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TRANSLATION OF BIPOLAR VALUED FUZZY SUBHEMIRING OF A HEMIRING

N. Selvak Kumaraen[1], K. Arjunan[2] & B. Ananth[3]

[1]*Associate Professor, Department of Mathematics, GRT Institute of Engineering and Technology, Tiruttani, Thiruvallur, Tamil Nadu, India*

[2]*PG and Research, Department of Mathematics, Alagappa Government Arts College, Karaikudi, Tamil Nadu, India*

[3]*Department of Mathematics, H.H.The Rajah's College, Pudukkottai, Tamil Nadu, India*

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ABSTRACT

In this paper, some definitions and new Theorems of a bipolar valued fuzzy subhemiring of a hemiring are presented. Using the definition of translation of bipolar valued fuzzy subhemiring of a hemiring, union, intersection and translation Theorems are introduced.

KEYWORDS: *Bipolar Valued Fuzzy Subset, Image, Preimage, Bipolar Valued Fuzzy Subhemiring, Translation*

INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [15]. Since its inception, the theory of fuzzy sets has developed in many directions and is finding applications in a wide variety of fields. In [5] Rosenfeld used this concept to develop the theory of fuzzy groups of a group. In fact, many basic properties in group theory are found to be carried over to fuzzy groups. Lee [9] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Anitha.M.S., Muruganantha Prasad & K. Arjunan[1] defined as bipolar valued fuzzy subgroups of a group. In this paper, we introduce the concept of bipolar valued fuzzy translation of bipolar valued fuzzy subhemirings of a hemiring. Using these concepts, some results are established.

1. PRELIMINARIES

1.1. Definition

A bipolar valued fuzzy set (BVFS) of X is defined as an object of the form $A = \{ < x, A^+(x), A^-(x) > / x \in X\}$, where $A^+ : X \to [0, 1]$ and $A^- : X \to [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

1.2. Example

A = { < x, 0.8, –0.6 >, < y, 0.7, –0.5 >, < z, 0.9, –0.4 >} is a bipolar valued fuzzy subset of X = {x, y, z }.

1.3. Definition

Let S be a hemiring. A bipolar valued fuzzy subset B of S is said to be a bipolar valued fuzzy subhemiring of S (BVFSHR) if the following conditions are satisfied,

- $B^{+}(x+y) \geq \min\{ B^{+}(x), B^{+}(y) \}$
- $B^{+}(xy) \geq \min\{ B^{+}(x), B^{+}(y) \}$
- $B^{-}(x+y) \leq \max\{ B^{-}(x), B^{-}(y) \}$
- $B^{-}(xy) \leq \max\{ B^{-}(x), B^{-}(y) \}$ for all x and y in S.

1.4. Example:

Let S = Z_3 = { 0, 1, 2 } be a hemiring with respect to the ordinary addition and multiplication. Then A = { < 0, 0.8, – 0.9 >, < 1, 0.6, –0.8 >, < 2, 0.6, – 0.8 > } is a bipolar valued fuzzy subhemiring of S.

1.5. Definition

Let X and Y be any two sets. Let f : X→ Y be any function and let A be a bipolar valued fuzzy subset in X, V be a bipolar valued fuzzy subset in f(X) = Y, defined by $V^{+}(y) = \sup_{x\in f^{-1}(y)} A^{+}(x)$ and $V^{-}(y) = \inf_{x\in f^{-1}(y)} A^{-}(x)$, for all x in X and y in Y. A is called a preimage of V under f and is defined as $A^{+}(x) = V^{+}(f(x))$, $A^{-}(x) = V^{-}(f(x))$ for all x in X and is denoted by $f^{-1}(V)$.

