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ANALYSIS OF STRUCTURE AND KINEMATICS OF FOUR-BAR CRANK-ROCKER WALKING MECHANISM

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Abstract. Problems and prospects of using the walking mode for producing motion of robotic systems are considered. Advantages and spheres of the use of mobile robotic systems equipped with walking movers are substantiated. Preferences of cyclic (lever) walking mechanisms are analyzed. The kinematic parameters of four-bar crank-rocker walking mechanism, constructed on the basis of Chebyshov-Umnov mechanism, are accepted as the subject of research. The process of motion of the supporting foot of the walking mechanism is accepted as the object of research. The main aim of the investigation consists in carrying out structure and kinematics analysis of the mentioned mechanism with further derivation of analytical dependencies for calculating kinematic parameters of the supporting foot motion. The special features of the structure of four-bar walking mechanism and theoretical foot loci (paths) are considered. The form of the most reasonable path for further research is established. As a result of carried out structure analysis, it is ascertained that the mechanism consists of two structural groups: that of the first class and that of the second class second order first type. As a result of kinematics analysis, the analytical expressions for calculation of coordinates of the supporting foot hinge of the walking mechanism depending on geometric parameters of the mechanism and the angle of the crank rotation are deduced. The prospects of further investigation under the themes of the paper are analyzed. In particular, the goal function, which may be used for further optimization of structural parameters of the mechanism, is considered and the use of applied software for solving optimization problem is suggested. The optimization problem consists in evaluating such geometric parameters of the walking mechanism which allow the motion of the supporting foot in accordance with the prescribed (specified) path.

Introduction

The research of alternative movers for transportation vehicles and mobile technological complexes has been held since the time of wheel invention. The caterpillar and walking movers, created in the 19-th century, have steadfastly taken their places in modern technique [1; 2]. The research and development of walking movers and machines are being held in all advanced countries. At the same time because of structure complicity, dynamic unbalance and low power efficiency of walking movers, the introduction of walking machines and robots into industry and agriculture encounters dires difficulties [2; 3]. In spite of the fact that the number of established models of walking machines and robots may be estimated in several hundreds, the overwhelming majority of them are presented as laboratory mock-ups, which cannot be used in actual operating conditions and aren’t adapted for real-time use.

The designing of walking machines and robots is hindered by the complexities of practical realization of optimal motion algorithm that meet certain criterions. First of all, this problem is associated with difficulty of organization of the interaction between the components of the walking robot and also with the imperfection of the theory of optimal motion of walking machines. For using in various operation conditions and performing large range of tasks, which may be set for walking machines and robots, it is necessary to provide a number of important motion parameters: power efficiency, cross-country capability, roadblocks overcoming, maneuverability, limitation of maximal loads in the transmission, motion comfortableness (limitation of maximal accelerations of the machine body), shockless interaction between the pivot foot and the ground in predetermined positions etc. [1–4].

Walking motion is of unique interest due to its implications for human and animal pathology. Current walking robots require multiple actuators (motors) because they have multiple degrees of freedom. Furthermore, they require complex control strategies to enable stable walking. These two characteristics have resulted in walking prototypes that are complicated, expensive, heavy, and energetically inefficient [3]. That's why it is necessary to improve single-degree-of-freedom kinematic mechanisms that coordinate all of the robot's movement with the aim of providing high power efficiency, cross-country capability, roadblocks overcoming, maneuverability etc.

Problem statement

Ground-based transport vehicles should be able to move in various operational conditions depending on their designated purpose. In the case when these conditions require vehicles operating at the territory with randomly located roadblocks, the usage of the machines with traditional movers (drives) may be essentially complicated. Advanced scientific and research institutions carry out different investigations associated with walking equipment, which corresponds to the mentioned conditions: exoskeletons (Raytheon, USA; Cyberdyne, Daiwa Hous, Japan); anthropomorphic robots (HONDA, Japan; BostonDynamics, USA); walking cargo conveyors (Plustech, Finland; BostonDynamics, USA; Volgograd State Technical University, Russia) [1–11].

During the transportation of large-sized ladings along the roads with light soils and low bearing strength (transportation of heavyweight structures for industrial enterprises) or along the territories which belong to nonrenewal categories (peat (turf) extracting, self-propelled field systems of irrigation) it is reasonably to use walking vehicles with cyclic movers [1; 2]. Cyclic biped walking movers used in drives of mobile robotic systems are distinguished by the structure and control simplicity (one degree of freedom). However, these walking movers have several faults (absence of rectilinear supporting phase on the path of typical foot point (hinge), disproportionate of horizontal component of velocity of the center of gravity of the machine frame etc.), which decrease energy inefficiency of the walking machine (recurrent lifting and sinking, accelerating and braking of the frame center of gravity) [3; 5; 6]. That's why improvement of the cyclic walking mover with the aim of obtaining rectilinear phase on the path of the foot and constant velocity of the center of gravity of the machine frame during the supporting phase is very urgent and interesting problem [1; 2; 12–14].

Analysis of modern information sources on the subject of the article

The experience of development of transporting and technological systems with walking principle of moving shows us that it is possible to distinguish following actively forming directions of walking machines: exoskeletons, anthropomorphic robots, walking transporting and technological machines etc. There exist several types of walking movers, which differ from one another with the number of controlled degrees of freedom: in the form of open (unlocked) kinematic loops, orthogonal, pantographic, cyclic (lever), arachnoid, "horsy"; hexapod etc. [1–3; 5; 6; 8; 12–14].

For the vehicles of soil trafficability it is preferably to use cyclic movers [1; 2]. The control of cyclic mover is carried out by the only drive while power inputs for supporting of machine weight may be minimized with the help of certain law of the frame motion [1–3; 5; 12–14]. The typical feature of the biped walking machine consists in use of doubled (paired) walking movers composed of two walking mechanisms, which are kinematically interlocked [1; 2]. Usually in the doubled (paired) walking mover at every point of time, one of the walking mechanisms is in the support phase while the other one is in the return phase. Thus, every walking mover with one of its two foots always lean upon the ground. This increases the stability of the machine and simplifies the algorithms of its control [1; 2].

Distinctive features of four-bar cyclic mechanism, constructed on the basis of Chebyshov-Umnov mechanism, are the absence of rectilinear supporting phase on the path of typical foot point (hinge), disproportionate of horizontal component of velocity of the center of gravity of the machine frame etc. [1; 2]. That's why the research aimed at the improving of existent walking mechanisms by means of optimization of their geometric parameters and use of larger number of links is continuously carrying out. In particular, in publications [12–14] the results of investigation of structure and kinematics of four-bar and six-bar cyclic walking mechanisms of mobile robotic systems are presented. Further research concerned with previous results consists in optimization of geometric parameters of analyzed walking mechanisms.

