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THE APPLICATION OF THE CES UTILITY APPROACH TO THE ESTIMATION OF BILATERAL FLOWS IN A TRADE MATRIX

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The estimation and forecasting of trade shares is attempted in a world framework for major trading countries on the basis of data on bilateral flows for the period 1965–1980 for manufactures and chemicals [Standard International Trade Classification (SITC), (5) to (8)]. The assumptions of separability and homotheticity are made to obtain an operational Hickman-Lau model from Armington's approach in which products are distinguished not only by their kind but by their place of production. The problem of heteroscedasticity is encountered and estimates of elasticities of substitution are obtained using the most general form of the model. The results are compared with the Gana, Hickman, Lau, and Jacobson (1979) study. The estimates of substitution elasticities based on 1965–1980 data are lower than the ones obtained using 1960–1970 data and it is argued that the flexible exchange rate (and the agents' responses) could be responsible for low substitution elasticities when the period after 1973 is included.

I. Introduction

The objectives of this paper are to adopt a CES approach to the estimation of exports to a given importing country, assuming that home demand and import demands are separable.[1] The paper builds upon the

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[1] This assumption is somewhat restrictive but in the international link framework the national country models determine domestic demand, import demands and export prices, while the trade share matrix in conjunction with import demands and export prices determine a country's import prices and exports. This process of iteration is carried on until the world system and country aggregates have converged.

general theory of demand for products that are distinguished not only by their kind, but by their place of production. This approach was first presented by Armington (1969). Armington's starting point was that demand was separable over groups of goods, not over supplies. Hence, he separated clothing from machinery, but within each group treated home and all foreign supplies entirely symmetrically. Armington used separability to isolate goods from each other in the first step and then, in his second step, sought a model to allocate each good among supplying countries.

Armington's model was put to operational use by Hickman and Lau (1973), and Gana, et al., (1979). This paper discusses the estimates based on the Hickman-Lau model for the whole world when the world is divided into 25 countries/regions, (Appendix C). The main purpose is to use the newly compiled World Trade Matrix at constant prices and compare the results on substitution elasticities (for each importing country) for some countries with the Gana, et al., (1979) estimates. The substitution elasticities are compared and contrasted amongst different import markets between two or more exporting countries. Moreover, the substitution elasticities of this study are compared with the other studies and changes in magnitudes are attributed to the price instability after 1973.

In both consumption and production theory, any pair of bilateral imports of a specific category of goods may be characterized as substitutes or complements. The substitution elasticity between two exporting countries in a given import market is of enormous importance. The study proposes to estimate the elasticity in a given market and compares elasticities of substitution in different import markets. The market where the substitution elasticity is higher tends to adjust its quantities, in response to price, faster than where the substitution elasticity is smaller. If the substitution elasticity is greater than one, the improvement in competitiveness will yield an increased share of that country in the import market. This approach also has its uses in the World Link Model.

The paper is organized into five sections. In section II, the Hickman-Lau model, based on Armington (1969), and its limitations, namely the assumptions of homotheticity and separability, are discussed. In section III, estimated equations are presented and used, under different assumptions, for the analysis of bilateral flows. In section IV, the results for three countries which are importing from their major trading partners are analysed, and a comparative analysis on substitution elasticities is presented. In section V, the predictions from estimated equations are attempted, while in section VI, some conclusions are drawn. There are three appendices, one on the data, the second on the derivation of the Hickman-Lau equations, and the third on the details of countries/regions.

II. Armington's Model

Armington specified a two-stage budgeting approach and assumed that the elasticities of substitution in each market are constant and the elasticity of substitution between any two products (Italian and French machinery) competing in the Canadian market is the same as between any other pair of products (German and U.S. machinery) in the same market. In his paper Armington confuses the issue of goods and markets but the description of Armington's model below keeps to the convention used in this paper as a whole, i.e., that one is considering imports by a country j from exporting countries i. Thus, he specified:

$$x_j = \phi_j(x_{1j}, x_{2j} \ldots x_{mj}) \tag{1.1}$$

where x_j is the total imports of country j and the x_{ij} are imports by country j from country i, with x_j a CES aggregate,

$$x_j = [b_{1j} x_{1j}^{-\rho j} + b_{2j} x_{2j}^{-\rho j} + \ldots + b_{mj} x_{mj}^{-\rho j}]^{-(1/\rho j)} \tag{1.2}$$

The CES utility maximization approach can be presented in two alternative forms. Maximize u = V(m*) subject to pm* = M, implying that utility is to be maximized, subject to a given outlay on imports or, minimize M = pm* subject to V(m*) = u, meaning that the cost of imports is to be minimized, subject to a given utility level. It is the latter alternative which is used in the allocation of imports to exporting countries m*.

