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# BOUNDARY-VALUE PROBLEMS WITH GENERALIZED GLUING CONDITIONS FOR A LOADED DIFFERENTIAL EQUATION WITH A PARABOLIC-HYPERBOLIC OPERATOR

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In the present work we study the unique solvability of local boundary value problems with generalized gluing conditions for the third order differential equation with a loaded parabolic-hyperbolic operator.

*Key words: loaded equation; equations of mixed type; parabolic-hyperbolic operator; integral equations; gluing condition.*



УДК 517.956

# КРАЕВЫЕ ЗАДАЧИ С ОБОБЩЕННЫМИ УСЛОВИЯМИ СКЛЕИВАНИЯ ДЛЯ НАГРУЖЕННОГО ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ С ПАРАБОЛИЧЕСКО-ГИПЕРБОЛИЧЕСКИМ ОПЕРАТОРОМ

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В настоящей работе исследуем однозначная разрешимость локальных краевых задач с обобщенными условиями склеиваниями для дифференциального уравнения третьего порядка с нагруженным параболо-гиперболическим оператором.

*Ключевые слова: нагруженное уравнения; уравнения смешанного типа; параболо-гиперболический оператор; интегральное уравнение; условия склеивание.*

## Introduction

The theory of mixed type equations is one of the principal parts of the general theory of partial differential equations. The interest for these kinds of equations arises intensively due to both theoretical and practical uses of their applications. Currently, the concept of mixed-type equations has expanded to include all possible combinations of two or three classic types of equations. In this the necessity of consideration the theory of parabolic-hyperbolic type equations for the first time was specified in 1956 by I.M.Gelfand [1].

On the other hand, recently, in connection with intensive research on problems of mathematical biology [2], optimal control of the agro-economical system [3, 4], long-term forecasting and regulating the level of ground waters and soil moisture [3], and also in the study of inverse problems [4], the numerical solution of integro-differential equations [4], the linearization of nonlinear equations [5] it has become necessary to investigate a new class of equations called "loaded equations".

Basic questions of the theory of boundary value problems for partial differential equations are the same for the boundary value problems for the loaded differential equations. However, existence of the loaded operator does not always make it possible to apply directly the known theory of boundary value problems for non-loaded partial differential equations.

Boundary value problems for mixed type equations are considered in mixed domains. To determine the solution of boundary value problem in the whole domain, the desired function and its derivatives should be glued continuously or by a special gluing condition on the lines, which separates various parts of the mixed domain. Special gluing conditions sometimes generalize continuous gluing conditions and they are used because of their physical meaning [6], [7]. Here we consider the boundary value problems (such as Tricomi) for the linear loaded differential equation of third order, with the parabolic-hyperbolic operators with the following gluing conditions

$$u(x,+0)=\alpha_1(x)u(x,-0)+\gamma_1(x), \quad 0<x<1, \tag{1}$$

$$\frac{\partial u(x,+0)}{\partial y}=\beta_1(x)\frac{\partial u(x,-0)}{\partial y}+\alpha_2(x)u(x,-0)+\gamma_2(x), \quad 0<x<1, \tag{2}$$

$\alpha_1(x)\beta_1(x)\neq 0$. This special gluing condition, is usual in the theory represents the equality of temperatures and streams on the boundary of oscillation bodies with different tenses [6], [7].

## The main results

### Statement of the problem and main functional relations

Let consider the loaded equation

$$(a\frac{\partial}{\partial x}+c)Lu=0 \tag{3}$$

$$Lu\equiv u_{xx}-\frac{1-sgny}{2}u_{yy}-\frac{1+sgny}{2}u_y-\lambda u-\mu u(x,0)\,,$$

for $(x,y) \in \Omega$, where $\Omega$ is a simple connected domain bounded by $y>0$ with segments $AA_0, BB_0, A_0B_0$, $(A(0,0),\ B(1,0), A_0(0,h), B_0(1,h))$ and by $y<0$ with characteristics $AC:$ $x+y=0,\ BC: x-y=1$ of equation (3). We let

$$\Omega_1 = \Omega \cap \{y>0\},\ \Omega_2 = \Omega \cap \{y<0\}, \quad I = \{(x,y): 0<x<1,\ y=0\}.$$

Then $a, c, \lambda, \mu$ are given real parameters in $\Omega_i$, i.e. $\lambda = (-1)^{i+1}\lambda_i, \mu = \mu_i$ for $i=1,2$ respectively and $a \neq 0$.