Statement of purpose and problems of research

The main aim of this investigation is carrying out the structure and kinematics analysis of four-bar walking mechanism of mobile robotic system, which is constructed on the basis of Chebyshov-Umnov mechanism. As a result of carried out investigation the analytical expressions for calculating kinematic parameters of the mechanism links and, particularly, supporting foot. The deduced expressions may be used while carrying out further structure and parametric synthesis and optimization of four-bar crank-rocker walking mechanism with the aim of providing the motion of the supporting foot in accordance with the prescribed (specified) path.

Four-bar crank-rocker walking mechanism and theoretical foot paths

Determinate linkage motion results when the number of independent input angular motions is two less than the number of links. All links are assumed to be rigid members and are pin-connected to one another. Freedom of relative angular motion exists between any two members at the pin joint. The minimum number of links which will permit relative motion between links is four. In the majority of applications one of the links (the line of centers) is stationary while a second link (the driving crank) is driven from an outside motion source. The motion of the remaining two links is a function of the geometry of the linkage and the motion of the driving crank and the line of centers [4].

A mechanism is an assembly of rigid bodies, referred to as links, that are connected via joints. Joints restrict the movement of two links relative to one another. The most common joint type is called a revolute joint. Revolute joints allow purely rotational motion along the joint axis. Another type of joint is the prismatic joint, also known as a slider joint. The prismatic joint allows one link to slide in only one coordinate direction relative to another link. There is also a spherical joint. A spherical joint allows rotation of a joint in all three directions relative to the other joint. Revolute and prismatic joints allow one degree of freedom between the links they connect, while a spherical joint allows three degrees of freedom [3].

From the point of view of reliability, it would be preferable to design the legs of a walking robot with only revolute pairs and avoid prismatic pairs. Since reduction of complexity is one of the prime objectives of a designer, the four-bar linkage would be ideal for a leg, provided a suitable coupler curve is available. The ideal coupler curve would be in the form of a "*D*" with the straight side of the *D* facing away from the mechanism envelop. In addition, a nearly constant velocity in the straight portion of the *D* would be preferable [5]. Unfortunately, no four-bar coupler curve satisfies these criteria.

Fig. 1 represents the principal diagram of the four-bar crank-rocker walking mechanism. The crank or driver is link O_1A. It rotates about a stationary center at O_1, which is located on the machine frame. Link O_2B is the rocker or oscillating follower, and it rotates about another stationary center at O_2, which is also located on the machine frame. Crank O_1A can make a complete revolution while the crank O_2B can only oscillate. The linkage is operating as so-called crank-rocker linkage. Link ABC is called the coupler, and it is connected to the foot at point C. The connecting link ABC in general moves in combined translation and rotation. Link O_1O_2 is the frame of the machine [4; 6].

The foot of a walking mechanism is the part of the mechanism that comes in direct contact with the ground (Fig. 1). As the crank turns, the foot traces out a cyclical path relative to the body of the walker. This path is known as the locus. In generating this path the vehicle is imagined to be stationary, and the terrain or roadway to be moving backward at the vehicle velocity. The locus can be divided into four parts: the support, lift, return, and lower phases. These phases are illustrated in Fig. 2. Throughout the support phase, the foot is ideally in contact with the ground. During the lift the foot is moving toward its maximum height in the locus. During the return, the foot reaches its maximum height off the ground and moves in the same direction as the body of the walker. Finally, during the lower the foot descends in height until it makes contact with the ground [7]. The shape of the lift and lower portions of the curve is useful in determining the action of the foot in initial and final contact with the terrain. Additionally, the lower phase determines the step height of the walker. The shape of the return portion of the curve is immaterial except as it affects the peak acceleration of the cycle [6].

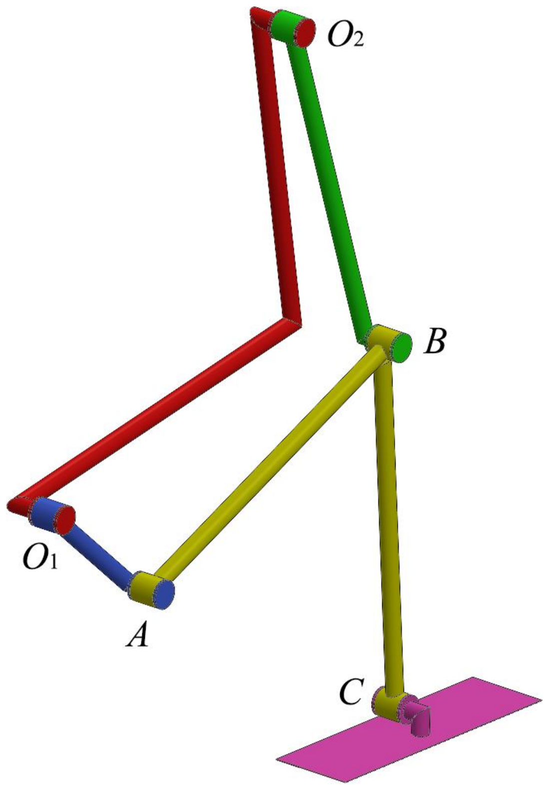

Fig. 1. *Principal diagram of the four-bar crank-rocker walking mechanism*

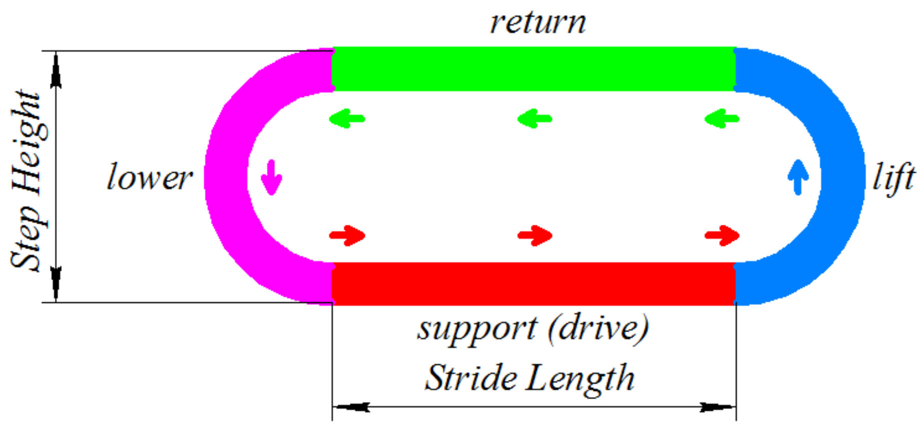

Fig. 2. *Phases of a foot locus*

Five theoretical loci are illustrated in Fig. 3. The symmetry of locus in Fig. 3, ***a*** means that inertial effects on the crank will me minimized by properly offsetting the relative phase of the legs in the walker. Its lift and lower phases resemble that of a wheel; ideally, their acceleration characteristics should resemble that of a wheel. However, it will not make for an energy efficient walking mechanism because the length of the support phase is small [7]. In fact this illustrates that a wheel may be classified as a special case of a walking mechanism.