Given (1.2), the x_{ij} have the form

$$x_{ij} = b_{ij}^{\sigma j} x_j (\frac{P_{ij}}{P_j})^{-\sigma j}, \text{ where } \sigma_j = \frac{1}{1+\rho j} \tag{1.3}$$

Where σ_j is the elasticity of substitution in the jth market and b_{ij} is a constant. x_{ij} is a function of the demand (x_j) by the jth country and of the ratio of the export price index (from country i to j) to the price index of imports of country j. Equation (1.3) is somewhat restrictive because of the assumptions on the ϕ_j function in equation (1.2). First, the marginal rate of substitution between any two suppliers competing in the jth market is independent of demand for any other supplier in that market and, secondly, the ϕ_j is linearly homogeneous.

Armington's specification is remarkably economical in the use of parameters. Assuming that we know the change in total demand for a good, Armington provides an allocation over foreign and domestic suppliers that involves a price change for each supplier using a single parameter (ρ).

Furthermore, once total demand is split between home supplies and total imports i.e., once the import function has been estimated, the value of that single parameter is known and the allocation is entirely determined from this parameter. Also, in Armington's model, exporters' shares of particular markets cannot change except through price changes. The elasticities of exports of a given ith country with respect to total imports of the jth country are assumed to be unity for all suppliers.[2]

One minor modification to Armington's model is made by Hickman and Lau (1973). They approximate the Armington's scheme by a first order Taylor's expansion about the point where the price indices are unity, using 1975 as the base year (see Appendix B). This permits Hickman and Lau to express imports from any source as a function of total imports (the sum rather than a CES aggregation), a base weighted price index, and a time trend. This is a considerable improvement in terms of applicability, for the independent variables are now readily available. There is also an improvement in terms of statistical fit, and the trend removes some of the worst excesses of the homotheticity assumption by allowing import shares to vary over the long run. However, all the fundamental objections to Armington's model, separable imports and home supplies, unit elasticities on total imports and grossly restricted price responses are left in the model. There is only one parameter with respect to relative prices. Thus, Armington's approach suffers from two very restrictive assumptions, namely homotheticity and separability [Winters, (1984)].

The CES model is a demand oriented model. Supply effects that are of importance on a domestic import market are twofold: there is a supply from domestic producers that tends to provoke import substitution while there is also the foreign export supply. In a complete Walrasian equilibrium model equating demand and supply schedules, the supply shortages from the domestic side during a period of increased business activity would lead to an increased demand for imports. In the absence of a full equilibrium model such effects are introduced directly into the import demand equations via capacity utilization. This was not possible to account for as no information was available for 25 countries of the world.

The introduction of an explicit supply equation next to the demand equation poses econometric problems but such problems are no less present if there is only a demand equation. In the latter case, it is well known that unless the export supply elasticity is infinite, estimation by

[2] This implies that the x_{ij} functions are homethetic.

ordinary least squares of the demand equation, in the absence of a supply equation, introduces simultaneous equations bias. Assuming that export supply is infintely elastic, the demand equations are identified.

A further generalisation of Armington's CES approach is by Barten (1971). Barten explicitly adopts separability between home demand and imports and between different sources of imports using the following import function to generate his allocation equations:

$$M = (\sum_i \gamma_i \; m_i^{\beta_i})^{\epsilon} \tag{1.4}$$

where M is the index of total imports; m_i is imports from country i; γ_i, β_i and ϵ are non-negative parameters.

Equation (1.4) is a strongly separable function. Hence, imports from i are separable from those from j, which is just as implausible as the separation of domestic and imported goods. Barten (1971) has derived allocation elasticities first. The substitution elasticity is a fixed proportion of the allocation elasticity. The adding-up condition, that the sum of allocated imports is equal to total imports, is violated.

Barten's approach[3] retains separability between domestic and foreign goods, and between different types of foreign goods, and, as a consequence of the latter, retains also the highly restrictive price responses.

