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SUBSTANTIATION OF PARAMETERS AND ANALYSIS OF OPERATIONAL CHARACTERISTICS OF OSCILLATING SYSTEMS OF VIBRATORY FINISHING MACHINES

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Abstract. The calculation diagrams of oscillating systems and operation features of vibratory finishing machines are considered. The mathematical models of three-mass and four-mass oscillating systems are presented. The amplitude values of the oscillating masses displacements are derived. The functions of inertial and stiffness parameters optimization are formed. The optimization problems are solved with a help of MathCAD software. On the basis of synthesized inertial and stiffness parameters, the amplitude-frequency characteristics of the oscillating systems of vibratory finishing machines are formed and analyzed. In order to verify the validity of the proposed theoretical approaches, the simulation of the mathematical model of the oscillating system motion is carried out by means of numerical solving of the system of differential equations of the oscillating masses motion.

The proposed structural diagrams and the operation schemes of the vibratory finishing machine, as well as the derived analytical formulas may be used by designers, researchers and technologists while improving existent and developing new equipment for vibro-finishing treatment.

Keywords: lap; vibratory finishing machine; lapping; oscillations; oscillating system; inertial parameters; stiffness; amplitude-frequency characteristic.

Introduction and Problem Statement

The modelling of operation of mechanical oscillating systems requires the presence of all inertial parameters of the masses and the stiffness parameters of the elastic systems [1]. The mechanical parameters of the given structure determine the motion conditions of the mechanical oscillating systems and define their peculiarities and advantages. Nevertheless, most of the parameters can be defined on the basis of solutions of the differential equations of the system motion in certain operational conditions [2]. To obtain qualitatively new relations of parameters that ensure the high efficiency of mechanical oscillating systems, it is necessary to simplify the differential equations of the mechanical oscillating systems motion by imposing on the system essentially new operational conditions that will be reflected in certain values of inertial and stiffness parameters.

This is extremely important in the case when a new vibratory finishing equipment is being developed, for which there are no known analytical expressions for determining the required mechanical parameters. Therefore, the obtaining of analytical expressions for the rational distribution of mechanical parameters is the final result of the scientific research at this stage of investigations.

The designing and development of modern vibratory finishing machines are impossible without the dynamic analysis of their mechanical oscillating systems [1]–[5], which is based on differential equations of motion that provide a deep analysis of the processes occurring in mechanical systems, reliably define the necessary analytical dependencies, and make it possible to understand the physics of the process. Applying the systems modelling using differential equations can guarantee the obtaining of predicted results.

Analysis of Modern Information Sources on the Subject of the Article

In the publications [2]–[6], [13], the stages of designing and the techniques of calculation of inter-resonant vibratory finishing machines with electromagnetic drive and in-phase laps motion are considered. The majority of such machines are developed using the effect of “zero-stiffness”.

The scientists in publications [7] and [8] describe the fundamental principles of vibration treatment and propose the mathematical models allowing the roughness prediction of the working surface of laps. They substantiate the parameters of the proposed models of vibratory treatment with a help of experimental results and propose the algorithm of constructing the optimal process of vibration finishing treatment. The new mathematical model of the vibratory lapping machine of the “bowl” type is considered. This model is used for analysis of dynamic behaviour of the mechanical oscillating system taking into account both free and forced oscillations. The influence of the basic parameters on the productiveness of the proposed machine operation is analysed.

In the work [9], the structure of the resonant vibratory machine with electromagnetic drive with circumferential lateral oscillations of the laps is proposed. The authors assume that the stiffness of the vibration isolators is relatively small, so it is not considered in calculations. For the proposed structure of the vibratory machine, the differential equations of motion are derived using the corresponding generalized coordinates. The expression for calculation of the kinetic energy consisting of the energy of lateral motion of the oscillating masses and the energy of their rotation round the centers of masses are derived using the Lagrange method. The analysis of the proposed analytical model has shown that it may be reduced to single-mass system with one degree of freedom with a sufficient degree of accuracy. This allows sufficient simplifying the derivation of the analytical dependencies for calculation of the basic parameters of the structure and the operational modes of the machine drive. In addition, this allow to decrease the amount of further experimental investigations while setting up and regulating of the process of finishing treatment on such machines.

In the publication [10], the analysis of kinematic diagrams of various drives of existing plane-finishing machines is carried out. The peculiarities of cycloidal trajectories of the tools motion on the testing equipment for finishing of flat surfaces are investigated. This equipment allow the regulation of the parameters of cycloidal trajectories during the treatment. It is established that one can purposely change the network density of the tracks of the tools cycloidal trajectories and form the required operational properties of the microrelief roughness of the surface being treated.

In the investigations presented in [11], the kinematic model and the motion simulation of the vibration treatment are considered. The description of the motion is verified by the digital video recorder with a help of high-speed camera. It is shown that middle kinematic line of path is the circumferential spiral motion trajectory depended on the relation between the angular velocity and the translational motion speed, as well as on the direction of rotation and the structure of the vibratory machine. The model of material disposal intensity has been developed on the basis of the proposed kinematic model. The optimal placement of fixators and orientation of the part have been substantiated by means of kinematic analysis.

The publication [12] presents the invention of the device for double-sided treatment of flat parts. One of the working discs can move with a help of the specific driving mechanism and have the device for organizing the parts being treated.

Analyzing the information sources related with vibration finishing treatment, we can state that the problems of modelling the operation of vibratory finishing machines and substantiation of the mechanical parameters of their oscillating systems are not fully solved. Thus, in the paper [14], there was made an effort to develop the mathematical models of three- and four-mass oscillating systems of vibratory finishing machines with circumferential trajectories of laps motion. The present paper will continue the investigations started earlier in [14] and will consist of the research results related with the dynamic analysis of the oscillating systems of the vibratory finishing machines, optimization of their inertial and stiffness parameters, construction of amplitude-frequency characteristics and displacement-time graphical dependencies for each mass of the oscillating system.

Formation of Assumptions and Problems of Research

Let us consider the three-mass and four-mass oscillating systems of vibratory finishing machines. Let us present them as mechanical systems consisting of absolutely rigid bodies connected by elastic elements of the certain stiffness. Such assumptions allow significant simplification of the material presentation and ensuring the sufficient accuracy of the obtained results. Let us start the investigation from the analysis of the mathematical models of the oscillating systems. Herewith, we will consider that the law of changing the stiffness in elastic elements does not exceed out the ranges of linearity and corresponds to the Hooke's law. This is absolutely is justified if we will consider small oscillations of mechanical oscillating systems in near-resonance modes.