The loci of Fig. 3, ***b*** and ***c*** represent paths which can be generated by four-bar linkages. In some cases a loop exists at the pointed end. Unsatisfactory performance may be expected from both of these because of the high accelerations at the pointed ends, and because of the manner in which the foot contacts or finishes contact with the roadway at the pointed ends. Locus ***b*** is especially undesirable because of its low step height; it may have problems stepping over irregularities in terrain or cause dragging if it makes contact with the ground during the lower [6; 7].

The rectangular locus in Fig. 3, ***e*** offers distinct advantages. It gets the foot off the ground quickly and in the most direct manner at the end of stride and replaces the foot in the same manner at the beginning of stride. But time is required for the lower phase of the action as well as for the drive phase. If the foot should encounter the roadway before the lower phase is completed the vehicle would either pause for the remaining interval, or sliding would occur [6]. Furthermore, where velocities are at all appreciable a locus having corners is abhorred by nature. The result would be infinitely large accelerations resulting in unsatisfactory performance and life.

The locus of Fig. 3, ***d*** is an ideal one [6]. It has a long, straight, stride. The return portion of the locus is direct, and promises to give minimum accelerations. The lift and lower phases of the action resemble that of a wheel and should have acceleration characteristics not too different from a wheel. Furthermore, if the locus is symmetrical about both horizontal and vertical center lines, then the possibility exists of getting balanced inertia forces by properly phasing a number of walking mechanisms on like paths. If the path is symmetrical then vehicle reversibility is no problem. And finally, the shape of the lift and lower phases seems to be the best compromise between the circular locus of Fig. 3, ***a*** and the rectangular one of Fig. 3, ***b***.

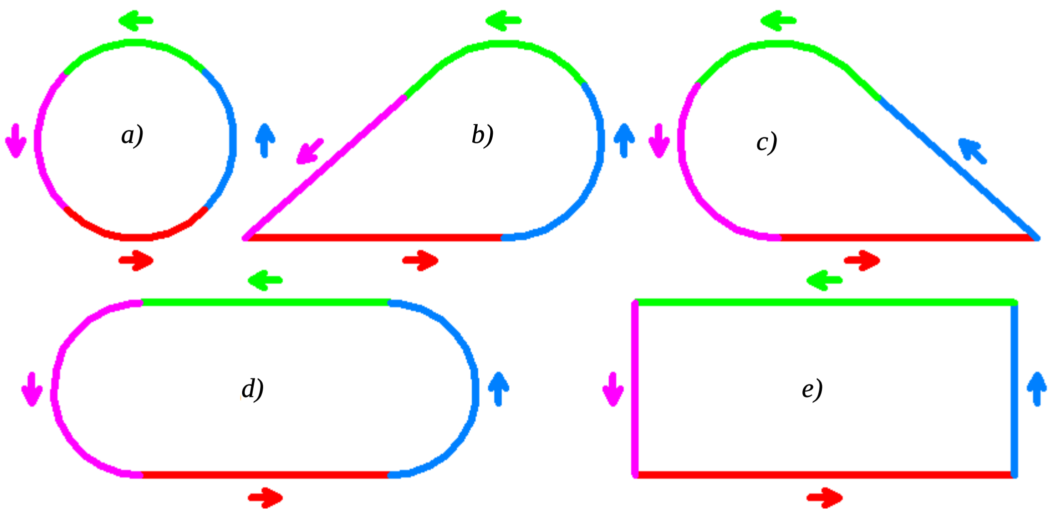

Fig. 3. *Theoretical foot loci*

The problem of providing the ideal foot locus includes two major points. The first point is that reduced up-and-down movement of the center of gravity during walking increases the energy efficiency of the walking machine. Improvement of the energy efficiency is an important factor to be taken into consideration especially for energy self-sustaining mobile robot. The second point is that a gravitationally decoupled actuator system becomes possible by adding a leg length changing mechanism, which also relates to energy saving. Although it is ideal to make the foot trajectory of the stance phase exactly straight, it is generally difficult to get such a trajectory using a limited number of actuators [8]. Therefore, for a practical application, an approximate straight line instead of an exact straight line is sufficient to realize gravitationally decoupled actuator system, if degree of an approximate straight line is fine.

Analysis of the mechanism structure

The number of degrees of freedom of a mechanism is specifically referred to as the mobility. The mobility of a mechanism is the number of coordinates needed to specify the positions of all members of the mechanism relative to a particular member chosen as the base or frame [3]. Said another way, if a mechanism has a single degree of freedom, the entire configuration of the mechanism is known if one link angle is defined. If a mechanism has two degrees of freedom, the entire configuration of the mechanism is known if two link angles are defined, and so on. If all the motions of a mechanism are confined to parallel planes, then the mechanism is said to be planar. Planar mechanisms may be visually represented very easily on a 2-D surface since their motion is limited to parallel planes [3].

The simplest mechanism to reasonably produce the walking motion was determined to be a four-bar linkage. The mechanism, as shown in Fig. 4, ***a***, has four links and four binary joints. Three bars are movable (O_1A, O_2B, ABC) and another bar (0) is considered to be the reference or base link. The base link O_1O_2 is the body of the mechanism. The crank or driver is link O_1A. It rotates about a stationary center at O_1, which is located on the machine frame. Link O_2B is the rocker or oscillating follower, and it rotates about another stationary center at O_2, which is also located on the machine frame. Crank O_1A can make a complete revolution while the crank O_2B can only oscillate. The linkage is operating as so-called crank-rocker linkage. Link ABC is called the coupler, and it is connected to the foot at point C. The connecting link ABC in general moves in combined translation and rotation [4; 6].