III. The Hickman-Lau Linearized Model

Hickman and Lau (1973) derived the operational version of Armington's model in which

$$x_{ij} = a_{ij}^o \, m_j - \sigma_j \, (P_{ij} - P_j)^m \, a_{ij}^o \, m_j \tag{2.1}$$

where $a_{ij}^o = \dfrac{x_{ij}^o}{m_j^o}$ (fixed weighted average of the export prices of all countries in the jth market)

$$P_j^m = \sum_{k=1}^{n} a_{kj}^o \; P_{kj}$$

σ_j = substitution elasticity between the imports received from two countries in the jth market;

[3] This approach neither satisfies conditions for a perfect aggregation nor those for the first step of the budgeting procedure.

x_{ij} = value of exports at constant prices from country i to j;
m_j = value of imports of country j.

Equation (2.1) satisfies the adding-up property i.e.,

$$\sum_{i=1}^{n} x_{ij} = m_j \tag{2.2}$$

If (2.1) is summed across i, we obtain

$$\sum_{i=1}^{n} x_{ij} = m_j = \sum_{i=1}^{n} a^o_{ij}\, m_j - \sigma_j \left(\sum_{i=1}^{n} a^o_{ij}\, P_{ij} - \sum_{i=1}^{n} a^o_{ij}\, P^m_j\right) m_j \tag{2.1A}$$

Now,

$$\sum_i a^o_{ij} = 1,\ \sum_{i=1}^{n} a^o_{ij}\, P_{ij} = P^m_j,\ \sum_{i=1}^{n} x_{ij} = m_j \tag{2.3}$$

This adding-up property ensures that predicted total world exports will equal actual total world imports provided that the appropriate c.i.f. – f.o.b. valuation adjustments are made.

Hickman-Lau added a trend term to the equation system (2.1) and derived a bilateral and aggregate export equation in the following form in which r_{ij} is the trend coefficient.

$$x_{ijt} = a^o_{ij}\, m_{jt} - \sigma_j\, x^o_{ij}\, (P_{ij} - P^m_j)_t + \sigma_j\, x^o_{ij}\, r_{ij} t \tag{2.4}$$

and the aggregated exports equation

$$x_{it} = \sum_{j=1}^{n} a^o_{ij}\, m_{jt} - \bar{\sigma}_i\, x^o_{it}\, (P_{it} - P^c_{it}) + \bar{\sigma}_i\, x^o_{it}\, \bar{r}_i t \tag{2.5}$$

where

$$\bar{\sigma}_i = \sum_{j=1}^{n} \sigma_j\, \lambda_{ij} \tag{2.6}$$

and where

$$\lambda^o_{ij} = \frac{x^o_{ij}}{x^o_i} \tag{2.7}$$

$$P^c_{it} = \sum_{j=1}^{n} \frac{\sigma_j}{\bar{\sigma}_i}\, \lambda^o_{ij}\, P^m_{jt} \tag{2.8}$$

$$P_{it} = \sum_{j=1}^{n} \frac{\sigma_j}{\bar{\sigma}_i} \lambda_{ij}^{o} P_{ijt} \tag{2.9}$$

$$\bar{r}_i = \sum_{j=1}^{n} \frac{\sigma_j}{\bar{\sigma}_i} \lambda_{ij}^{o} r_{ij} \tag{2.10}$$

The model equation (2.1) was further modified by adopting the partial adjustment hypothesis with static price expectations. This, however, violates the additivity criterion. Dynamics were then introduced by postulating that the formation of price expectations are adaptive and instead of a partial adjustment hypothesis on bilateral flows, an instantaneous adjustment was assumed, with adaptive expectations on prices. The derived equation was:

$$x_{ij} - a_{ij}^{o} m_{jt} = \sigma_j x_{ij}^{o} (\delta_j r_{ij} - \beta_{ij}) + \sigma_j x_{ij}^{o} [(1-\delta_j) r_{ij} - \gamma_{ij}] t - \sigma_j (1-\delta_j) x_{ij}^{o} (P_{ij} - P_j^{m})_t + \delta_j (x_{ij} - a_{ij}^{o} m_j)_{t-1} \tag{2.11}$$

The parameters δ_j and σ_j are identifiable, but others are not. In particular, it is not possible to estimate both the trend coefficient of tastes (r_{ij}) and the trend coefficient of price expectations (γ_{ij}). If either trend coefficient is assumed to be zero for all i and j the other is identifiable. If $r_{ij} = 0$, for example, so that tastes are assumed constant over time and the trend drops out of (2.4), the second term of (2.11) will identify β_{ij} and the third term will identify γ_{ij}. Similarly, assuming $\gamma_{ij} = 0$ permits identification of β_{ij} and r_{ij}. The adaptive expectations model is consistent with the adding-up criterion provided the constant and trend coefficients are appropriately constrained in the estimation procedure.