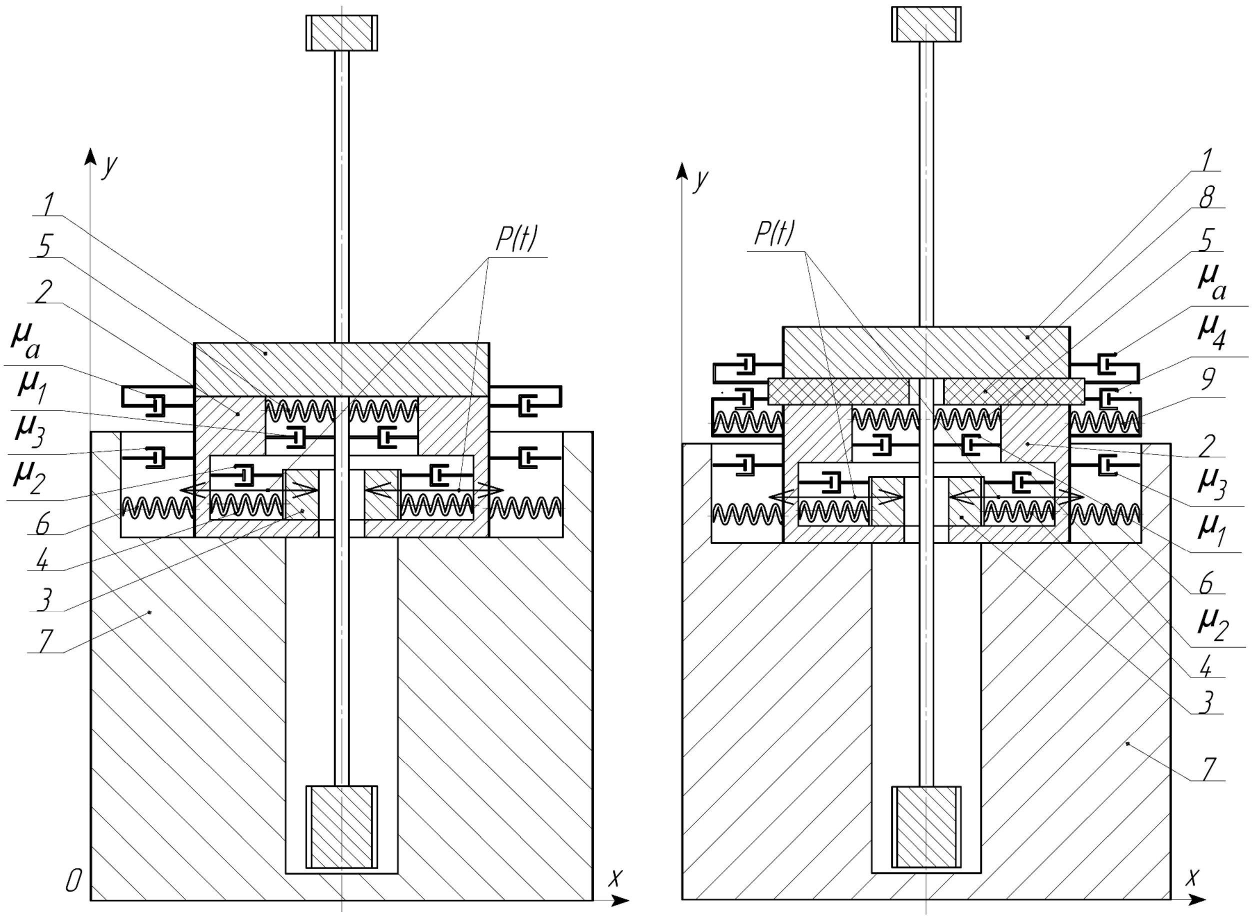
In further investigations, we will consider entirely sinusoidal law of changing the traction force in electromagnetic vibration exciters. In addition, we will consider only the sinusoidal form of the mass oscillations being the basic motion form of the mechanical systems, which are in resonance state. During a resonance, the mass, which is a part of a certain system, moves according to the sinusoidal law, even if the disturbing force has a non-sinusoidal harmonic form of the signal. In this mode, the mechanical oscillating system corresponds to one of the sinusoidal harmonics of the disturbing signal, the frequency of which coincides with the actual frequency of oscillations of the system itself. From the point of view of energy consumption, it is the most efficiently to use sinusoidal vibration disturbance signals in resonant mechanical oscillating systems. If the signal is periodic, but not sinusoidal, then its energy will appear during the system operation in the forms of heat release and parasitic fluctuations.

Calculation Diagrams of Oscillating Systems of Vibratory Finishing Machines

Let us consider the three-mass (Fig. 1, *a*) [2], [5], [6], [14] and four-mass (Fig. 1, *b*) [14] structures of the vibratory finishing machine as the basic mechanical oscillating systems for further investigations, in which the translational oscillations are implemented. In the three-mass system (Fig. 1, *a*), the active 1, intermediate 2 and lower reactive 3 masses with inertial parameters of m_a, m_n and m_p, correspondingly, perform the translational oscillations along the horizontal axis x according to the generalized coordinates x_1, x_2 and x_3, correspondingly. In the four-mass system (Fig. 1, *b*), the mass $m_в$ of the carrier 8 with parts being treated is also taken into account. This mass performs translational oscillations along the horizontal axis x according the generalized coordinate x_4.

The active mass 1 is set into motion due to the kinematic excitement from the intermediate mass 2. The excitement of the forced oscillations is performed due to the sinusoidal force $P(t) = P \cdot \sin(wt + e)$ (P is the amplitude value of the disturbing force; t is time; e is the phase shift (difference) between the force and the displacement; w is the angular frequency of the disturbing force) applied between the intermediate 2 and reactive 3 masses. The active 1 and the intermediate 2 masses, as well as the intermediate 2 and the reactive 3 masses are connected with each other by elastic systems 5 and 4, correspondingly, with stiffnesses c_1 and c_2. In the picture, theses elastic systems are schematically represented by the twisted springs. The structures of the vibratory finishing machines are supported by the vibration isolators 6 with the stiffness of $c_{iз}$ placed between the intermediate masses 2 and the fixed frames of the machines. Let us adopt that the dissipative forces also act in the system. For this, into the dynamic model, we introduce the coefficients of viscous resistance (friction) m_1, m_2, m_3, which are proportional to the motion speed of the corresponding masses and represent the hysteresis phenomena in the elastic systems 4, 5, and 6, correspondingly. In addition, we take into account the coefficients m_a, m_n and m_p describing the viscous friction during the oscillating masses 1, 2 and 3 relative motion.

