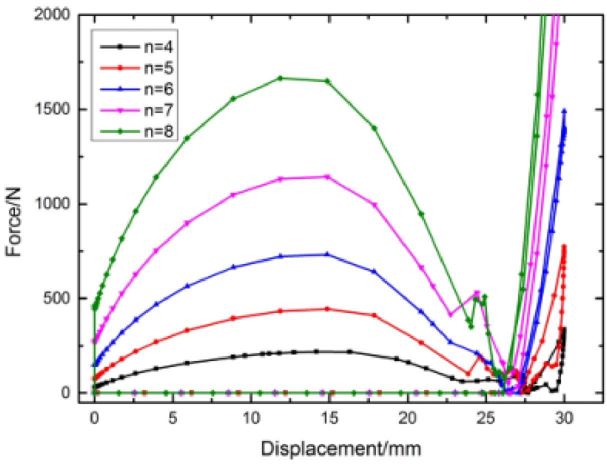
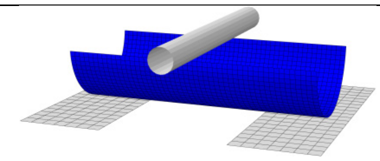
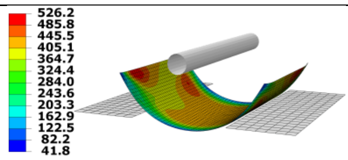


Figure 6. The load-displacement curves of 100-25-170-nanti45_shell.

Through Figure 6, we find that the load on antisymmetrical laminated cylindrical shells increases with the increase of radial displacement at the initial loading stage, and then decreases to a certain minimum after reaching the critical load (maximum value) of steady state transition. The displacement of different layers does not change much when reaching the critical load. Continuing loading found that, after a small displacement change, the load quickly increased to a peak. During the unloading phase, the initial unloading phase is similar to the post-load and the displacement varies very little, but the load is rapidly reduced from peak to zero, indicating that the model has reached the second steady state. The load remains zero and no longer changes as the displacement decreases in the late unloading period.

The load-displacement curves of a rigid cylindrical indenter are obtained by numerical simulation of the above five models, as shown in Figure 6. The load-displacement curves of a rigid cylindrical indenter are obtained by numerical simulation of the above five models, as shown in Figure 6. According to the principle of force interaction, by comparing the load-displacement curves of the five models, it is found that the critical load of steady-state transition increases gradually with the number of layers n from 4 to 8, and the displacement of the steady-state transition is almost constant.

Table 2. The Von Mises stress nephogram of with different layers.

<table>
<tr><td colspan="3">Von Mises stress nephogram</td></tr>
<tr><td>n</td><td>the first steady state</td><td>the second steady state</td></tr>
<tr><td>4</td><td></td><td>
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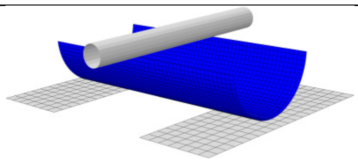
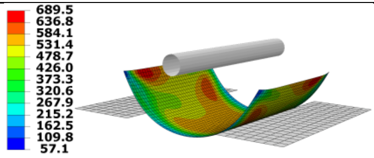
<tr><td>5</td><td></td><td>
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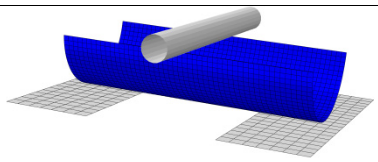
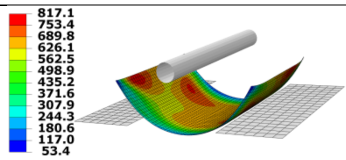
<tr><td>6</td><td></td><td>
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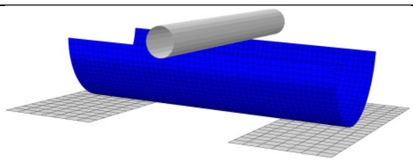
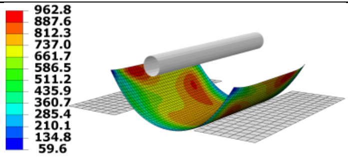
<tr><td>7</td><td></td><td>
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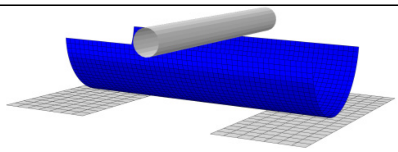
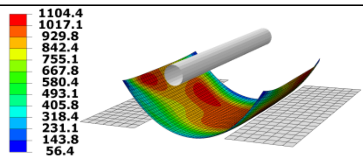
<tr><td>8</td><td></td><td>
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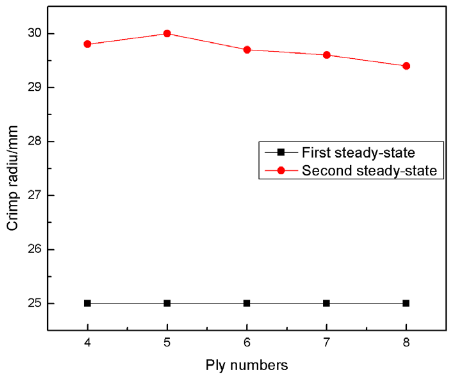

Figure 7. The steady-state curling radius of different layers.

Figure 7 shows the effect of different longitudinal lengths on the second steady-state crimp radius of cylindrical shells, from which we can see that the second steady-state crimp radii of the above five models are 29.829 mm, 30.0312 mm, 29.7718 mm, 29.7445 mm, 29.3278 mm, respectively. We find that the second steady-state crimp radius changes very little, which indicates that the number of ply layers of antisymmetrical laminated cylindrical shells has little effect on the second steady crimp radius.

5 Conclusion

An accurate bistable model of antisymmetrical laminated cylindrical shells is established by using finite element simulation software. It is found that the critical load of steady-state transition increases gradually with the number of layers n from 4 to 8, and the displacement of the steady-state transition is almost constant. It is easy to find that the residual stress is produced by coupling during the transition from the first steady state to the second steady state. It find that there is no Von Mises stress in the first steady-state model and residual stress is generated in the second steady-state model. It should notice that the maximum Von Mises stress on the neutral surface increases with the increase of the number of layers. This is the same as the change trend of critical load of steady state transition with holding angle. It finds that the number of ply layers of antisymmetrical laminated cylindrical shells has little effect on the second steady crimp radius. It has certain reference value for theoretical analysis and experimental research based on two-point loading method.

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