The base year for all countries or regions is chosen to be 1975. All price indices are set equal to one in 1975. In addition the a_{ij} are taken from the 1975 constant dollar import share matrix. Equations (2.1), (2.4) and (2.11) are three models which have been used by the Hickman-Lau (1973) study. An equivalent form of (2.1) can be obtained by dividing both sides by m_j, the total imports of the jth country, to obtain estimating equations for import shares.

$$a_{ijt} = a_{ij}^{o} - \sigma_j a_{ij}^{o} (P_{ij} - P_j^{m})_t \tag{2.12}$$

This form is more suitable if heteroscedasticity of disturbances is encountered.

It is also recognized by researchers in international trade that the estimation of bilateral flow models (2.1), (2.4), (2.5), (2.11) or (2.12) requires data on bilateral export prices. P_{ij} is the ith country's price charged to the jth country and it cannot be assumed to be the same for all j's. Price discrimination and product differentiation are generally common practices in international trade.[4]

The assumption that $P_{ij} = P_i$ has to be made, which means, for example, that if Pakistan is a supplying country to the United Kingdom and the United States, she charges the same price for the products/goods supplied.

This is because the data on bilateral prices between exporting and importing country are not available. Export prices from ith country to any country is the same. Equation (2.12) has a similarity with the Moriguchi (1973) approach. Moriguchi uses the competitors' price namely PCM_{ij}, the price of imported goods that are competitive with country i's export in the jth market. He used the model

$$\log_e a_{ijt} = \alpha_i + \beta_i \log_e \left(\frac{PX_i}{PCM_{ij}}\right)_t + \gamma_i \left(\frac{SX_{it}}{M_{jt}}\right) \qquad (2.13)$$

where β_i corresponds to the elasticities of substitution of country i's exports in the world market (or in various import markets) and γ_i is the elasticity of certain non-price competitive factors that contribute to changes in trade shares. The non-price factor (SX_i/M_j) is the relative change in country i's total export capacity against country j's level of total imports. Moriguchi does not use the restriction that

$$\sum_{i=1}^{n} a_{ijt} = 1$$

for all j and t in the estimation but makes use of the restriction by distributing residuals of

$$\left(\sum_{i}^{n} \hat{a}_{ij} - 1\right)$$

[4] It is common evidence [Kravis and Lipsey, (1978)] that there is price discrimination between destinations and at least for petroleum products and some manufactured products, there are oligopolistic firms facing different elasticities of demand at home and in each foreign market. Profit maximization leads firms to charge lower prices in the more elastic demand conditions. Moreover, changes in comparative advantage and changing market price make it possible for one

where $\hat{a}_{ij}$ is the estimated import share. Ezaki (1978) estimates x_{ijt}, the bilateral flows for all trading countries except for the rest of the world, as a function of the level of economic activity in the importing country, and the export prices in relation both to the import prices and to the domestic price of the country. The relationship is ad-hoc and is not based on any allocation models derived from theory. Consistency in the Ezaki model is achieved by adapting for each country j its imports from the rest of world, which is equal to the exports of the rest of the world to that country, such that its total imports, m_j are equal to the amount generated by its country model. The exports from the rest of the world to the rest of the world are assumed to maintain a fixed proportion of world trade.

One further model in which $a_{ijt} - a_{ij}$ was used as a dependent variable is tried:

$$a_{ijt} - a^o_{ij} = - \sigma_j \, a^o_{ij} \, (P_i - P_j) - \sigma_j \, a^o_{ij} \, \gamma_{ij} t \qquad (2.14)$$

in which a time-trend is added to account for changes in tastes. These results are discussed in Parikh (1986). Equation (2.14) is an extension of (2.12).