Since the masses 2 (m_n) and 3 (m_p) do not interact between each other, according to [2], [5], [14] we may neglect the coefficients of dissipation m_n, m_p. At the same time, the coefficient m_a describing the viscous friction during the motion of the active 1 and the intermediate 2 oscillating masses, between which the working zone for charging the parts being treated in the vibro-finishing machine is located, should be taken into account. In such a case, the whole energy of the vibration exciters is being transformed into the heat energy of the parts lapping.



a *b*

Fig.1. *Calculation diagrams of vibratory finishing machines with three-mass (a) and four-mass (b) oscillating systems*

In addition, for the four-mass system (Fig. 1, b), we should take into account the coefficient of viscous friction m_4, which is proportional to the carrier 8 motion related to the motion of the lower lap 2 and also represents the hysteresis phenomena in the elastic element 9. The coefficient m_a, in this case, describes the viscous resistance of the mass 1 motion along the mass 8.

Differential Equations of the Oscillating Masses Motion

The mathematical model of the vibratory finishing machine, the calculation diagram of which is presented in Fig. 1, a, is developed on the basis of the three-mass oscillating system [5], [14], in which the translational oscillations of the masses are implemented in horizontal plane. The active 1 (m_a), intermediate 2 (m_n) and reactive 3 (m_p) masses perform translational oscillations along the horizontal axis x according the generalized coordinates x_1, x_2 and x_3 due to the influence of the disturbing force $p(t)$ [14]. The system of differential equations of the three-mass system motion has the following form:

$$\begin{cases} m_a \cdot \ddot{x}_1 + c_1 \cdot (x_1 - x_2) + (m_a + m_1) \cdot (\dot{x}_1 - \dot{x}_2) = 0; \\ m_n \cdot \ddot{x}_2 + c_1 \cdot (x_2 - x_1) + c_2 \cdot (x_2 - x_3) + c_{i3} \cdot x_2 + (m_a + m_1) \cdot (\dot{x}_2 - \dot{x}_1) + m_2 \cdot (\dot{x}_2 - \dot{x}_3) + \\ \quad + m_3 \cdot \dot{x}_2 = P \cdot \sin(wt + e); \\ m_p \cdot \ddot{x}_3 + c_2 \cdot (x_3 - x_2) + m_2 \cdot (\dot{x}_3 - \dot{x}_2) = -P \cdot \sin(wt + e). \end{cases} \qquad (1)$$

The mathematical model of the vibratory finishing machine developed on the basis of the four-mass oscillating system [14], in which the translational oscillations of the masses are implemented in horizontal plane, also takes into account the presence of the carrier 8 with the parts being treated between the upper 1 and lower 2 laps. This carrier with the parts forms the fourth mass [14]. The system of differential equations of the four-mass system motion has the following form:

$$\begin{cases} m_a \cdot \ddot{x}_1 + c_1 \cdot (x_1 - x_2) + m_a \cdot (\dot{x}_1 - \dot{x}_4) + m_1 \cdot (\dot{x}_1 - \dot{x}_2) = 0; \\ m_n \cdot \ddot{x}_2 + c_1 \cdot (x_2 - x_1) + c_2 \cdot (x_2 - x_3) + c_{i з} \cdot x_2 + c_3 \cdot (x_2 - x_4) + \\ + m_1 \cdot (\dot{x}_2 - \dot{x}_1) + m_2 \cdot (\dot{x}_2 - \dot{x}_3) + m_3 \cdot \dot{x}_2 + m_4 \cdot (\dot{x}_2 - \dot{x}_4) = P \cdot \sin(wt + e); \\ m_p \cdot \ddot{x}_3 + c_2 \cdot (x_3 - x_2) + m_2 \cdot (\dot{x}_3 - \dot{x}_2) = -P \cdot \sin(wt + e); \\ m_в \cdot \ddot{x}_4 + c_3 \cdot (x_4 - x_2) + m_4 \cdot (\dot{x}_4 - \dot{x}_2) + m_a \cdot (\dot{x}_4 - \dot{x}_1) = 0. \end{cases} \tag{2}$$

Determination of Amplitude Values of the Oscillating Masses Displacements

Using the general methods of solving the obtained systems of differential equations (1) and (2), let us define the analytical expressions of the masses motion by three independent degrees of freedom in the following form: $x_1 = X_1 \cdot e^{i \cdot w \cdot t}$, $x_2 = X_2 \cdot e^{i \cdot w \cdot t}$, $x_3 = X_3 \cdot e^{i \cdot w \cdot t}$, $x_4 = X_4 \cdot e^{i \cdot w \cdot t}$, where X_1, X_2, X_3, X_4 are the amplitude values of linear forced oscillations by the generalized coordinates x_1, x_2, x_3 and x_4, correspondingly. Substituting these expressions into (1) or (2) and cancelling the term e^{iwt} (where $i = \sqrt{-1}$) in each part of the equations system, after certain transformation [14], we obtain the dependencies for determination of the values X_1, X_2, X_3 of the three-mass oscillating system:

$$\begin{aligned} X_1 &= \frac{P \cdot k_{12} \cdot (k_{23} + k_{33})}{k_{12} \cdot k_{21} \cdot k_{33} - k_{11} \cdot k_{22} \cdot k_{33} + k_{11} \cdot k_{23} \cdot k_{32}}; \\ X_2 &= \frac{-P \cdot k_{11} \cdot (k_{23} + k_{33})}{k_{12} \cdot k_{21} \cdot k_{33} - k_{11} \cdot k_{22} \cdot k_{33} + k_{11} \cdot k_{23} \cdot k_{32}}; \\ X_3 &= \frac{P \cdot (k_{11} k_{32} - k_{12} k_{21} + k_{22} k_{11})}{k_{12} \cdot k_{21} \cdot k_{33} - k_{11} \cdot k_{22} \cdot k_{33} + k_{11} \cdot k_{23} \cdot k_{32}}, \end{aligned} \tag{3}$$