We investigate the following problem:

**Problem 1.** To find a regular solution of (3) from the class of functions

$$W_2 = \{u(x,y): u_x \in C(AA_0), u \in C(\bar{\Omega}_i) \cap C^1(\Omega_i \cup I \cup AC), i=1,2\};$$

satisfying boundary conditions

$$u(x,y)|_{AA_0} = \varphi_1(y),\ u(x,y)|_{BB_0} = \varphi_2(y),\ u_x(x,y)|_{AA_0} = \varphi_3(y),\ 0 \le y \le h, \tag{4}$$

$$u(x,y)|_{AC} = \psi_1(x),\ \left.\frac{\partial u(x,y)}{\partial n}\right|_{AC} = \psi_2(x),\ 0 \le x \le \frac{1}{2}, \tag{5}$$

and the gluing conditions (1) and (2), where $n$ is the interior normal, $\varphi_1(y), \varphi_2(y), \varphi_3(y), \psi_1(x)$ and $\psi_2(x)$ are given functions, such that $\varphi_1(0) = \psi_1(0)$.

**Theorem 2.1.** If the following conditions

$$\lambda_1 \ge 0, \varphi_j(y) \in C^1[0;h],\ \varphi_3(y) \in C[0;h] \cap C^1(0;h),\ j=\overline{1,2}, \tag{6}$$

$$\psi_1(x) \in C^1\left[0;\frac{1}{2}\right] \cap C^3\left(0;\frac{1}{2}\right),\ \psi_2(x) \in C\left[0;\frac{1}{2}\right] \cap C^2\left(0;\frac{1}{2}\right), \tag{7}$$

$$\alpha_1(x), \gamma_1(x) \in C^1[0;1] \cap C^3(0;1), \beta_1(x), \alpha_2(x), \gamma_2(x) \in C[0,1] \cap C^2(0;1), \tag{8}$$

are fulfilled, then Problem 1 has a unique solution.

Proof. We start by introducing the following notations:

$$u(x,+0) = \tau_1(x),\ u(x,-0) = \tau_2(x), \tag{9}$$

$$u_y(x,+0) = \nu_1(x),\ u_y(x,-0) = \nu_2(x), \tag{10}$$

$$u_{yy}(x,+0) = \mu_1(x),\ u_{yy}(x,-0) = \mu_2(x). \tag{11}$$

Then instead of (1), (2) we have

$$\tau_1(x) = \alpha_1(x)\tau_2(x) + \gamma_1(x), \tag{12}$$

$$\nu_1(x) = \beta_1(x)\nu_2(x) + \alpha_2(x)\tau_2(x) + \gamma_2(x). \tag{13}$$

Supposing

$$u(x,y) = \begin{cases} u_1(x,y),\ (x,y) \in \Omega_1, \\ u_2(x,y),\ (x,y) \in \Omega_2, \end{cases}$$

equation (3) can be represented in the form of two systems:

$$\left.\begin{array}{l} L_1u_1 \equiv u_{1xx} - u_{1y} - \lambda_1 u_1 - \mu_1 u_1 = \upsilon_1(x,y), \\ a\upsilon_{1x} + c\upsilon_1 = 0, \end{array}\right\} (x,y) \in \Omega_1, \tag{14}$$

$$\left.\begin{array}{l} L_2u_2 \equiv u_{2xx} - u_{2yy} + \lambda_2 u_2 - \mu_2 u_2 = \upsilon_2(x,y), \\ a\upsilon_{2x} + c\upsilon_2 = 0, \end{array}\right\} (x,y) \in \Omega_2, \tag{15}$$

where $\upsilon_1(x,y)$, $\upsilon_2(x,y)$ arbitrary continuous functions.