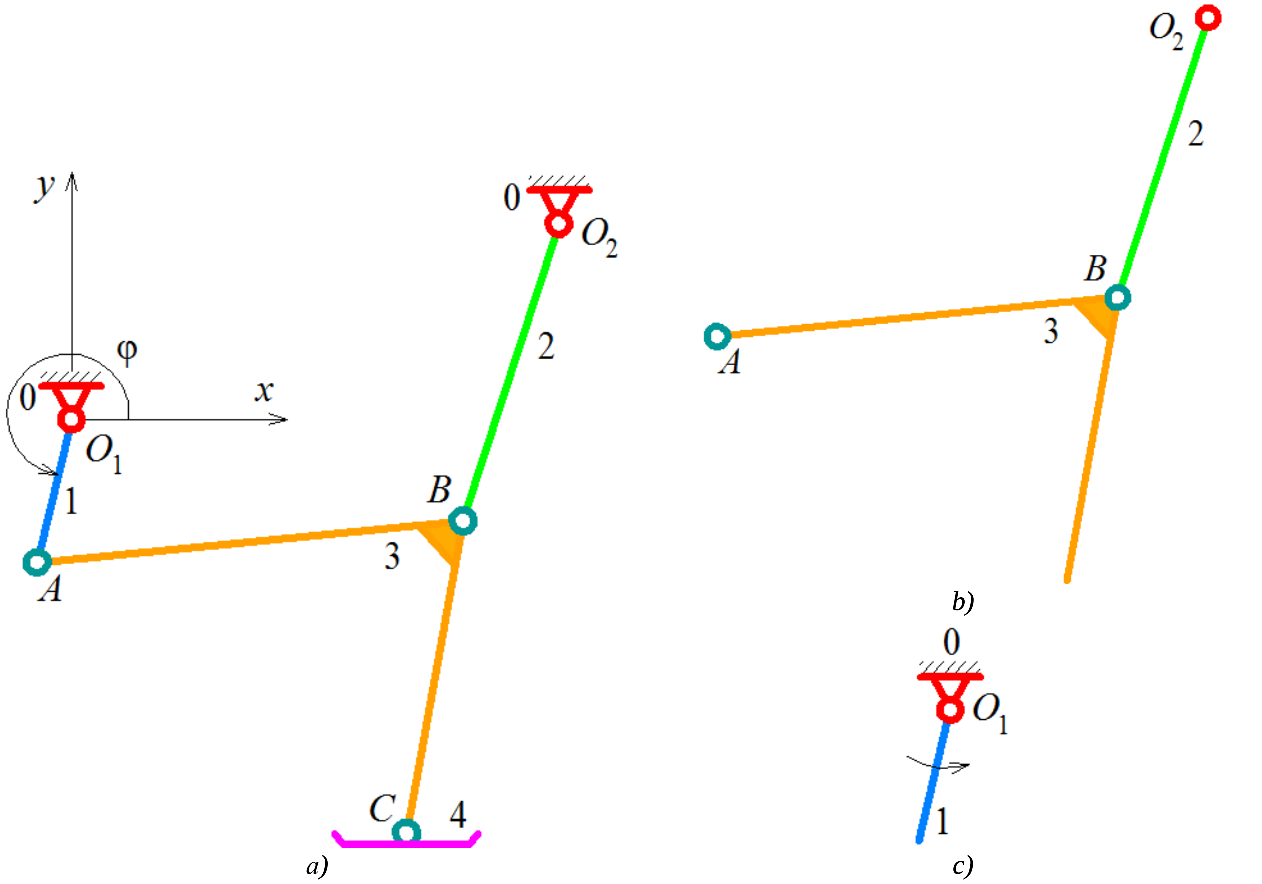

Fig. 4. Diagram of the mechanism structure (a) and its structural groups (b, c)

Walking mechanism (Fig. 4, *a*) consists of three movable links ($n = 3$): O_1A, O_2B, ABC and four kinematic couples (joints) of the V class ($p_5 = 4$): $O_1(0{,}1)$; $A(1{,}3)$; $O_2(0{,}2)$; $B(2{,}3)$. There are no higher kinematic couples in the mechanism, that is $p_4 = 0$. The degree of freedom in the hinge C is uncontrolled and serves for self-adapting of the foot 4 according to the supporting surface irregularities. That's why, the total number of degrees of freedom of the four-bar walking mechanism may be calculated with the help of the formula derived by Chebyshov [1; 9]:

$$W = 3 \cdot n - 2 \cdot p_5 - p_4 = 3 \cdot 3 - 2 \cdot 4 - 0 = 1. \tag{1}$$

Therefore, the walking mechanism has one degree of freedom and thus one input link and one independent coordinate, which exactly determines the position of all other parts of the mechanism. Let's accept the crank 1 with rotation axis O_1 (Fig. 4, *a*) as an input link for the walking mechanism. In that case the angle φ, which defines the deflection of the crank 1 from the horizontal axis directed from the point O_1 to the right (Fig. 4, *a*), may be concerned as the generalized coordinate for further kinematics and dynamics analysis. The positive direction of the angle φ reference is counterclockwise.

With the aim of choosing the rational method of further kinematics analysis of the walking mechanism (Fig. 4, *a*) let's expand it into the groups of Assur. Let's separate the loop which consists of two links 2 and 3 and three kinematic couples (joints) A, O_2, B (Fig. 4, *b*). After this we receive one input link 1 (Fig. 4, *c*), which is characterized by $W = 1$. Therefore, the mentioned loop may be concerned as the structural group of the second class second order first type [9]. Let's write down the structure formula of the four-bar walking mechanism:

$$I(\mathit{fixed\ pillar}\ \ 0,\ \mathit{crank}\ \ 1) \rightarrow II(\mathit{coupler}\ \ 3,\ \mathit{rocker}\ \ 2). \tag{2}$$

In order to analyze the walking mechanism, the equations of motion of the foot joint are needed. This means that the coordinates of the foot joint expressed as a function of the input link rotation, the lengths of each link in the mechanism, and the angle of the base link are needed [3]. Based on the results of structure analysis let's accept the method of closed vector loops for further kinematics analysis of the walking mechanism. This method was developed by V.A. Zinoviev and consists in expanding the mechanism into separate closed vector loops and deducing vector equations of closeness of each loop [9].

Kinematic analysis of the mechanism

The first step in deriving the equations of motion of the mechanism is to express each link as a vector. The kinematic diagram of the four-bar crank-rocker walking mechanism is shown in Fig. 5. It is known that a four-bar mechanism can be identified by 10 geometrical parameters. All the 10 parameters are free, but in this paper, only 7 of them will be defined as a result of the further optimization procedure (because the location of the crank pivot O_1 remains unchanged). The parameters are identified by [10]:

– the four link lengths are O_1A, O_2B, AB, $O_1O_2 = \sqrt{(x_{O_2} - x_{O_1})^2 + (y_{O_2} - y_{O_1})^2} \underset{\substack{x_{O_1}=0 \\ y_{O_1}=0}}{=} \sqrt{x_{O_2}^2 + y_{O_2}^2}$;

– the four revolute joint axes are centered at points O_1, A, B and O_2 which are identified by two Cartesian coordinates: $O_1(0;\ 0)$, $A(x_A;\ y_A)$, $B(x_B;\ y_B)$ and $O_2(x_{O_2};\ y_{O_2})$. The location of the driving crank pivot $O_1(0;\ 0)$ remains unchanged in our approach;

– the angles considered in the four-bar mechanism are referred to the right direction of the horizontal axis O_1x of the global coordinate system fixed to the mechanism frame (positive if counterclockwise) and are defined as follows: $\varphi = \gamma_1 + \gamma_{1_0}$ is the angle between vectors $\overrightarrow{O_1x}$ and $\overrightarrow{O_1A}$; γ_3 is the angle between vectors $\overrightarrow{O_1x}$ and $\overrightarrow{AB}$; γ_2 is the angle between vectors $\overrightarrow{O_1x}$ and $\overrightarrow{BO_2}$;

– the angle $\gamma_0 = \arccos\left(\dfrac{(x_{O_2} - x_{O_1})}{\sqrt{(x_{O_2} - x_{O_1})^2 + (y_{O_2} - y_{O_1})^2}}\right) \underset{\substack{x_{O_1}=0 \\ y_{O_1}=0}}{=} \arccos\left(\dfrac{x_{O_2}}{\sqrt{x_{O_2}^2 + y_{O_2}^2}}\right)$ represents the orientation of the link O_1O_2 with respect to the horizontal axis O_1x of the global coordinate system fixed to the mechanism frame (positive if counterclockwise);

– the starting inclination of the crank O_1A when the foot starts to move along the support (drive) phase is defined by the angle γ_{1_0}. For further optimization procedure let us define $\gamma_{1_0} = 0$;

– the coupler is defined as a rigid triangular body with vertices in *A*, *B* and *C*. The coupler $C(x_C;\ y_C)$ corresponds to the experimental reference point used to describe the relative motion of a point of the foot with respect to the mechanism frame. The parameter AC represents the length of the vector $\overrightarrow{AC}$ whereas the parameter β represents the angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{CB}$ (positive if counterclockwise).