where

$$k_{11} = c_1 - m_a \cdot w^2 + i \cdot (m_a + m_1) \cdot w; \; k_{31} = 0; \; k_{13} = 0;$$

$$k_{22} = c_1 + c_2 + c_{i з} - m_n \cdot w^2 + i \cdot (m_1 + m_2 + m_3 + m_a) \cdot w; \; k_{12} = k_{21} = -c_1 - i \cdot (m_a + m_1) \cdot w;$$

$$k_{23} = k_{32} = -c_2 - i \cdot m_2 \cdot w; \; k_{33} = c_2 - m_p \cdot w^2 + i \cdot m_2 \cdot w.$$

Similarly, in [14] there were derived the dependencies for determination of the values X_1, X_2, X_3, X_4 of the four-mass oscillating system:

$$\begin{aligned} X_1 &= -\frac{P \cdot (k_{12} \cdot k_{44} - k_{14} \cdot k_{42}) \cdot (k_{23} + k_{33})}{\begin{pmatrix} k_{11} \cdot k_{22} \cdot k_{33} \cdot k_{44} - k_{11} \cdot k_{23} \cdot k_{32} \cdot k_{44} - k_{11} \cdot k_{24} \cdot k_{33} \cdot k_{42} - k_{12} \cdot k_{21} \cdot k_{33} \cdot k_{44} + \\ + k_{12} \cdot k_{41} \cdot k_{33} \cdot k_{24} + k_{21} \cdot k_{14} \cdot k_{33} \cdot k_{42} - k_{22} \cdot k_{14} \cdot k_{33} \cdot k_{41} + k_{14} \cdot k_{23} \cdot k_{32} \cdot k_{41} \end{pmatrix}}; \\ X_2 &= \frac{P \cdot (k_{11} \cdot k_{44} - k_{14} \cdot k_{41}) \cdot (k_{23} + k_{33})}{\begin{pmatrix} k_{11} \cdot k_{22} \cdot k_{33} \cdot k_{44} - k_{11} \cdot k_{23} \cdot k_{32} \cdot k_{44} - k_{11} \cdot k_{24} \cdot k_{33} \cdot k_{42} - k_{12} \cdot k_{21} \cdot k_{33} \cdot k_{44} + \\ + k_{12} \cdot k_{41} \cdot k_{33} \cdot k_{24} + k_{21} \cdot k_{14} \cdot k_{33} \cdot k_{42} - k_{22} \cdot k_{14} \cdot k_{33} \cdot k_{41} + k_{14} \cdot k_{23} \cdot k_{32} \cdot k_{41} \end{pmatrix}}; \\ X_3 &= -\frac{\left[P \cdot \begin{pmatrix} k_{11} \cdot k_{22} \cdot k_{44} - k_{11} \cdot k_{24} \cdot k_{42} - k_{12} \cdot k_{21} \cdot k_{44} + k_{12} \cdot k_{41} \cdot k_{24} + \\ + k_{21} \cdot k_{14} \cdot k_{42} - k_{22} \cdot k_{14} \cdot k_{41} + k_{11} \cdot k_{32} \cdot k_{44} - k_{14} \cdot k_{32} \cdot k_{41} \end{pmatrix} \right]}{\begin{pmatrix} k_{11} \cdot k_{22} \cdot k_{33} \cdot k_{44} - k_{11} \cdot k_{23} \cdot k_{32} \cdot k_{44} - k_{11} \cdot k_{24} \cdot k_{33} \cdot k_{42} - k_{12} \cdot k_{21} \cdot k_{33} \cdot k_{44} + \\ + k_{12} \cdot k_{41} \cdot k_{33} \cdot k_{24} + k_{21} \cdot k_{14} \cdot k_{33} \cdot k_{42} - k_{22} \cdot k_{14} \cdot k_{33} \cdot k_{41} + k_{14} \cdot k_{23} \cdot k_{32} \cdot k_{41} \end{pmatrix}}; \end{aligned} \tag{4}$$

$$X_4 = -\frac{P \cdot (k_{11} \cdot k_{42} - k_{12} \cdot k_{41}) \cdot (k_{23} + k_{33})}{\begin{pmatrix} k_{11} \cdot k_{22} \cdot k_{33} \cdot k_{44} - k_{11} \cdot k_{23} \cdot k_{32} \cdot k_{44} - k_{11} \cdot k_{24} \cdot k_{33} \cdot k_{42} - k_{12} \cdot k_{21} \cdot k_{33} \cdot k_{44} + \\ + k_{12} \cdot k_{41} \cdot k_{33} \cdot k_{24} + k_{21} \cdot k_{14} \cdot k_{33} \cdot k_{42} - k_{22} \cdot k_{14} \cdot k_{33} \cdot k_{41} + k_{14} \cdot k_{23} \cdot k_{32} \cdot k_{41} \end{pmatrix}},$$

where

$$k_{11} = c_1 - m_a \cdot w^2 + i \cdot (m_a + m_1) \cdot w \,;\ k_{12} = k_{21} = -c_1 - i \cdot m_1 \cdot w \,;\ k_{14} = k_{41} = -i \cdot m_a \cdot w \,;$$

$$k_{22} = c_1 + c_2 + c_{i3} - m_n \cdot w^2 + i \cdot (m_1 + m_2 + m_3 + m_4) \cdot w \,;\ k_{23} = k_{32} = -c_2 - i \cdot m_2 \cdot w \,;$$

$$k_{33} = c2 - m_p \cdot w^2 + i \cdot m2 \cdot w \,;\ k_{24} = k_{42} = -c_3 - i m_4 w \,;\ k_{44} = c_3 - m_в \cdot w^2 + i \cdot (m_a + m_4) \cdot w \,;$$

$$k_{34} = k_{43} = k_{31} = k_{13} = 0 \,.$$

For the three-mass oscillating system, we have the following group of parameters: m_a, m_n, m_p, c_1, c_2, c_{i3}, P, w, e, m_1, m_2, m_3, m_a; and for the four-mass system, we have: m_a, m_n, m_p, $m_в$, c_1, c_2, c_3, c_{i3}, P, w, e, m_1, m_2, m_3, m_4, m_a.