By virtue of the second representation, system (15) are reduced to the following form in the domain $\Omega_2$

$$L_2 u = w_2(y) \exp\left(-\frac{c}{a}x\right), \tag{16}$$

and to change of variables $\xi = x + y, \eta = x - y$, we get

$$u_{2\xi\eta} + \frac{\lambda_2}{4}u_2 - \frac{\mu_2}{4}u_2\left(\frac{\xi+\eta}{2}, 0\right) = \frac{1}{4}w_2\left(\frac{\xi-\eta}{2}\right) \times$$

$$\times \exp\left(-\frac{c(\xi+\eta)}{2a}\right), \tag{17}$$

boundary-value conditions (5) are reduced to the form

$$u_2|_{\xi=0} = \psi_1\left(\frac{\eta}{2}\right), \ \ 0 \le \eta \le 1, \tag{18}$$

and

$$\left.\frac{\partial u_2}{\partial \xi}\right|_{\xi=0} = \frac{1}{\sqrt{2}}\psi_2\left(\frac{\eta}{2}\right), \ \ 0 < \eta < 1. \tag{19}$$

It is know that every regular solution of equation (17) in $\Omega_2$, with the boundary condition (18), and

$$\left(u_{2\xi} - u_{2\eta}\right)\big|_{\eta=\xi} = \nu_2(\xi), \ \ 0 < \xi < 1, \tag{20}$$

(problem Darboux) is represented as

$$u(x,y) = \int\limits_0^{x+y} \nu_2(\xi) J_0\left[\sqrt{\lambda_2\left(\xi - x - y\right)\left(\xi - x + y\right)}\right] d\xi +$$

$$+\psi_1(0) J_0\left[\sqrt{\lambda_2\left(x^2 - y^2\right)}\right] + \frac{1}{2}\int\limits_0^{x-y} \psi_1'\left(\frac{\eta}{2}\right) B\left(0, \eta;\ x+y, x-y\right) d\eta + \tag{21}$$

$$+\frac{1}{4}\int\limits_0^{x+y} d\xi \int\limits_{\xi}^{x-y} B\left(\xi, \eta;\ x+y, x-y\right)\left(\mu_2\tau_2\left(\frac{\xi+\eta}{2}\right) + w_2\left(\frac{\xi-\eta}{2}\right)\exp\left(-\frac{c(\xi+\eta)}{2a}\right)\right) d\eta,$$

were $B(\xi,\eta;x+y,x-y)$ is the Riemann-Hadamard function [9], $J_0[z]$ is the Bessel function[10].

Using boundary condition (19), we take into consideration the property of $B(\xi,\eta;\xi_1,\eta_1)$ [8] we get

$$\mu_2\int\limits_0^\eta \tau_2\left(\frac{t}{2}\right)dt+\int\limits_0^\eta w_2\left(-\frac{t}{2}\right)\exp\left(-\frac{c}{2a}\right)tdt=2\sqrt{2}\psi_2\left(\frac{\eta}{2}\right)+$$

$$+\lambda_2\int\limits_0^\eta \psi_1\left(\frac{t}{2}\right)dt-2\psi_1'(0)-4\nu_2(0)-\lambda_2\eta\psi_1(0), \tag{22}$$