In order to define the objective function of the further optimization procedure, two subproblems have to be solved at each considered stance phase stage: the four-bar mechanism configuration has to be obtained by solving the mechanism's closure equations; from the obtained four-bar mechanism configuration, the *C* point positions have to be computed [10].

With the vectors described in the Fig. 5, the vector loop equation is formed:

$$\overrightarrow{O_1A} + \overrightarrow{AC} = \overrightarrow{O_1O_2} + \overrightarrow{O_2B} + \overrightarrow{BC}. \tag{3}$$

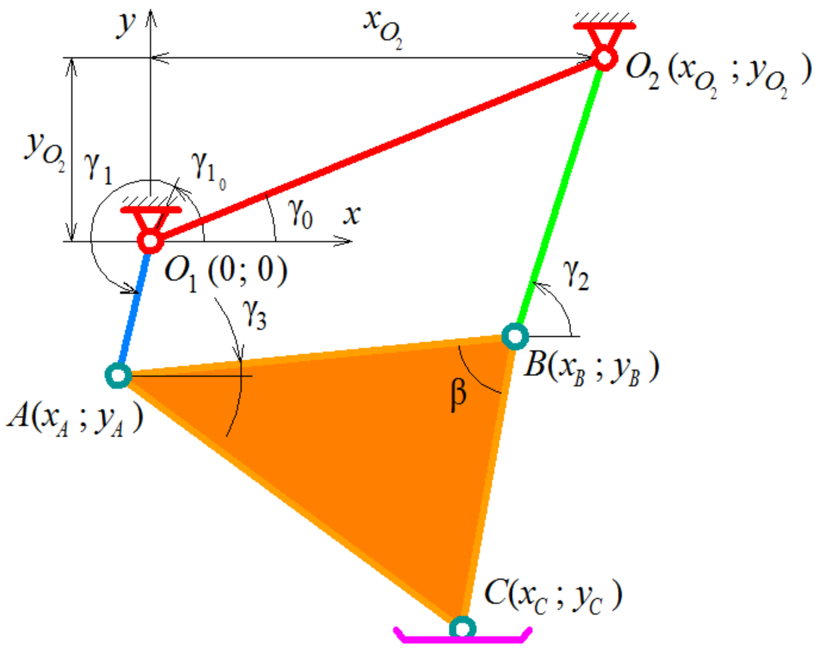

***Fig. 5.** Kinematic diagram of the walking mechanism*

The $\overrightarrow{O_2B}$ term is then isolated on the left-hand side, giving the following equation:

$$\overrightarrow{O_2B}=\overrightarrow{O_1A}+\overrightarrow{AC}-\overrightarrow{O_1O_2}-\overrightarrow{BC}. \tag{4}$$

Next, the vectors of (3) are described in their *x*- and *y*-coordinates as seen in these two equations:

$$\begin{aligned} &O_1A\cdot\cos\left(\gamma_1+\gamma_{1_0}\right)+AC\cdot\cos\left(\beta_1-\gamma_3\right)=O_1O_2\cdot\cos\left(\gamma_0\right)-O_2B\cdot\cos\left(\gamma_2\right)-BC\cdot\cos\left(\gamma_3+\beta\right);\\ &O_1A\cdot\sin\left(\gamma_1+\gamma_{1_0}\right)-AC\cdot\sin\left(\beta_1-\gamma_3\right)=O_1O_2\cdot\sin\left(\gamma_0\right)-O_2B\cdot\sin\left(\gamma_2\right)-BC\cdot\sin\left(\gamma_3+\beta\right), \end{aligned} \tag{5}$$

where β_1 is the angle between vectors $\overrightarrow{AC}$ and $\overrightarrow{AB}$; BC defines the length of the vector $\overrightarrow{BC}$. The angle β_1 and the length BC may be obtained as functions of AC, AB and β:

$$\begin{aligned} &BC=AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2};\\ &\beta_1=\arcsin\left(\frac{BC\cdot\sin(\beta)}{AC}\right)=\arcsin\left(\frac{\left(AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2}\right)\cdot\sin(\beta)}{AC}\right). \end{aligned} \tag{6}$$

On the basis of (4), (5) and (6) the following equations may be written:

$$\begin{aligned} O_2B\cdot\cos(\gamma_2)=&-\left(AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2}\right)\cdot\cos(\gamma_3+\beta)-O_1A\cdot\cos\left(\gamma_1+\gamma_{1_0}\right)-\\ &-AC\cdot\cos\left(\arcsin\left(\frac{\left(AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2}\right)\cdot\sin(\beta)}{AC}\right)-\gamma_3\right)+O_1O_2\cdot\cos(\gamma_0);\\ O_2B\cdot\sin(\gamma_2)=&-\left(AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2}\right)\cdot\sin(\gamma_3+\beta)-O_1A\cdot\sin\left(\gamma_1+\gamma_{1_0}\right)+\\ &+AC\cdot\sin\left(\arcsin\left(\frac{\left(AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2}\right)\cdot\sin(\beta)}{AC}\right)-\gamma_3\right)+O_1O_2\cdot\sin(\gamma_0). \end{aligned} \tag{7}$$

In order to eliminate the unknown γ_3 variable, the two equations (7) are squared and then added together. Completion of this step produces the following equation:

$$
\begin{aligned}
(O_2B)^2 &= (O_1O_2)^2 + \left(AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2}\right)^2 + (O_1A)^2 + (AC)^2 - \\
&- 2\cdot O_1O_2\cdot\left(AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2}\right)\cdot\cos(\gamma_0-\gamma_3-\beta) - \\
&- 2\cdot O_1O_2\cdot O_1A\cdot\cos(\gamma_0-\gamma_1-\gamma_{1_0}) - 2\cdot O_1O_2\cdot AC\times \\
&\times\cos\left(\gamma_0+\arcsin\left(\frac{\left(AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2}\right)\cdot\sin(\beta)}{AC}\right)-\gamma_3\right)+ \\
&+ 2\cdot\left(AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2}\right)\cdot O_1A\cdot\cos(\gamma_3+\beta-\gamma_1-\gamma_{1_0})+ \\
&+ 2\cdot\left(AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2}\right)\cdot AC\times \\
&\times\cos\left(\beta+\arcsin\left(\frac{\left(AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2}\right)\cdot\sin(\beta)}{AC}\right)\right)+ \\
&+ 2\cdot O_1A\cdot AC\cdot\cos\left(\gamma_1+\gamma_{1_0}+\arcsin\left(\frac{\left(AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2}\right)\cdot\sin(\beta)}{AC}\right)-\gamma_3\right).
\end{aligned}
\tag{8}
$$