The values of the active m_a and the intermediate m_n masses (the masses of the upper and lower lap, correspondingly), as well as the mass $m_в$ of the carrier with parts being treated, are to be given by the designer. The frequency w and the phase difference e of the disturbing force are defined by the parameters of the electromagnetic drive. Let us consider the damping coefficients m_1, m_2, m_3, m_4, m_a as known ones for further calculations.

Formation of Objective Functions of Problems of Optimization of Inertial and Stiffness Parameters of Oscillating Systems

According to the results of theoretical investigations, we have five determinative parameters: the inertial parameters of three oscillating masses and the stiffness coefficients of two elastic systems, putting out of the account the dissipation parameters of the mechanical system. Thus, the calculation of the oscillating systems of the vibratory finishing machines may be reduced to determination of the values of the reactive mass m_p and the stiffness coefficients c_1, c_2, c_3, c_{i3} when the amplitude value of the disturbing force P is known. That's why, for the three-mass system of the vibratory finishing machine we obtain 4 unknown parameters, and for the four-mass system we have 5 unknown parameters.

The stiffness c_{i3} can be chosen as relatively small one taking into account the condition that the mechanical oscillating system mounted (installed) on the vibration isolators must have the self-frequency $w_в$ that is n times smaller than the angular frequency of forced oscillations w:

$$w_в = \frac{w}{n}$$

The larger is the value n, the weaker (safer) is the mechanical interaction between the oscillating system and the fixed frame of the machine. Its minimal value is $n = 3 \cdots 4$ [2]. Vibration isolators are to be calculated according to the single-mass scheme consisting of one rigid body with the mass equal to the total mass of the whole structure and the elastic element of certain stiffness representing the isolator. Thus, the stiffness coefficients of vibration isolators for three-mass and four-mass systems can be determined as:

$$c_{i3} = (m_a + m_n + m_p) \cdot \left(\frac{w}{n}\right)^2 ;$$
$$c_{i3} = (m_a + m_n + m_p + m_в) \cdot \left(\frac{w}{n}\right)^2 . \qquad (5)$$

The equation (5) allow to define the dependencies between the stiffness of the vibration isolators c_{i3} and the reactive mass m_p. Based on this, we can state that for the three-mass system we have 3 unknown parameters, and for the four mass system we have 4 unknown parameters.

In order to ensure the highest efficiency of the laps dressing (during the “lap over lap” dressing) and the vibration treatment of parts located inside the carrier, it is necessary to maximize the amplitude values of displacements of the active and intermediate oscillating masses (i.e., the masses of the upper and the lower laps) and to ensure their antiphase motion. Herewith, the carrier should be almost unmovable in order to ensure the dressing uniformity of the upper and the lower edge surfaces of the parts being treated. That’s why, the first summand of the objective function of optimization problem has the form of:

– for the three-mass system:

$$\Delta_1 = k_1 \cdot \left| X_1\left(m_p, c_1, c_2\right) \right| + k_2 \cdot \left| X_2\left(m_p, c_1, c_2\right) \right|;$$

– for the four-mass system: (6)

$$\Delta_1 = k_1 \cdot \left| X_1\left(m_p, c_1, c_2, c_3\right) \right| + k_2 \cdot \left| X_2\left(m_p, c_1, c_2, c_3\right) \right| + k_3 \cdot \frac{1}{\left| X_4\left(m_p, c_1, c_2, c_3\right) \right|},$$

where k_1, k_2, k_3 are the weight coefficients of each of the summands of the objective function.

In order to minimize the traction force of the electromagnets and, correspondingly, the power consumption for driving the vibratory finishing machine, it is necessary to ensure the minimal value of the air gap between the armatures of each of the pairs of electromagnets. Thus, the second summand of the objective function has the following form:

$$\Delta_2 = k_4 \cdot \frac{1}{\left| X_2\left(m_p, c_1, c_2\right) - X_3\left(m_p, c_1, c_2\right) \right|}, \tag{7}$$

where k_4 is the weight coefficient of this summand of the objective function.

Therefore, taking into account the dependencies (6) and (7), the general form of the objective function of the problem of optimization of inertial and stiffness parameters is as follows:

– for the three-mass system:

$$\Delta = \Delta_1 + \Delta_2 = \begin{bmatrix} k_1 \cdot \left| X_1\left(m_p, c_1, c_2\right) \right| + k_2 \cdot \left| X_2\left(m_p, c_1, c_2\right) \right| + \\ + k_4 \cdot \dfrac{1}{\left| X_2\left(m_p, c_1, c_2\right) - X_3\left(m_p, c_1, c_2\right) \right|} \end{bmatrix} \Rightarrow \max;$$

– for the four-mass system: (8)

$$\Delta = \Delta_1 + \Delta_2 = \begin{bmatrix} k_1 \cdot \left| X_1\left(m_p, c_1, c_2, c_3\right) \right| + k_2 \cdot \left| X_2\left(m_p, c_1, c_2, c_3\right) \right| + \\ + k_3 \cdot \dfrac{1}{\left| X_4\left(m_p, c_1, c_2, c_3\right) \right|} + k_4 \cdot \dfrac{1}{\left| X_2\left(m_p, c_1, c_2\right) - X_3\left(m_p, c_1, c_2\right) \right|} \end{bmatrix} \Rightarrow \max.$$

Numerical Solution of Optimization Problems

For further solving of optimization problems, let us prescribe the parameters of the implemented structure of the vibratory finishing machine [2], [5], [14]: $m_a = 50$ kg, $m_n = 50$ kg, $m_в = 1.5$ kg, $c_{i3} \approx 0\ \frac{\text{Н}}{\text{м}}$, $w \approx 314\ \frac{\text{rad}}{\text{s}}$, and let us neglect the dissipation coefficients.