Modifications to the Hickman-Lau approach were suggested by Dramais and Waelbroeck (1975) who introduced capacity variables and emphasised the role of non-price factors in the estimation of trade shares. As there is no such information on capacity variables for the 25 countries/regions of the world used here they have not been employed. Instead, the basic Hickman-Lau model has been used without the knowledge of bilateral price deflators but many alternatives have been tried in order to take into account the heteroscedasticity of the error term.

IV. Estimation, Results and a Comparative Analysis

In this section the results of the Hickman-Lau model, and a comparative analysis of substitution elasticities are presented.

The following three models for all importing countries/regions under three different assumptions of heteroscedasticity were specified and estimated.

$$x_{ij} = a^o_{ij} \, m_j - \sigma_j \, x^o_{ij} \, (P_i - P^m_j) + \epsilon_{ij} \qquad (3.1)$$

source of supply to be selling at lower prices over prolonged periods of time. In addition, lack of knowledge, uncertainty regarding the reliability of new suppliers, the reluctance to give up a satisfactory relationship with suppliers, and commitments for a given type of equipment may also explain the existence of price differences.

$$x_{ij} = a^o_{ij}\ m_j - \sigma_j\ x^o_{ij}\ (P_i - P^m_j) + \sigma_j\ x^o_{ij}\ r_{ij}t + \epsilon_{ij} \qquad (3.2)$$

$$\begin{aligned} x_{ij} = a^o_{ij}\ m_j - \sigma_j\ x^o_{ij}\,\beta_{ij} - \sigma_j\,(1-\delta_j)\ x^o_{ij}\ (P_i - P^m_j) \\ - \sigma_j\ x^o_{ij}\ \gamma_{ij}t + \delta_j\,(x_{ij} - a^o_{ij}m_j)_{-1} + \epsilon_{ij} \end{aligned} \qquad (3.3)$$

The adding-up condition $\sum\limits_i x^o_{ij} = m_j$ (3.4)

must also be satisfied. There are two further restrictions necessary for estimating (3.3). They are:

$$\sum_i x^o_{ij}\,\beta_{ij} = 0 \text{ and } \sum_i x^o_{ij}\ \ \gamma_{ij} = 0 \qquad (3.5)$$

σ_j is the elasticity of substitution between any two countries supplying the jth market and is the same for any supplying country. Equation (3.3) is more general than equations (3.1) or (3.2), and this is the first model used as it is the most general Hickman-Lau version (3.3), with constraints, so that the adding-up condition is satisfied.

First, the 1975 import shares were computed and the trade flows of exporting countries to a given market whose import share for 1975 exceeded 0.01 were specified. Countries with entries below 0.01 are in the rest of the world group. In each equation, the dependent variable is exports of country i to an importing country j, at constant prices. Despite the elimination of small entries, the problem of the residual variance not being homoscedastic is still present and, hence, three assumptions about the error variance are made:

Variance $(\epsilon_{ijt}) = S^2_j$ for all i, t (3.6)

Variance $(\epsilon_{ijt}) = S^2_j\ x^{o\,2}_{ij}$ for all i, t (3.7)

Variance $(\epsilon_{ijt}) = S^2_j\ x^o_{ij}$ for all i, t (3.8)

It is also assumed that $\sum\limits_{i=1}^{n} e_{ijt} = 0$ for all j, t. (3.9)

Equation (3.3) was estimated subject to the restrictions of (3.5) using three different assumptions of heteroscedasticity for the error variance-covariance matrix. If the residual variance-covariance matrix was positive definite and the iterative Zellner estimation procedure satisfied the convergence criteria, the estimates were reported. The models under assumptions (3.7) and (3.8) were most successful, in terms of R^2, *a priori* signs and convergence. Convergence was not achieved in every case but in most cases one of the three assumptions provided estimates for the Hickman-Lau model. Where the equation (3.3) failed on convergence criterion, equation (3.2) was used under three different assumptions of heteroscedasticity. If that failed, then equation (3.1) was used with the relevant adding-up condition and the heteroscedasticity assumption. In each of the models there are equality restrictions on the σ and δ's as they are the same in the jth market. Both (3.1) and (3.2) do not have a constant term as derived in the appendix based on cost minimization and hence in the estimation, the constant term is suppressed.