Differentiating (22) with respect to $\eta$, taking account of $\nu_2(0)=u_{2y}(0,0)=u_{1y}(0,0)=\varphi_1'(0)$ and $\varphi_1'(0)=\frac{1}{2}\left[\sqrt{2}\psi_2(0)-\psi_1'(0)\right]$, we find the function $w_2(-\frac{\eta}{2})$:

$$w_2\left(-\frac{\eta}{2}\right)=\{-\mu_2\tau_2\left(\frac{\eta}{2}\right)+\sqrt{2}\psi_2'\left(\frac{\eta}{2}\right)+\lambda_2\psi_1\left(\frac{\eta}{2}\right)-\lambda_2\psi_1(0)\}\exp\left(\frac{c}{2a}\right)\eta, \tag{23}$$

where $w_2\left(-\frac{\eta}{2}\right)$ in $-\frac{1}{2}\le-\frac{\eta}{2}\le 0$. As in characteristic triangle be realized inequality $0\le\xi\le\eta$, then $-\frac{1}{2}\le\frac{\xi-\eta}{2}\le 0$. Therefore in place of $w_2\left(-\frac{\eta}{2}\right)$ we can take $w_2\left(\frac{\xi-\eta}{2}\right)$. Substituting into (21) expression of $w_2\left(\frac{\xi-\eta}{2}\right)$ with regards (23) and

$$u(\xi,\eta)|_{\eta=\xi}=\tau_2(\xi),$$

and after some transformation we find main functional relation between the function $\tau_2(x)$ and $\nu_2(x)$ on $AB$ in the domain $\Omega_2$:

$$\tau_2(x)=\int\limits_0^x \nu_2(t)J_0\left[\sqrt{\lambda_2}(x-t)\right]dt+M(x)+ \tag{24}$$

$$+\frac{\mu_2}{2}\int\limits_0^x dt\int\limits_t^y J_0\left[\sqrt{\lambda_2(t-x)(s-x)}\right]\left[\tau_2\left(\frac{t+s}{2}\right)-\tau_2\left(\frac{s-t}{2}\right)\exp\left(-\frac{c}{2a}(t+s)\right)\right]ds,$$

where

$$M(x)=2\psi_1\left(\frac{x}{2}\right)-\psi_1(0)J_0\left[\sqrt{\lambda_2}x\right]-\lambda_2\int\limits_0^x x\bar{J}_1\left[\sqrt{\lambda_2 x(x-t)}\right]\psi_1\left(\frac{t}{2}\right)dt+ \tag{25}$$

$$+\frac{1}{2}\int\limits_0^x dt\int\limits_t^x J_0\left[\sqrt{\lambda_2(t-x)(s-x)}\right]\left[\sqrt{2}\psi_2'\left(\frac{s-t}{2}\right)+\lambda_2\psi_1\left(\frac{s-t}{2}\right)-\lambda_2\psi_1(0)\right]e^{-\frac{ct}{a}}ds.$$

Hence, after some transformations we have:

$$\tau_2(x)-\mu_2\int\limits_0^x \Pi(x,t)\tau_2(t)dt=\int\limits_0^x \nu_2(t)J_0\left[\sqrt{\lambda_2}(x-t)\right]dt+M(x), \tag{26}$$

where

$$\Pi(x,t)=\begin{cases}\int\limits_0^t J_0\sqrt{\lambda_2(s-x)\,(2t-s-x)}dt+\int\limits_0^{x-2t} J_0\left[\sqrt{\lambda_2(s-x)(2t+s-x)}\right]*\\ *\exp\left(-\frac{c}{a}(t+s)\right)dt,\quad 0<t\leq\frac{x}{2},\\ \int\limits_{2t-x}^{t} J_0\left[\sqrt{\lambda_2(s-x)(2t-s-x)}\right]dt,\quad \frac{x}{2}\leq t<x.\end{cases}$$

Representation (26) is the main functional relation in $\Omega_2$. Present we need to get second relation of betweenness these functions. To this end equation (3) for $y>0$, bearing in mind (14) rewrite in view of

$$L_1u_1=w_1(y)\exp\left(-\frac{c}{a}x\right),\tag{27}$$

Passing to the limit in (27) at $y\to+0$ taking into consideration necessary conditions problem, (9), (10), [8] we have:

$$\tau_1''(x)-\nu_1(x)-(\lambda_1+\mu_1)\,\tau_1(x)=w_1(0)\exp\left(-\frac{c}{a}x\right),\tag{28}$$

where $w_1(0)$ is an unknown constant to be defined. Equality (28) is the second functional relation between $\tau(x)$ and $\nu(x)$, transferred from the domain $\Omega_1$ to AB.