Let us numerically solve the problem of optimization of inertial and stiffness parameters of the three-mass oscillating system of the vibratory finishing machine with a help of the function of searching the local maximum in MathCAD software (Fig. 2). As a result, we obtain the values of the parameters to be defined: $m_p = 2.07$ kg, $c_1 = 2.515 \cdot 10^6\ \frac{\text{N}}{\text{m}}$, $c_2 \approx 0\ \frac{\text{N}}{\text{m}}$. The obtained values are almost the same as the values determined in [2] and [5], and ensure one of the most efficient modes of “lap over lap” dressing. Herewith, the oscillating system uses the effect of “zero-stiffness” when the reactive mass and the lower lap move in-phasely with equal amplitudes. Due to the existence of in-phase motion, the air gap in electromagnetic vibration exciter can be minimal because in such a case there is no relative oscillations amplitudes between two adjacent masses. This allows significant decreasing of the value of current flowing in the winding of the vibration exciter, as well as the power consumed.

$ma := 50 \quad mn := 50 \quad P := 600 \quad ciz := 0 \quad \omega := 314 \quad k1 := 0.333 \quad k2 := 0.333 \quad k4 := 0.333$

$$X1(mp, c1, c2) := \left[\frac{P \cdot \omega^2 \cdot c1 \cdot mp}{c2^2 \cdot \left(c1 - \omega^2 \cdot ma\right) + c1^2 \cdot \left(c2 - \omega^2 \cdot mp\right) - \left(c1 - \omega^2 \cdot ma\right) \cdot \left(c2 - \omega^2 \cdot mp\right) \cdot \left(c1 - mn \cdot \omega^2 + c2 + ciz\right)}\right]$$

$$X2(mp, c1, c2) := \left[\frac{P \cdot \omega^2 \cdot mp \cdot \left(c1 - \omega^2 \cdot ma\right)}{c2^2 \cdot \left(c1 - \omega^2 \cdot ma\right) + c1^2 \cdot \left(c2 - \omega^2 \cdot mp\right) - \left(c1 - \omega^2 \cdot ma\right) \cdot \left(c2 - \omega^2 \cdot mp\right) \cdot \left(c1 - mn \cdot \omega^2 + c2 + ciz\right)}\right]$$

$$X3(mp, c1, c2) := \left[-\frac{P \cdot \left(\omega^2 \cdot c1 \cdot ma - c1 \cdot ciz + \omega^2 \cdot c1 \cdot mn + \omega^2 \cdot ciz \cdot ma - \omega^4 \cdot ma \cdot mn\right)}{c2^2 \cdot \left(c1 - \omega^2 \cdot ma\right) + c1^2 \cdot \left(c2 - \omega^2 \cdot mp\right) - \left(c1 - \omega^2 \cdot ma\right) \cdot \left(c2 - \omega^2 \cdot mp\right) \cdot \left(c1 - mn \cdot \omega^2 + c2 + ciz\right)}\right]$$

$$\Delta(mp, c1, c2) := \left(k1 \cdot \left|X1(mp, c1, c2)\right| + k2 \cdot \left|X2(mp, c1, c2)\right|\right) + k4 \cdot \frac{1}{\left|X2(mp, c1, c2) - X3(mp, c1, c2)\right|}$$

$FF := \mathrm{Maximize}(\Delta, mp, c1, c2)$

$$FF = \begin{pmatrix} 2.07 \\ 2.515 \times 10^{6} \\ 2.051 \times 10^{-10} \end{pmatrix}$$

$mp := FF_0 \quad c1 := FF_1 \quad c2 := FF_2$

***Fig. 2.** The example of solving the problem of optimization of inertial and stiffness parameters of the three-mass oscillating system of the vibratory finishing machine in MathCAD software*

Having defined the optimal parameters of the three-mass oscillating system of the vibratory finishing machine, which ensure the efficient “lap over lap” dressing, it is necessary to determine the stiffness c_3, when the carrier with parts being treated is placed between the upper and the lower laps. To do this, let us use the optimization function (8) for the four-mass oscillating system. Numerically solving the optimization problem with a help of the function of the local maximum searching in MathCAD software (Fig. 3), we obtain the following value of stiffness of the elastic element between the lower lap and the carrier: $c_3 \approx 0 \ \frac{\text{N}}{\text{m}}$. Therefore, we may state that there should be an air gap between the lower lap and the carrier having the size, which not exceeds the difference between the amplitudes of their vibrations (i.e., the amplitude of the lower lap because the carrier should be almost unmovable). In such a case, there is ensured a uniformity of dressing of the upper and the lower edge surfaces of the parts being treated.

Construction of Amplitude-Frequency Characteristics and Displacement-Time Dependencies

Having numerically solved of the optimization problem (8) and determining the values of inertial and stiffness parameters of the oscillating systems if the vibratory finishing machines, based on the equations (3) and (4), let us construct their amplitude-frequency characteristics (Fig. 4) and time dependencies of displacements of the oscillating masses (Fig. 5, *a*). In order to confirm the theoretical approaches obtained in the present article, there was carried out a numerical solving of the system of differential equations (1) with a help of the RADAUS method in MathCAD software (Fig. 5, *b*).

Analyzing the obtained graphical dependencies of the oscillations amplitudes of masses of the vibratory finishing machine within the range of frequencies close to resonance (Fig 4), we can see the equality of the amplitudes of oscillations of the intermediate mass (lower lap) and the reactive mass (electromagnets armature). This states about their in-phase motion and the realization of “zero-stiffness” effect. The amplitude of oscillations of the active mass (upper lap) is in the antiphase to the amplitudes of the other two masses, and by the absolute value, it is almost equal to them. (Fig. 4). This is confirmed by the results of modelling (simulation) of the oscillating masses motion using the analytical dependencies (3) and by the results of numerical solving of the system of differential equations of the oscillating masses motion in the MathCAD software (Fig. 5). Approximately at 0.5 s after the system is started (switched on), one can see the synchronous motion of the intermediate and the reactive mass in in-phase to the motion of the active mass (Fig. 5, b). Herewith, the amplitudes of oscillations of the intermediate and the reactive masses become equal approximately at 1 s after the system is started (Fig. 5).