Wolfram Mathametica 10 was used to perform the outlined procedure and then find the expression for γ_3. The final expression for γ_3 is:

$$
\gamma_3 = 2\cdot\arctan\left[\frac{\left[\begin{array}{c}2\cdot AB\cdot O_1O_2\cdot\sin(\gamma_0)-2\cdot AB\cdot O_1A\cdot\sin(\gamma_1+\gamma_{1_0})+ \\ +\sigma\sqrt{\begin{array}{c}\left[\begin{array}{c}(O_2B+AB)^2-\left(O_1A\cdot\cos(\gamma_1+\gamma_{1_0})-O_1O_2\cdot\cos(\gamma_0)\right)^2- \\ -\left(O_1A\cdot\sin(\gamma_1+\gamma_{1_0})-O_1O_2\cdot\sin(\gamma_0)\right)^2\end{array}\right]\times \\ \times\left[\begin{array}{c}\left(O_1A\cdot\cos(\gamma_1+\gamma_{1_0})-O_1O_2\cdot\cos(\gamma_0)\right)^2-(O_2B-AB)^2+ \\ +\left(O_1A\cdot\sin(\gamma_1+\gamma_{1_0})-O_1O_2\cdot\sin(\gamma_0)\right)^2\end{array}\right]\end{array}}\end{array}\right]}{\left[\begin{array}{c}\left(AB-O_1A\cdot\cos(\gamma_1+\gamma_{1_0})+O_1O_2\cdot\cos(\gamma_0)\right)^2- \\ -O_2B^2+\left(O_1A\cdot\sin(\gamma_1+\gamma_{1_0})-O_1O_2\cdot\sin(\gamma_0)\right)^2\end{array}\right]}\right].
\tag{9}
$$

The value of σ can either be "+1" or "-1", so there are two solutions for γ_3. These two different solutions represent two different assembly modes of the mechanism. This can also be stated as there being two distinct configurations possible using the four links [3]. In order to determine which configuration was desired for the purpose of this research, the equations of motion for each case were solved, and then qualitatively observed via a plot. One linkage configuration created an assembly similar to that shown in Fig. 5. This configuration corresponded to $\sigma = +1$. This was the desirable configuration. The other linkage configuration created an assembly distinctly different than the configuration shown in Fig. 5.

Once the angle γ_3 is computed the following equations, which describe the position of the coupler point C as a function of the mentioned parameters according to the symbols shown in Fig. 5, may be univocally determined by:

$$
\begin{aligned}
x_C &= O_1A\cdot\cos\left(\gamma_1+\gamma_{1_0}\right)+AC\cdot\cos\left(\beta_1-\gamma_3\right)=O_1A\cdot\cos\left(\gamma_1+\gamma_{1_0}\right)+AC\times\\
&\times\cos\left(\begin{array}{l}\arcsin\left(\dfrac{\left(AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2}\right)\cdot\sin(\beta)}{AC}\right)-\\
-2\cdot\arctan\left[\dfrac{\left[\begin{array}{c}2\cdot AB\cdot O_1O_2\cdot\sin(\gamma_0)-2\cdot AB\cdot O_1A\cdot\sin\left(\gamma_1+\gamma_{1_0}\right)+\\
+\sqrt{\begin{array}{l}\left[\begin{array}{c}\left(O_2B+AB\right)^2-\left(O_1A\cdot\cos\left(\gamma_1+\gamma_{1_0}\right)-O_1O_2\cdot\cos(\gamma_0)\right)^2-\\-\left(O_1A\cdot\sin\left(\gamma_1+\gamma_{1_0}\right)-O_1O_2\cdot\sin(\gamma_0)\right)^2\end{array}\right]\times\\
\times\left[\begin{array}{c}\left(O_1A\cdot\cos\left(\gamma_1+\gamma_{1_0}\right)-O_1O_2\cdot\cos(\gamma_0)\right)^2-\left(O_2B-AB\right)^2+\\+\left(O_1A\cdot\sin\left(\gamma_1+\gamma_{1_0}\right)-O_1O_2\cdot\sin(\gamma_0)\right)^2\end{array}\right]\end{array}}\end{array}\right]}{\left[\begin{array}{c}\left(AB-O_1A\cdot\cos\left(\gamma_1+\gamma_{1_0}\right)+O_1O_2\cdot\cos(\gamma_0)\right)^2-\\-O_2B^2+\left(O_1A\cdot\sin\left(\gamma_1+\gamma_{1_0}\right)-O_1O_2\cdot\sin(\gamma_0)\right)^2\end{array}\right]}\right]\end{array}\right);\\
y_C &= O_1A\cdot\sin\left(\gamma_1+\gamma_{1_0}\right)-AC\cdot\sin\left(\beta_1-\gamma_3\right)=O_1A\cdot\sin\left(\gamma_1+\gamma_{1_0}\right)-AC\times\\
&\times\sin\left(\begin{array}{l}\arcsin\left(\dfrac{\left(AB\cdot\cos(\beta)+\sqrt{AB^2\cdot\cos^2(\beta)-AB^2+AC^2}\right)\cdot\sin(\beta)}{AC}\right)-\\
-2\cdot\arctan\left[\dfrac{\left[\begin{array}{c}2\cdot AB\cdot O_1O_2\cdot\sin(\gamma_0)-2\cdot AB\cdot O_1A\cdot\sin\left(\gamma_1+\gamma_{1_0}\right)+\\
+\sqrt{\begin{array}{l}\left[\begin{array}{c}\left(O_2B+AB\right)^2-\left(O_1A\cdot\cos\left(\gamma_1+\gamma_{1_0}\right)-O_1O_2\cdot\cos(\gamma_0)\right)^2-\\-\left(O_1A\cdot\sin\left(\gamma_1+\gamma_{1_0}\right)-O_1O_2\cdot\sin(\gamma_0)\right)^2\end{array}\right]\times\\
\times\left[\begin{array}{c}\left(O_1A\cdot\cos\left(\gamma_1+\gamma_{1_0}\right)-O_1O_2\cdot\cos(\gamma_0)\right)^2-\left(O_2B-AB\right)^2+\\+\left(O_1A\cdot\sin\left(\gamma_1+\gamma_{1_0}\right)-O_1O_2\cdot\sin(\gamma_0)\right)^2\end{array}\right]\end{array}}\end{array}\right]}{\left[\begin{array}{c}\left(AB-O_1A\cdot\cos\left(\gamma_1+\gamma_{1_0}\right)+O_1O_2\cdot\cos(\gamma_0)\right)^2-\\-O_2B^2+\left(O_1A\cdot\sin\left(\gamma_1+\gamma_{1_0}\right)-O_1O_2\cdot\sin(\gamma_0)\right)^2\end{array}\right]}\right]\end{array}\right).
\end{aligned}
\tag{10}
$$