1.6. Definition

Let A = $\langle A^{+}, A^{-} \rangle$ be a bipolar valued fuzzy subset of X and α in [0, 1– sup { $A^{+}(x)$}], β in [–1– inf { $A^{-}(x)$}, 0]. Then T = $\langle T^{+}, T^{-} \rangle$ is called a bipolar valued fuzzy translation of A if $T^{+}(x) = {T^{+A}}_{\alpha_i}(x) = A^{+}(x)+ \alpha$, $T^{-}(x) = T_{\beta}^{-A}(x) = A^{-}(x) + \beta$, for all x in X.

1.7 Example

Consider the set X = { 0, 1, 2, 3, 4 }. Let A = { (0, 0.5, –0.1), (1, 0.4, –0.3), (2, 0.6, –0.05), (3, 0.45, –0.2), (4, 0.2, –0.5) } be a bipolar valued fuzzy subset of X and α = 0.1, β = –0.1. Then the bipolar valued fuzzy translation of A is T = $T^{A}_{(0.1, -0.1)}$ = { (0, 0.6, –0.2), (1, 0.5, –0.4), (2, 0.7, –0.15), (3, 0.55, –0.3), (4, 0.3, –0.6) }.

2. PROPERTIES

2.1. Theorem

If M and N are two bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A of a hemiring R, then their intersection M∩N is also a bipolar valued fuzzy translation of A.

Proof: Let x and y belong to R. Let M = $T^{A}_{(\alpha,\beta)}$ = { ⟨ x, $A^{+}(x) + \alpha$, $A^{-}(x) + \beta$ ⟩ / x∈R } and N = $T^{A}_{(\gamma,\delta)}$ = { ⟨ x, $A^{+}(x) + \gamma$, $A^{-}(x) + \delta$ ⟩ / x∈R } be two bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A = ⟨ A^{+}, A^{-} ⟩ of R. Let C = M∩N and C = { ⟨ x, $C^{+}(x)$, $C^{-}(x)$ ⟩ / x∈R }, where $C^{+}(x)$ = min { $A^{+}(x) + \alpha$, $A^{+}(x) + \gamma$ } and $C^{-}(x)$ = max { $A^{-}(x) + \beta$, $A^{-}(x)+\delta$ }.

Case (i): $\alpha \le \gamma$ and $\beta \le \delta$. Now $C^{+}(x)$ = min{$M^{+}(x)$, $N^{+}(x)$ } = min{$A^{+}(x)+\alpha$, $A^{+}(x)+\gamma$}= $A^{+}(x)+\alpha$ = $M^{+}(x)$ for all x in R. And $C^{-}(x)$ = max { $M^{-}(x)$, $N^{-}(x)$ } = max { $A^{-}(x) + \beta$, $A^{-}(x) + \delta$ }= $A^{-}(x) + \delta$ = $N^{-}(x)$ for all x in R. Therefore C = $T^{A}_{(\alpha,\delta)}$ = { ⟨ x, $A^{+}(x) + \alpha$, $A^{-}(x)+ \delta$ ⟩ / x∈R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R.

Case (ii): $\alpha \ge \gamma$ and $\beta \ge \delta$. Now $C^{+}(x)$ = min { $M^{+}(x)$, $N^{+}(x)$ } = min { $A^{+}(x) + \alpha$, $A^{+}(x)+ \gamma$ } = $A^{+}(x) + \gamma$ = $N^{+}(x)$ for all x in R. And $C^{-}(x)$ = max { $M^{-}(x)$, $N^{-}(x)$ } = max { $A^{-}(x) + \beta$, $A^{-}(x) + \delta$ } = $A^{-}(x) + \beta$ = $M^{-}(x)$ for all x in R. Therefore C = $T^{A}_{(\gamma,\beta)}$ = { ⟨ x, $A^{+}(x)+ \gamma$, $A^{-}(x)+ \beta$ ⟩ / x∈R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R.

Case (iii): $\alpha \le \gamma$ and $\beta \ge \delta$. Clearly C = $T^{A}_{(\alpha,\beta)}$ = { ⟨ x, $A^{+}(x) + \alpha$, $A^{-}(x)+\beta$ ⟩ / x∈R} is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R.