$$ma := 50 \quad mn := 50 \quad P := 600 \quad ciz := 0 \quad \omega := 314 \quad k1 := 0.25 \quad k2 := 0.25 \quad k3 := 0.25 \quad k4 := 0.25$$

$$c1 := 2.515 \cdot 10^{6} \quad c2 := 0 \quad mp := 2.07 \quad mv := 1.5$$

$$X1(c3) := \left[\frac{P\cdot\omega^2\cdot c1\cdot mp\cdot\left(c3-\omega^2\cdot mv\right)}{c3^2\cdot\left(c1-\omega^2\cdot ma\right)\cdot\left(c2-\omega^2\cdot mp\right)+c2^2\cdot\left(c1-\omega^2\cdot ma\right)\cdot\left(c3-\omega^2\cdot mv\right)\ldots+\left[c1^2\cdot\left(c2-\omega^2\cdot mp\right)\cdot\left(c3-\omega^2\cdot mv\right)\ldots+\left[-\left[\left(c1-\omega^2\cdot ma\right)\cdot\left(c2-\omega^2\cdot mp\right)\cdot\left(c3-\omega^2\cdot mv\right)\cdot\left(c1-mn\cdot\omega^2+c2+c3+ciz\right)\right]\right]\right]}\right]$$

$$X2(c3) := \left[\frac{P\cdot\omega^2\cdot mp\cdot\left(c1-\omega^2\cdot ma\right)\cdot\left(c3-\omega^2\cdot mv\right)}{c3^2\cdot\left(c1-\omega^2\cdot ma\right)\cdot\left(c2-\omega^2\cdot mp\right)+c2^2\cdot\left(c1-\omega^2\cdot ma\right)\cdot\left(c3-\omega^2\cdot mv\right)\ldots+\left[c1^2\cdot\left(c2-\omega^2\cdot mp\right)\cdot\left(c3-\omega^2\cdot mv\right)\ldots+\left[-\left[\left(c1-\omega^2\cdot ma\right)\cdot\left(c2-\omega^2\cdot mp\right)\cdot\left(c3-\omega^2\cdot mv\right)\cdot\left(c1-mn\cdot\omega^2+c2+c3+ciz\right)\right]\right]\right]}\right]$$

$$X3(c3) := \left[\frac{P\cdot\left[c1\cdot c3\cdot ciz+\omega^4\cdot c3\cdot ma\cdot mn+\omega^4\cdot c1\cdot ma\cdot mv+\omega^4\cdot c3\cdot ma\cdot mv+\omega^4\cdot c1\cdot mn\cdot mv\ldots+\left[\omega^4\cdot ciz\cdot ma\cdot mv-\omega^6\cdot ma\cdot mn\cdot mv-\omega^2\cdot c1\cdot c3\cdot ma-\omega^2\cdot c1\cdot c3\cdot mn\ldots+\left(0-\omega^2\cdot c1\cdot c3\cdot mv-\omega^2\cdot c3\cdot ciz\cdot ma-\omega^2\cdot c1\cdot ciz\cdot mv\right)\right]\right]}{c3^2\cdot\left(c1-\omega^2\cdot ma\right)\cdot\left(c2-\omega^2\cdot mp\right)+c2^2\cdot\left(c1-\omega^2\cdot ma\right)\cdot\left(c3-\omega^2\cdot mv\right)\ldots+\left[c1^2\cdot\left(c2-\omega^2\cdot mp\right)\cdot\left(c3-\omega^2\cdot mv\right)\ldots+\left[-\left[\left(c1-\omega^2\cdot ma\right)\cdot\left(c2-\omega^2\cdot mp\right)\cdot\left(c3-\omega^2\cdot mv\right)\cdot\left(c1-mn\cdot\omega^2+c2+c3+ciz\right)\right]\right]\right]}\right]$$

$$X4(c3) := \left[\frac{P\cdot\omega^2\cdot c3\cdot mp\cdot\left(c1-\omega^2\cdot ma\right)}{c3^2\cdot\left(c1-\omega^2\cdot ma\right)\cdot\left(c2-\omega^2\cdot mp\right)+c2^2\cdot\left(c1-\omega^2\cdot ma\right)\cdot\left(c3-\omega^2\cdot mv\right)\ldots+\left[c1^2\cdot\left(c2-\omega^2\cdot mp\right)\cdot\left(c3-\omega^2\cdot mv\right)\ldots+\left[-\left[\left(c1-\omega^2\cdot ma\right)\cdot\left(c2-\omega^2\cdot mp\right)\cdot\left(c3-\omega^2\cdot mv\right)\cdot\left(c1-mn\cdot\omega^2+c2+c3+ciz\right)\right]\right]\right]}\right]$$

$$\Delta(c3) := k1\cdot\left|X1(c3)\right|+k2\cdot\left|X2(c3)\right|+k3\cdot\frac{1}{\left|X4(c3)\right|}+k4\cdot\frac{1}{\left|X2(c3)-X3(c3)\right|}$$

$$FF := \mathrm{Maximize}(\Delta, c3)$$

$$FF = 6.476\times 10^{-10} \qquad c3 := FF$$

Fig. 3. *The example of solving the problem of optimization of inertial and stiffness parameters of the four-mass oscillating system of the vibratory finishing machine in MathCAD software*