It is well known that closed curves similar to the ideal locus (Fig. 3, *d*) can be generated by a point on the coupler of a four-bar linkage. It is not difficult to cause the coupler to generate a line which is very nearly straight. Furthermore, the length of the straight-line portion of the locus can be quite large in comparison with the crank dimensions. As an additional incentive the possibility exists of finding a linkage in which the straight-line portion of the loop is generated rather slowly while the crank is turning through a major portion (say 60 to 80 per cent) of its total crank angle. This means that the crank may turn through say 270° during stride and use the remaining angle of 90° for the return events of the cycle. Thus, for slow-speed walking machines where inertia forces are not important considerations, the four-bar linkage holds out the promise of giving a vehicle with only a few legs, driven by a simple power source, and having a long stride [6]. Although there are an infinite number of possible solutions to such a problem, the rewards appear to be great. Consequently, a great deal of effort was put into the synthesis of such a linkage.

Scopes of further investigation

In the pre-computer era, kinematic analysis and synthesis was accomplished primarily by intuitive and graphical techniques. The computer offers a tremendous advantage to the mechanism designer by accomplishing tedious calculations quickly and offer very attractive alternatives to the classical techniques. One of the difficulties that has arisen is the inability of the designer to keep up with the computer which can spew literally reams of computer output, which needs to be further analyzed. Computer graphics offers a timely solution for this input, output problem [11].

Structural error is the error involved in the point C actual coordinates deviation from the desired curve. The error in point tracing can be expressed as [11]:

$$Er = \sum_{i=1}^{N} \sqrt{\left(x_{C_i} - x_i\right)^2 + \left(y_{C_i} - y_i\right)^2}\ , \tag{11}$$

where x_{C_i} and y_{C_i} are the desired coordinates at the ***i***-th position of the point *C*; x_i and y_i are the actual coordinates at the ***i***-th position of the point *C*; *N* is the number of positions desired.

If there is the equation (10) that describes the point *C* motion as a function of each link length and the input link rotation, the *fmincon* function in MATLAB may be used to optimize the mechanism parameters. In order to use the *fmincon* function, the user must first supply an objective function. This objective function must be a function of a value or values that are being optimized. For the purpose of this research, the objective function in the femur optimization was defined as (11) [3].

Let us identify the length of the longest link of the mechanism (Fig. 5) as l, the length of the shortest link as s, and the lengths of the other two links as p and q. The following relations, stated without proof, are valid [11]: a Grashof four-bar is one in which the sum of the lengths of the longest and the shortest links is less than the sum of the lengths of the other two links, that is $l + s < p + q$. The crank-rocker mechanisms is possible in case when the shortest link is the crank, the frame being either adjacent link.

The objective of further investigations is to synthesize a four-bar linkage whose coupler point C will generate, as closely as possible, a given curve, and whose crank rotation will be as close as possible to desired values. The relationship between the link lengths turns out to be very important from the point of view of establishing whether the crank (link O_1A) is capable of a complete rotation and the follower (link O_2B) can only oscillate (Fig. 5).

Having deduced the dependence $y_C = F(x_C)$ from the equations (10), we are able to determine such geometrical parameters of the mechanism which allow the motion of the hinge *C* (Fig. 5) in accordance with the prescribed (specified) path $y_C = F_{pr}(x_C)$. It is well-known that the simplest method of approximation of a function is interpolation. Interpolation provide for coincidence of the values theoretically derived function $y_C = F(x_C)$ with the prescribed one $y_C = F_{pr}(x_C)$ in ***n*** points of the considered interval of the argument *x* changing. Here *n* is a number of unknown parameters (or interpolation nodes).

In this case, analytical solution of the problem of determination of unknown parameters of approximate function $y_C = F(x_C)$ may be reduced to solving n equations which are generated by equalization of the value of the difference $\Delta = \left|F(x_C) - F_{pr}(x_C)\right|$ to zero in n points:

$$\Delta(x_{Ci}) = \left|F(x_{Ci}) - F_{pr}(x_{Ci})\right| = 0 . \tag{12}$$

Usually, the segment of approximation is being divided into the interpolation nodes with a help of Chebyshov formula [15]. In our case, we have seven points:

$$x_{C_i} = x_{C0} + \frac{a_0 + a_m}{2} + \frac{a_0 - a_m}{2} \cdot \cos\left(\frac{2 \cdot i - 1}{2 \cdot n} \cdot \pi\right), \tag{13}$$

where $i = \overline{1, n}$; n is total number of unknown parameters, which should be determined; $[a_0;\, a_m]$ is a segment in which the approximation is being held. It determines the x coordinate of the start and finish position of the point C.

Having substituted (13) into (12) and specified the function $F_{pr}(x_C)$, which should be described by the hinge C, we obtain the system of nonlinear equations. For solving of this system, we may use the method of steepest descent [16]. Let us define a function $\Phi(O_1A; O_2B; AB; O_1O_2; \gamma_0; \beta; BC) = \sum_{i=1}^{n} F(x_{Ci})^2$ and deduce analytical expressions for partial derivatives:

$$\begin{gathered}
\frac{\partial \Phi}{\partial (O_1A)} = 2 \cdot \sum_{i=1}^{n} F(x_{Ci}) \cdot \frac{\partial F(x_{Ci})}{\partial (O_1A)}, \\
\ldots \\
\frac{\partial \Phi}{\partial (BC)} = 2 \cdot \sum_{i=1}^{n} F(x_{Ci}) \cdot \frac{\partial F(x_{Ci})}{\partial (BC)}.
\end{gathered} \tag{14}$$

Let us obtain the solution of the system of nonlinear equations by means of the following formulae:

$$\begin{gathered}
(O_1A)_1 = O_1A - \frac{\dfrac{\partial \Phi}{\partial (O_1A)} \cdot \Phi}{\left(\dfrac{\partial \Phi}{\partial (O_1A)}\right)^2 + \ldots + \left(\dfrac{\partial \Phi}{\partial (BC)}\right)^2}, \\
\ldots \\
(BC)_1 = BC - \frac{\dfrac{\partial \Phi}{\partial (BC)} \cdot \Phi}{\left(\dfrac{\partial \Phi}{\partial (O_1A)}\right)^2 + \ldots + \left(\dfrac{\partial \Phi}{\partial (BC)}\right)^2}.
\end{gathered} \tag{15}$$

Having substituted the expressions (14) for partial derivatives into the system (15), we may determine $(O_1A)_1$, $(O_2B)_1$, $(AB)_1$, $(O_1O_2)_1$, $(\gamma_0)_1$, $(\beta)_1$ and $(BC)_1$. If $\left|(O_1A)_1 - O_1A\right| < \varepsilon$, …, $\left|(BC)_1 - BC\right| < \varepsilon$, where ε is specified value of the calculation error, we should stop the computation. Otherwise, we should suppose that $O_1A = (O_1A)_1$, …, $BC = (BC)_1$ and continue the computation by the formula (15).