Case (iv): $\alpha \ge \gamma$ and $\beta \le \delta$. Clearly C = $T^{A}_{(\gamma,\delta)}$ = { ⟨ x, $A^{+}(x)+ \gamma$, $A^{-}(x)+ \delta$ ⟩ / x∈R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R. In other cases are true, so in all the cases, the intersection of any two bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A of R is a bipolar valued fuzzy translation of A.

2.2. Theorem

The intersection of a family of bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A of a hemiring R is a bipolar valued fuzzy translation of A.

Proof: Using the Theorem 2.1, we can prove easily.

2.3. Theorem

Union of any two bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A of a hemiring R is a bipolar valued fuzzy translation of A.

Proof: Let x and y belong to R. Let M = $T^{A}_{(\alpha,\beta)}$ = { ⟨ x, $A^{+}(x) + \alpha$, $A^{-}(x) + \beta$ ⟩ / x∈R } and N = $T^{A}_{(\gamma,\delta)}$ = { ⟨ x, $A^{+}(x) + \gamma$, $A^{-}(x) + \delta$ ⟩ / x∈R } be two bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A = ⟨ A^{+}, A^{-} ⟩ of R. Let C = M∪N and C = { ⟨ x, $C^{+}(x)$, $C^{-}(x)$ ⟩ / x∈R }, where $C^{+}(x)$ = max { $A^{+}(x) + \alpha$, $A^{+}(x) + \gamma$ } and $C^{-}(x)$ = Min { $A^{-}(x)+ \beta$, $A^{-}(x)+ \delta$ }.

Case (i): $\alpha \le \gamma$ and $\beta \le \delta$. Now $C^{+}(x)$ = max { $M^{+}(x)$, $N^{+}(x)$ } = max{$A^{+}(x)+\alpha$, $A^{+}(x)+ \gamma$}= $A^{+}(x)+ \gamma$ = $N^{+}(x)$ for all x and y in R. And $C^{-}(x)$ = Min { $M^{-}(x)$, $N^{-}(x)$ } = Min { $A^{-}(x)+ \beta$, $A^{-}(x)+ \delta$} = $A^{-}(x)+ \beta$ = $M^{-}(x)$ for all x in R.

Therefore C = $T^{A}_{(\gamma,\beta)}$ = { ⟨ x, $A^{+}(x) + \gamma$, $A^{-}(x) + \beta$ ⟩ / x∈R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R.

Case (ii): $\alpha \geq \gamma$ and $\beta \geq \delta$. Now $C^{+}(x)$ = max { $M^{+}(x)$, $N^{+}(x)$ } = max { $A^{+}(x) + \alpha$, $A^{+}(x) + \gamma$} = $A^{+}(x) + \alpha = M^{+}(x)$ for all x in R. And $C^{-}(x)$ = min{ $M^{-}(x)$, $N^{-}(x)$ } = min {$A^{-}(x)+\beta$, $A^{-}(x)+ \delta$} = $A^{-}(x)+ \delta = N^{-}(x)$ for all x in R. Therefore C = $T^{A}_{(\alpha,\delta)}$ = { ⟨ x, $A^{+}(x)+ \alpha$, $A^{-}(x)+ \delta$ ⟩ / x∈R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R.

Case (iii): $\alpha \leq \gamma$ and $\beta \geq \delta$. Clearly C = $T^{A}_{(\gamma,\delta)}$ = { ⟨ x, $A^{+}(x) + \gamma$, $A^{-}(x)+\delta$ ⟩ / x∈R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R.

Case (iv): $\alpha \geq \gamma$ and $\beta \leq \delta$. Clearly C = $T^{A}_{(\alpha,\beta)}$ = { ⟨ x, $A^{+}(x) + \alpha$, $A^{-}(x)+\beta$⟩ / x∈R } is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring A of R. In other cases are true, so in all the cases, union of any two bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A of R is a bipolar valued fuzzy translation of A.