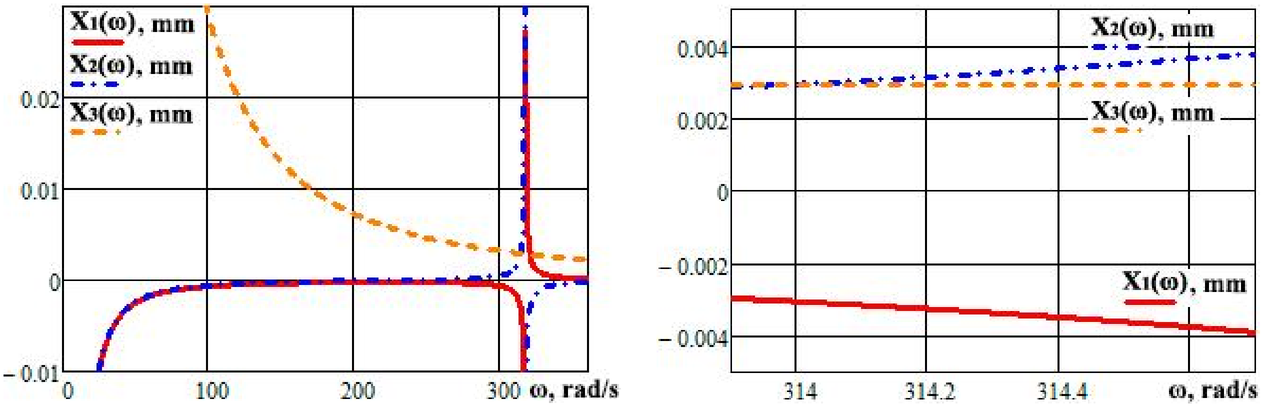
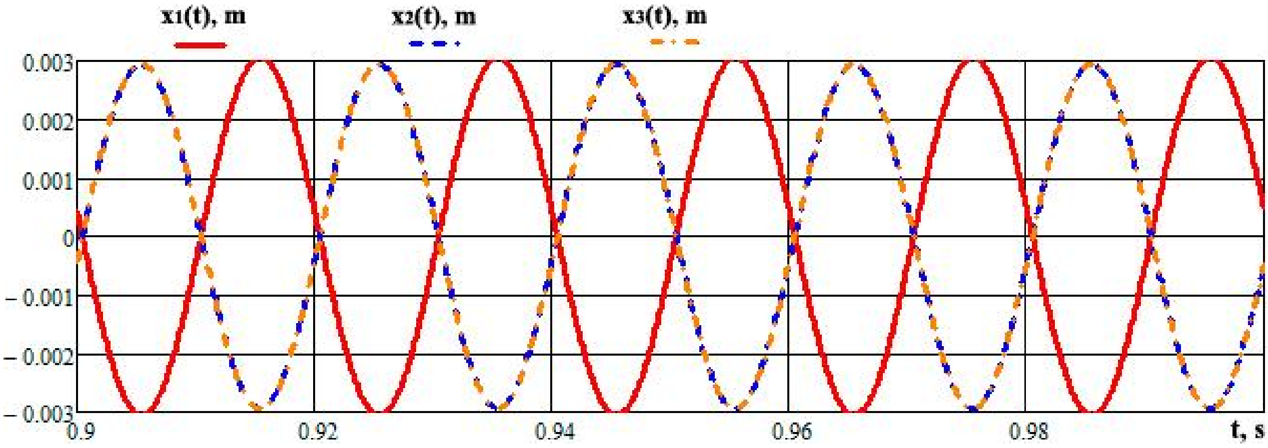
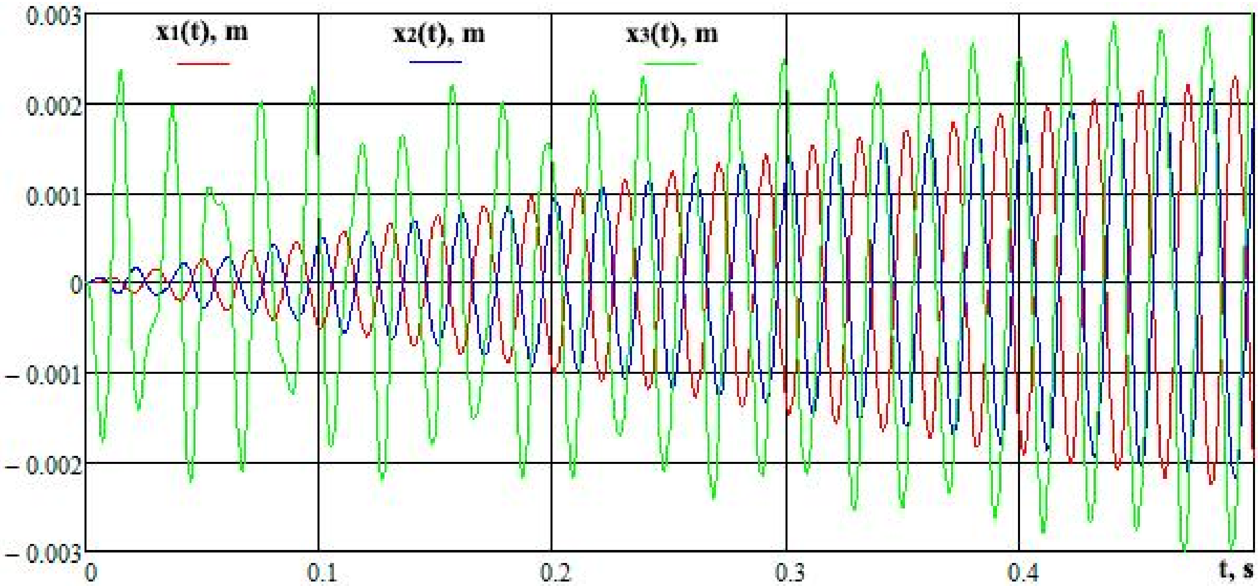


Fig. 4. *Amplitude-frequency characteristics of the three-mass oscillating system of the vibratory finishing machine: 1 – activve mass; 2 – intermediate mass; 3 – reactive mass*



a



b

Fig. 5. *Time dependencies of the oscillating masses displacements: a – constructed on the basis of the formulas (3); b – constructed by the results of the numerical solving of the system of differential equations (1) in MathCAD software; 1 – activve mass; 2 – intermediate mass; 3 – reactive mass*

Therefore, the obtained values of the inertial and stiffness parameters of the oscillating systems ensure the highest efficiency of the vibratory finishing machine operation [2], [5]. This is explained by the presence of the additional (extra) resonance peak, which is inherent to three-mass mechanical oscillating systems. In this case, taking into account the assumed consumption about $c_{i3} \Rightarrow 0 \ \frac{\text{N}}{\text{m}}$, the second resonance peak coincides with the zero-frequency of the forced oscillations (Fig. 4).

Conclusions

In comparison with the traditionally constructed mechanical oscillating systems, in the present paper, we constructed the planar principal and calculation diagrams of the vibratory finishing machine, which graphically describes the full pattern of the oscillating masses motion caused by the forced disturbance. The considered diagrams allow the further significant simplification of the existence structures of the oscillating systems of the vibratory finishing machines and the construction of their spatial diagrams.

The presented mathematical models of the three-mass and the four-mass oscillating systems allow the determination of the amplitude values of the oscillating masses displacements according to the prescribed dressing modes at the stage of designing the vibratory finishing machine.

On the basis of the derived mathematical models, the dynamics of the non-stationary processes of the machine operation was analyzed; the amplitude-frequency characteristics of the corresponding structures were constructed; the inertial and stiffness parameters of the oscillating systems were substantiated and the prospects of further investigations are considered.

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