Results

For the United States, there are 13 countries which have a significant share in her imports (Table 1). Canada, Japan and the EEC were major partners of the U.S.A. and their share in U.S. imports exceeded 10 per cent in 1975. The long-run elasticity of substitution is 1.502, (0.852/0.57) with the correct sign. The time trend coefficients indicate that Japan, other developed countries, Latin America, Taiwan, the Republic of Korea, and Singapore have increased their share with respect to the base year, 1975, while the EEC, other European countries, and Canada seem to have lost out in the United States market. The model with the heteroscedasticity version (3.7) yields plausible results with a correct sign of substitution elasticity.

For the EEC, the general model with a time trend and lagged variable (adaptive expectation in prices) performs well, (Table 2). The heteroscedasticity encountered necessitated the deflation of both the left-hand and right-hand side variables by $\sqrt{x_{ij}^{o}}$. The long-run elasticity of substitution between any two competitors in the EEC market is 1.438 with the correct sign. The time trend coefficient indicates that the share of Canada, the United States and African countries in EEC imports is falling while Japan has gained in the EEC market.

In the Japanese market, there are 12 important suppliers, (Table 3). The model with a lagged dependent variable did not produce plausible estimates of substitution elasticities and the model with a time trend has been used. The substitution elasticity between two exporters is 0.343 in the Japanese market, which is fairly low. The exports of the United States, Africa, CMEA and centrally planned economies have declined between 1965 and 1980 while those of Hong Kong, China, Latin America

TABLE 1

Hickman-Lau Model under Heteroscedasticity Assumption

Importing Country: USA

Exporting Country	α^{o}_{ij}	β^{*}_{ij}	$\sigma_j(1-\delta_j)$	γ^{*}_{ij}	δ_j	R^2/SEE*
Canada	0.24566	−0.07044 * 0.02207	−0.00852 * 0.00003	−0.02019 * 0.00421	0.43271 0.00335	−0.29070 0.07757
Japan	0.21378	0.06187 * 0.01840	”	0.01085 * 0.00351	”	0.60780 0.06467
EEC	0.28073	−0.04682 * 0.01287	”	−0.01644 * 0.00246	”	0.94820 0.04527
Other European	0.05723	−0.11490 * 0.01708	”	−0.01857 * 0.00326	”	0.90630 0.06004
Other Developed	0.02057	0.10749 * 0.04973	”	0.03756 * 0.00947	”	0.43760 0.17491
Latin American	0.03765	0.16863 * 0.05892	”	0.02248 * 0.01123	”	0.48100 0.20718
Taiwan	0.03401	0.27730 * 0.04083	”	0.07285 * 0.00778	”	0.93570 0.14359
Hong Kong	0.03418	0.00460 0.03482	”	0.00797 0.00664	”	0.15730 0.12242
India	0.00551	0.14085 * 0.06814	”	−0.01988 0.01301	”	0.31860 0.23953
Republic of Korea	0.02874	0.08435 * 0.02432	”	0.04456 * 0.00463	”	0.92070 0.08557
Malaysia	0.00637	0.03305 * 0.05264	”	0.00854 0.01003	”	0.30210 0.18512
Singapore	0.00966	0.11425 * 0.04629	”	0.06917 * 0.00882	”	0.89700 0.16284
CMEA + Centrally Planned	0.00653	0.11392 * 0.05617	”	0.02069 0.01072	”	0.55520 0.19744
Rest of the World	0.02000	−0.87415	”	−0.21959	”	

System $R^2 = 0.9999$. $\chi^2 = 141.42$. log L = 338.63. Figures in the second line below the regression coefficient are standard errors. *Standard Error of Estimate.

and the Republic of Korea have increased over this period since their trend coefficients are positive and significant.

A large number of stochastic specifications have been estimated and a selected set of estimates is presented for the three countries. The criteria for selection are: (i) the estimated values of the adaptive expecta-