2.4. Theorem

The union of a family of bipolar valued fuzzy translations of bipolar valued fuzzy subhemiring A of a hemiring R is a bipolar valued fuzzy translation of A.

Proof: Using the Theorem 2.1, we can prove easily.

2.5. Theorem

A bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring A of a hemiring R is a bipolar valued fuzzy subhemiring of R.

Proof: Assume that T = ⟨ T^{+}, T^{-} ⟩ is a bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring A = ⟨ A^{+}, A^{-} ⟩ of a hemiring R. Let x and y in R. We have $T^{+}(x+y) = A^{+}(x+y) + \alpha \geq$ min{ $A^{+}(x)$, $A^{+}(y)$ } + α = min { $A^{+}(x) + \alpha$, $A^{+}(y) + \alpha$ } = min { $T^{+}(x)$, $T^{+}(y)$ }. Therefore $T^{+}(x+y) \geq$ min { $T^{+}(x)$, $T^{+}(y)$ } for all x and y in R. And $T^{+}(xy) = A^{+}(xy) + \alpha \geq$ min{ $A^{+}(x)$, $A^{+}(y)$ } + α = min { $A^{+}(x) + \alpha$, $A^{+}(y) + \alpha$ }= min { $T^{+}(x)$, $T^{+}(y)$ }. Therefore $T^{+}(xy) \geq$ min { $T^{+}(x)$, $T^{+}(y)$ } for all x and y in R. Also $T^{-}(x+y) = A^{-}(x+y) +\beta \leq$ max { $A^{-}(x)$, $A^{-}(y)$ }+ β = max { $A^{-}(x) + \beta$, $A^{-}(y) + \beta$ } = max { $T^{-}(x)$, $T^{-}(y)$ }. Therefore $T^{-}(x+y) \leq$ max{ $T^{-}(x)$, $T^{-}(y)$ } for all x and y in R. And $T^{-}(xy) = A^{-}(xy) +\beta \leq$ max{ $A^{-}(x)$, $A^{-}(y)$ }+ β = max { $A^{-}(x) + \beta$, $A^{-}(y) + \beta$ }= max { $T^{-}(x)$, $T^{-}(y)$ }. Therefore $T^{-}(xy) \leq$ max { $T^{-}(x)$, $T^{-}(y)$ } for all x and y in R. Hence T is a bipolar valued fuzzy subhemiring of R.

2.6. Theorem

Let (R, +, .) and (R^{l}, +, .) be any two hemirings and f be a homomorphism. Then the homomorphic image of a bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring A of R is also a bipolar valued fuzzy subhemiring of R^{l}.

Proof: Let V = (V^+, V^-) = f($T^{A}_{(\alpha,\beta)}$), where $T^{A}_{(\alpha,\beta)}$ is a bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring A = (A^+, A^-) of R. We have to prove that V is a bipolar valued fuzzy subhemiring of R^l. For all f(x) and f(y) in R^l, we have V^+[f(x)+f(y)] = V^+ [f(x+y)] ≥ $T^{+A}{}_{\alpha}$(x+y) = A^+(x+y)+ α ≥ min{ A^+(x), A^+(y)}+ α = min{A^+(x)+α, A^+(y)+α} = min{ $T^{+A}{}_{\alpha}$(x), $T^{+A}{}_{\alpha}$(y) }

which implies that V^+[f(x)+f(y)] ≥ min { V^+(f(x)), V^+(f(y)) } for all f(x) and f(y) in R^l. And V^+[f(x)f(y)] = V^+ [f(xy)] ≥ $T^{+A}{}_{\alpha}$(xy) = A^+(xy)+ α ≥ min{ A^+(x), A^+(y) }+α = min{A^+(x)+α, A^+(y)+α }= min{ $T^{+A}{}_{\alpha}$(x), $T^{+A}{}_{\alpha}$(y) }