## Uniqueness and existence of the solution

From (26) and (28) bearing in mind (12), (13), taking account of

$$\tau_2(0)=\psi_1(0),\,\tau_2(x)=\psi_1(0)+\int\limits_0^x\tau_2'(t)dt,\,\tau_2'(0)=\frac{\varphi_3(0)}{\alpha_1(0)}-\frac{\alpha_1'(0)}{\alpha_1(0)}\psi_1(0)-\frac{\gamma_1'(0)}{\alpha_1(0)},\tag{29}$$

using integration by parts and after some transformations we have

$$\tau_2'(x)-\int\limits_0^x\Pi_1(x,t)\tau_2'(t)dt=w_1(0)M_2(x)+M_1(x),\tag{30}$$

in which

$$\Pi_1(x,t)=\frac{\beta_1(x)}{\alpha_1(x)}\{1-\frac{(\alpha_1(t)\beta_1(t))'}{\beta_1^2(t)}J_0\left[\sqrt{\lambda_2}(x-t)\right]+\sqrt{\lambda_2}\frac{\alpha_1(t)}{\beta_1(t)}J_1\left[\sqrt{\lambda_2}(x-t)\right]-$$

$$-\int\limits_t^x\left[\frac{J_0\left[\sqrt{\lambda_2}(x-s)\right]}{\beta_1(s)}(\alpha_1''(s)-(\lambda_1+\mu_1)\,\alpha_1(s)-\alpha_2(s))+\mu_2\Pi(x,s)\right]ds\},\tag{31}$$

$$M_1(x)=\frac{\beta_1(x)}{\alpha_1(x)}\{\frac{J_0\left[\sqrt{\lambda_2}x\right]}{\beta_1(0)}\left(\varphi_3(0)-\gamma_1'(0)-\alpha_1'(0)\psi_1(0)\right)-M(x)+$$

$$+\psi_1(0)\left(1-\int\limits_0^x\left[\frac{J_0\left[\sqrt{\lambda_2}(x-t)\right]}{\beta_1(t)}(\alpha_1''(t)-(\lambda_1+\mu_1)\,\alpha_1(t)-\alpha_2(t))+\mu_2\Pi(x,t)\right]dt\}\right)+$$

$$-\int_0^x \left[\frac{J_0\left[\sqrt{\lambda_2}(x-t)\right]}{\beta_1(t)}(\gamma_1''(t)-(\lambda_1+\mu_1)\gamma_1(t)-\gamma_2(t))\right]dt,$$

$$M_2(x)=\frac{\beta_1(x)}{\alpha_1(x)}\int_0^x \frac{J_0\left[\sqrt{\lambda_2}(x-t)\right]}{\beta_1(t)}\exp\left(-\frac{c}{a}t\right)dt,$$

(30) is the second kind Volterra type integral equation. From the representations of the functions $\Pi_1(x,t)$, and $M_i(x)$ applying known properties of the Bessel function [10], taking account of (6)-(8) and based on the general theory of integral equations, one can easily be sure that (30) has a unique solution, which is represented as

$$\tau_2(x)=M_1^*(x)+w_1(0)M_2^*(x), \tag{32}$$

where

$$M_1^*(x)=\psi_1(0)+\int_0^x\left(M_1(t)+\int_0^t R(t,s)M_1(s)ds\right)dt,$$

$$M_2^*(x)=\int_0^x\left[M_2(t)+\int_0^t R(t,s)M_2(s)ds\right]dt,$$

where $R(x,t)$ is the resolvent of the kernel $\Pi_1(x,t)$. Hence, by virtue of the condition (12), $\tau_1(1)=\varphi_2(0)$, with respect to $\alpha_1(1)\neq 0$, $w_1(0)$ are determined uniquely.
Since the Problem 1 reduced to the equivalent integral equation (30), from the unique solvability of the equation (30) we can conclude that Problem 1 has a unique solution. The solution $u_2(\xi,\eta)$ of the Problem 1 in $\Omega_2$ is determined by the formula (21), where the function $\tau_2(x)$ can be defined by (33) and the function $\nu_2(x)$ using the representation of the integral operator $C_{0x}^{0,\sqrt{\lambda}}$[12], by the formula (26).