The values of maximal deviation of the curve which is being described by the point C of the connecting rod from the prescribed path $y_C = F_{pr}(x_C)$ also may be calculated by analytical dependencies (as functions of the parameters of the mechanism) on the basis of numerical iterative method.

It is natural that the solution of the system of nonlinear equations depends on the initial values of O_1A, O_2B, AB, O_1O_2, γ_0, β and BC. Therefore, by varying the parameters values in accessible regions, every time we obtain different values of maximal deviations $\delta_{\max}$ from the prescribed curve (path). Choosing among them the smallest one δ_{min}, we may consider the corresponding geometrical parameters as desired quantities. Principal flow diagram of the algorithm is shown in Fig. 6.

The value of deviation essentially depends on the length of the sector in which the argument *x* is being changed: the longer sector is the larger deviation appears. Nevertheless, the tendency of saturation of the growth of the deviation value with the increase of displacement of the driving link of the mechanism is being also retraced.

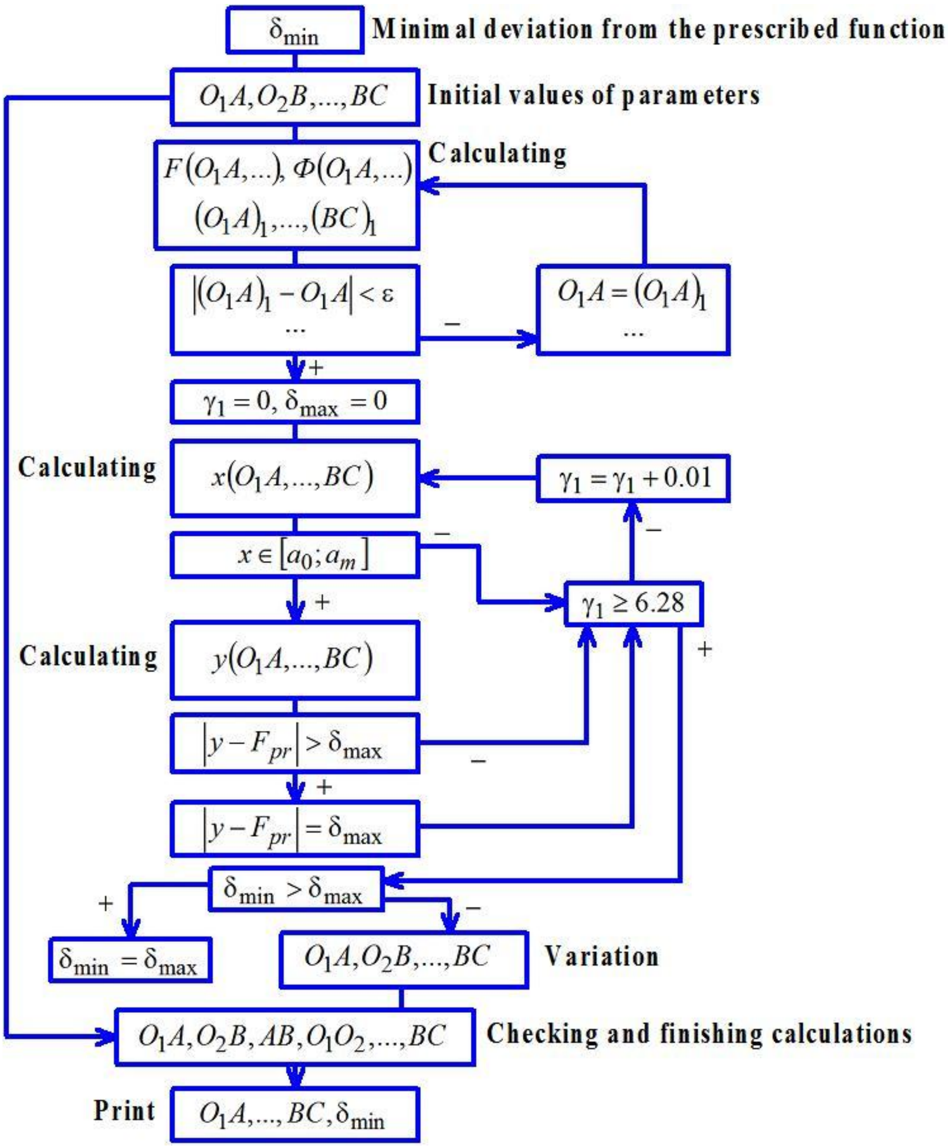

Fig. 6. *Principal flow diagram of the algorithm of optimization of geometrical parameters of the four-bar crank-rocker walking mechanism*

Conclusions

Problems and prospects of using the walking mode for producing motion of robotic systems are considered. Advantages and spheres of use of mobile robotic systems equipped with walking movers are substantiated. Preferences of cyclic (lever) walking mechanisms are analyzed. The kinematic parameters of four-bar crank-rocker walking mechanism, constructed on the basis of Chebyshov-Umnov mechanism, are accepted as the subject of research. The process of motion of the supporting foot of the walking mechanism is accepted as the object of research. The main aim of the investigation consists in carrying out structure and kinematics analysis of mentioned mechanism with further derivation of analytical dependencies for calculating kinematic parameters of the supporting foot motion.

The special features of the structure of four-bar walking mechanism (Fig. 1) and theoretical foot loci (paths) (Fig. 3) are considered. The form of the most reasonable path (Fig. 3, d) for further research is established. As a result of carried out structure analysis it is ascertained that the mechanism consists of two structural groups: of the first class and of the second class second order first type (Fig. 4). As a result of kinematics analysis the analytical expressions (10) for calculation of coordinates of the supporting foot hinge of the walking mechanism depending on geometric parameters of the mechanism and the angle of the crank rotation are deduced.

The prospects of further investigation under the themes of the paper are analyzed. In particular, the goal function (11), which may be used for further optimization of structural parameters of the mechanism, is considered and the use of applied software for solving optimization problem is suggested. The optimization problem consists in evaluating such geometric parameters of the walking mechanism which allow the motion of the supporting foot in accordance with the prescribed (specified) path.

Hence, presented mathematical statement and developed algorithm for optimization of geometrical parameters of the four-bar crank-rocker walking mechanism is recommended for using in further investigations with the application of computer-aided design systems.

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