which implies that V^+[f(x)f(y)] ≥ min { V^+(f(x)), V^+(f(y)) } for all f(x) and f(y) in R^l. Also V^-[f(x)+f(y)] = V^-[f(x+y)] ≤ T_{β}^{-A}(x+y) = A^-(x+y)+ β ≤ max{ A^-(x), A^-(y) }+β = max { A^-(x)+β, A^-(y)+β} = max { T_{β}^{-A}(x), T_{β}^{-A}(y) }

which implies that V^-[f(x)+f(y)] ≤ max { V^-(f(x)), V^-(f(y)) } for all f(x) and f(y) in R^l. And V^-[f(x)f(y)] = V^-[f(xy)] ≤ T_{β}^{-A}(xy) = A^-(xy)+ β ≤ max {A^-(x), A^-(y) }+ β = max { A^-(x)+β, A^-(y)+β} = max { T_{β}^{-A}(x), T_{β}^{-A}(y) } which implies that V^-[f(x)f(y)] ≤ max { V^-(f(x)), V^-(f(y)) } for all f(x) and f(y) in R^l. Therefore V is a bipolar valued fuzzy subhemiring of R^l.

2.7. Theorem

Let (R, +, .) and (R^l, +, .) be any two hemirings and f be a homomorphism. Then the homomorphic pre-image of bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring V of R^l is a bipolar valued fuzzy subhemiring of R.

Proof: Let T = $T^{V}_{(\alpha,\beta)}$ = f(A), where $T^{V}_{(\alpha,\beta)}$ is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring V = (V^+, V^-) of R^l. We have to prove that A = (A^+, A^-) is a bipolar valued fuzzy subhemiring of R. Let x and y in R. Then A^+(x+y) = T_{α}^{+V} (f(x+y)) = T_{α}^{+V} (f(x)+f(y)) = V^+[f(x)+f(y)]+ α ≥ min { V^+(f(x)), V^+(f(y)) }+ α = min {V^+(f(x)) + α, V^+(f(y)) + α } = min { T_{α}^{+V} (f(x)), T_{α}^{+V} (f(y))} = min { A^+(x), A^+(y) } which implies that A^+(x+y) ≥ min{A^+(x), A^+(y)} for all x, y in R. And A^+(xy) = T_{α}^{+V} (f(xy)) = T_{α}^{+V} (f(x)f(y)) = V^+[f(x)f(y)] + α ≥ min { V^+(f(x)), V^+(f(y)) }+ α = min { V^+(f(x)) + α, V^+(f(y)) + α } = min { T_{α}^{+V} (f(x)), T_{α}^{+V} (f(y)) } = min { A^+(x), A^+(y) } which implies that A^+(xy) ≥ Min { A^+(x), A^+(y) } for all x and y in R. Also A^-(x+y) = T_{β}^{-V} (f(x+y)) = T_{β}^{-V} (f(x)+f(y)) = V^-[f(x)+f(y)] +β ≤ max {V^-(f(x)), V^-(f(y)) }+β = max{V^-(f(x))+β, V^-(f(y)) + β } = max{ T_{β}^{-V}(f(x)), T_{β}^{-V} (f(y)) } = max { A^-(x), A^-(y) } which implies A^-(x+y) ≤ max{ A^-(x), A^-(y) } for all x and y in R. And A^-(xy) = T_{β}^{-V} (f(xy)) = T_{β}^{-V}(f(x)f(y)) = V^-[f(x)f(y)] +β ≤ max { V^-(f(x)), V^-(f(y)) }+ β = max { V^-(f(x)) + β, V^-(f(y)) + β } = max { T_{β}^{-V} (f(x)), T_{β}^{-V}(f(y)) } = max { A^-(x), A^-(y) } which implies A^-(xy) ≤ max{ A^-(x), A^-(y) } for all x and y in R. Therefore A is a bipolar valued fuzzy subhemiring of R.

2.8. Theorem

Let (R, +, .) and (R^l, +, .) be any two hemirings and f be a anti-homomorphism. Then the anti-homomorphic image of a bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring A of R is also a bipolar valued fuzzy subhemiring of R^l.