For determination function $u_1(x,y)$ in domain $\Omega_1$ problem 1 reduce to problem: (4),

$$u_1(x,0)=\tau_1(x),$$

for equation

$$\left(a\frac{\partial}{\partial x}+c\right)\left(u_{1xx}-u_{1y}-\lambda_1 u_1\right)=a\mu_1\tau_1'(x)+c\mu_1\tau_1(x). \tag{33}$$

Introduce new unknown function $\upsilon(x,y)$, by inversion formula

$$u_1(x,y)=e^{-\lambda_1 y}\upsilon(x,y). \tag{34}$$

Problem in this equivalent way reduced to Problem A:

$$\left(a\frac{\partial}{\partial x}+c\right)(\upsilon_{xx}-\upsilon_y)=F(x,y), \tag{35}$$

$$\upsilon(0,y)=e^{\lambda_1 y}\varphi_1(y),\ \ \upsilon(1,y)=e^{\lambda_1 y}\varphi_2(y),$$

$$\upsilon_x(0,y) = e^{\lambda_1 y}\varphi_3(y),$$

$$\upsilon(x,0) = \tau_1(x),$$

here $F(x,y) = -a\mu_1 e^{\lambda_1 y}\tau_1'(x)c\mu_1 e^{\lambda_1 y}\tau_1(x)-$ well-known function.

Unique solvability problem A proved in [11, §2, chapter 4]. We can conclude from these that $u(x,y) \in C(\bar{\Omega}_i) \cap C^1(\Omega_i \cup I) \cap C^{3,1}(\Omega_1) \bigcap C^{3,2}(\Omega_2)$, i.e., there exists a regular solution of Problem 1.

Analogously, we can investigate the following problem:

**Problem 2.** To find a regular solution (3) from the class of functions

$$W_2 = \{u(x,y) : u_x \in C(AA_0), u \in C(\bar{\Omega}_i) \cap C^1(\Omega_i \cup I \cup BC), i = 1,2\}, \tag{36}$$

satisfying boundary conditions (4),

$$u(x,y)|_{BC} = \tilde{\psi}_1(x), \ \left.\frac{\partial u(x,y)}{\partial n}\right|_{BC} = \tilde{\psi}_2(x), \ \frac{1}{2} \le x \le 1, \tag{37}$$

together with the gluing conditions (1) and (2), where $n$ is the interior normal, $\varphi_1(y)$, $\varphi_2(y)$, $\varphi_3(y)$, $\tilde{\psi}_1(x)$ and $\tilde{\psi}_2(x)$ are given functions, moreover $\varphi_2(0) = \tilde{\psi}_1(0)$, $\alpha_1(x)\beta_1(x) \neq 0$.

Similarly as in Theorems 2.1, we can obtain the following expression.

**Theorem 2.2.** Let $\alpha_1(x), \gamma_1(x) \in C^1[0;1] \cap C^3(0;1)$, $\beta_1(x), \alpha_2(x), \gamma_2(x) \in C^1[0,1] \cap C^2(0;1)$,

$$\varphi_j(y) \in C^1[0;h], \ \varphi_3(y) \in C[0;h] \cap C^1[0;h), \ j = \overline{1,2},$$

$$\tilde{\psi}_1(x) \in C^1\left[\frac{1}{2};1\right] \cap C^3\left(\frac{1}{2};1\right), \ \tilde{\psi}_2(x) \in C\left[\frac{1}{2};1\right] \cap C^2\left(\frac{1}{2};1\right),$$

then there exists a unique solution to the Problem 2 in the domain $\Omega$.

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