Proof: Let V = (V^+, V^-) = f($T^A_{(\alpha,\beta)}$), where $T^A_{(\alpha,\beta)}$ is a bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring A = (A^+, A^-) of R. We have to prove that V is a bipolar valued fuzzy subhemiring of R^l. For all f(x) and f(y) in R^l, we have V^+[f(x)+f(y)] = V^+ [f(y+x)] ≥ $T^{+A}{}_\alpha$ (y+x) = A^+(y+x) + α ≥ min{ A^+(y), A^+(x) }+ α = min {A^+(x)+α, A^+(y)+α } = min { $T^{+A}{}_\alpha$ (x), $T^{+A}{}_\alpha$ (y) } which implies that V^+[f(x)+f(y)] ≥ min { V^+(f(x)), V^+(f(y)) } for all f(x) and f(y) in R^l. And V^+[f(x)f(y)] = V^+ [f(yx)] ≥ $T^{+A}{}_\alpha$ (yx) = A^+(yx) + α ≥ min{ A^+(y), A^+(x) }+α = min {A^+(x)+α, A^+(y)+α } = min { $T^{+A}{}_\alpha$ (x), $T^{+A}{}_\alpha$ (y) } which implies that V^+[f(x)f(y)] ≥ min { V^+(f(x)), V^+(f(y)) } for all f(x) and f(y) in R^l. Also V^-[f(x)+f(y)] = V^-[f(y+x)] ≤ T_β^{-A}(y+x) = A^-(y+x) + β ≤ max { A^-(y), A^-(x) }+ β = max { A^-(x)+ β, A^-(y) + β }= max { T_β^{-A}(x), T_β^{-A}(y) } which implies that V^-[f(x) + f(y)] ≤ max { V^-(f(x)), V^-(f(y)) } for all f(x) and f(y) in R^l. And V^-[f(x)f(y)] = V^-[f(yx)] ≤ T_β^{-A}(yx) = A^-(yx) + β ≤ max { A^-(x), A^-(y) } + β = max { A^-(x) + β, A^-(y) + β}= max { T_β^{-A}(x), T_β^{-A}(y) } which implies that V^-[f(x)f(y)] ≤ max { V^-(f(x)), V^-(f(y)) } for all f(x) and f(y) in R^l. Therefore V is a bipolar valued fuzzy subhemiring of R^l.

2.9. Theorem

Let (R, +, .) and (R^l, +, .) be any two hemirings and f be an anti-homomorphism. Then the anti-homomorphic pre-image of bipolar valued fuzzy translation of a bipolar valued fuzzy subhemiring V of R^l is a bipolar valued fuzzy subhemiring of R.

Proof: Let T = $T^V_{(\alpha,\beta)}$ = f(A), where $T^V_{(\alpha,\beta)}$ is a bipolar valued fuzzy translation of bipolar valued fuzzy subhemiring V = (V^+, V^-) of R^l. We have to prove that A = (A^+, A^-) is a bipolar valued fuzzy subhemiring of R. Let x and y in R. Then A^+(x+y) = T_α^{+V} (f(x+y)) = T_α^{+V} (f(y)+f(x)) = V^+[f(y)+f(x)] + α ≥ min { V^+(f(y)), V^+(f(x)) }+ α = min { V^+(f(x)) + α, V^+(f(y)) + α } = min{ T_α^{+V} (f(x)), T_α^{+V} (f(y))} = min{A^+(x), A^+(y) } which implies that A^+(x+y) ≥ min { A^+(x), A^+(y) } for all x, y in R. And A^+(xy) = T_α^{+V} (f(xy)) = T_α^{+V} (f(y)f(x)) = V^+[f(y)f(x)] + α ≥ min { V^+(f(y)), V^+(f(x)) }+ α = min { V^+(f(x)) + α, V^+(f(y))+ α } = min { T_α^{+V} (f(x)), T_α^{+V} (f(y)) } = min { A^+(x), A^+(y) } which implies that A^+(xy) ≥ min { A^+(x), A^+(y) } for all x and y in R. Also A^-(x+y) = T_β^{-V} (f(x+y)) = T_β^{-V} (f(y)+f(x)) = V^-[f(y)+f(x)] +β ≤ max { V^-(f(y)), V^-(f(x)) } + β = max { V^-(f(x)) + β, V^-(f(y)) + β } = max { T_β^{-V} (f(x)), T_β^{-V} (f(y)) } = max { A^-(x), A^-(y) } which implies A^-(x+y) ≤ max{ A^-(x), A^-(y) } for all x and y in R. And A^-(xy) = T_β^{-V} (f(xy)) = T_β^{-V} (f(y)f(x)) = V^-[f(y)f(x)] +β ≤ max { V^-(f(y)), V^-(f(x)) } + β = max { V^-(f(x)) + β, V^-(f(y)) + β } = max { T_β^{-V} (

f(x)), T_{β}^{-V} (f(y)) } = max { A^{-}(x), A^{-}(y) } which implies A^{-}(xy) ≤ max{ A^{-}(x), A^{-}(y) } for all x and y in R. Therefore A is a bipolar valued fuzzy subhemiring of R.

REFERENCES

1. *Anitha.M.S., Muruganantha Prasad & K.Arjunan, Notes on Bipolar-valued fuzzy subgroups of a group, Bulletin of Society for Mathematical Services and Standards, Vol. 2 No. 3 (2013), pp. 52-59.*
2. *Anitha.M.S, K.L.Muruganantha Prasad & K.Arjunan, Homomorphism and anti-homomorphism of Bipolar valued fuzzy subgroups of a group, International Journal of Mathematical Archive-4(12), 2013, 1-4.*
3. *Anthony.J.M and H.Sherwood, fuzzy groups Redefined, Journal of mathematical analysis and applications, 69(1979),124 -130.*
4. *Arsham Borumand Saeid, Bipolar-valued fuzzy BCK/BCI-algebras, World Applied Sciences Journal 7 (11) (2009), 1404-1411.*
5. *Azriel Rosenfeld, fuzzy groups, Journal of mathematical analysis and applications 35(1971), 512-517.*
6. *F.P.Choudhury, A.B.Chakraborty and S.S.Khare, A note on fuzzy subgroups and fuzzy homomorphism, Journal of mathematical analysis and applications, 131(1988), 537 -553.*
7. *W.L.Gau and D.J. Buehrer, Vague sets, IEEE Transactons on Systems, Man and Cybernetics, 23(1993), 610-614.*
8. *Kyoung Ja Lee, Bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI-algebras, Bull. Malays.Math. Sci. Soc. (2) 32(3) (2009), 361–373.*
9. *K.M.Lee, Bipolar-valued fuzzy sets and their operations. Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand, (2000), 307-312.*
10. *K.M.Lee, Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets and bipolarvalued fuzzy sets. J. fuzzy Logic Intelligent Systems, 14 (2) (2004), 125-129.*
11. *Mustafa Akgul, some properties of fuzzy groups, Journal of mathematical analysis and applications, 133(1988), 93 -100.*
12. *Samit Kumar Majumder, Bipolar valued fuzzy sets in Γ-semigroups, Mathematica Aeterna, Vol. 2, no. 3(2012), 203 – 213.*
13. *Shanmugapriya.V and Arjunan.K., Some translators in bipolar valued fuzzy subsemiring of a semiring, Bulletin of Mathematics and Statistic Research, Vol.4, Iss.4, 2016, 118- 123.*
14. *Young Bae Jun and Seok Zun Song, Subalgebras and closed ideals of BCH-algebras based on bipolar-valued fuzzy sets, Scientiae Mathematicae Japonicae Online, (2008), 427-437.*
15. *L.A.Zadeh, fuzzy sets, Inform. And Control, 8(1965), 338-353.*