TABLE 2

Hickman-Lau Model under Heteroscedasticity Assumption

Importing Country: EEC

Exporting Country	α^{o}_{ij}	β^{*}_{ij}	$a_j(1-\delta_j)$	γ^{*}_{ij}	δ_j	R^2/SEE*
Canada	0.00894	– 0.04120 0.03904	– 0.00961 0.00035	– 0.05500* 0.00851	0.30276 0.03357	0.37630 0.13706
USA	0.08545	– 0.01757 0.01661	"	– 0.01710* 0.00359	"	0.52390 0.05825
Japan	0.03421	0.09812* 0.01916	"	0.02976* 0.00419	"	0.91470 0.06701
Other European	0.12263	– 0.01035* 0.00561	"	0.00210 0.00114	"	0.73560 0.01911
Other Developed	0.01418	0.03565 0.02552	"	– 0.00272 0.00535	"	0.31800 0.08938
Latin American	0.01085	0.05191 0.03047	"	– 0.01013 0.00644	"	0.24770 0.10706
African Countries	0.01095	0.12514* 0.05216	"	– 0.05310* 0.01113	"	0.70900 0.18138
Hong Kong	0.01013	– 0.02340 0.02489	"	0.00400 0.00522	"	– 0.04330 0.08724
CMEA + Centrally Planned	0.02626	– 0.02690 0.01452	"	– 0.00137 0.00306	"	0.86280 0.05039
Rest of the World (includes intra EEC trade)	0.67640	– 0.19140	"	0.10356	"	

System R^2 = 0.9996. χ^2 = 118.42. log L = 219.18. Figures in the second line below the regression coefficient are standard errors. *Standard Error of Estimate.

TABLE 3

Hickman-Lau Model under Heteroscedasticity Assumption

Importing Country: Japan

Exporting Country	α_{ij}^{0}	Time Trend	σ_j	R^2/SEE*
USA	0.334240	– 0.01275* 0.00301	– 0.00343* 0.00013	0.43980 0.06318
Australia	0.023110	– 0.01037 0.01256	,,	0.66360 0.26385
EEC	0.230990	0.01444 0.00379	,,	0.63820 0.07963
Other European	0.060488	0.00564 0.00312	,,	0.25900 0.06542
Other Developed	0.029470	– 0.00195 0.00926	,,	0.21180 0.19447
Latin American	0.028380	0.03276* 0.01536	,,	0.42980 0.32235
Africa	0.020120	– 0.11968* 0.02624	,,	0.42290 0.55060
Taiwan	0.032556	0.07095* 0.01622	,,	0.76920 0.34024
Hong Kong	0.018440	0.02563* 0.01022	,,	0.04710 0.21420
Republic of Korea	0.078920	0.05518 0.00961	,,	0.57230 0.20177
CMEA + Centrally Planned	0.033510	– 0.02896* 0.00695	,,	0.29600 0.14582
China	0.025800	0.03159* 0.00990	,,	0.28110 0.20773

log L = 236.20. Note: Since a constant term is not used, System R^2 and χ^2 are not valid. Figures in the second line below the regression coefficient are standard errors. *Standard Error of Estimate tion parameter must be less than unity; (ii) the residual variance-covariance must be positive definite on every iteration; (iii) the estimates must converge with the convergence criterion of 0.001; (iv) the estimated elasticity of substitution must have the correct sign.

The conventional goodness of fit statistics have been given in Tables 1, 2 and 3. But R^2's and standard errors of estimates are not comparable across different stochastic specifications as the variables are scaled differently in each of the models. When heteroscedasticity is taken into consideration, weighted least squares estimates are obtained and this means that both the left hand side and the right hand side variables are scaled differently from the original equation. Moreover, the R^2's are not comparable for the forms including and excluding a constant term and the concept of a coefficient of determination is not well defined when the regression is forced through the origin. So, in addition, system R^2 and χ^2 statistics are reported. The system R^2 is very high because the determinant of the residual variance-covariance matrix is generally small.

Comparative Analysis

The CES approach was used for 25 countries of the world. Summary results on the elasticities of substitution are shown in Table 4. The analysis of these results suggests that the substitution elasticities for the United States, Japan and the EEC were comparatively smaller than those obtained using 1960–1970 data by Gana, et al., (1975). The substitution elasticities must be negative while they are positive for Canada, other European countries and India. Also, heteroscedasticity indicates specification error.

Italianer has provided, estimates of substitution elasticities similar to the ones presented in Table 4.[5] His estimates are available for each of the EEC countries using bilateral price data and are contrasted with those of Gana, et al. They reveal that, excepting the Italian market, all the estimates are lower using 1965–1980 data as compared to 1960–1970 data using the CES linearized version. The reasons for this might be that world trade has not grown fast enough after 1973 and the inclusion of data after 1973 in the estimation of elasticities of substitution seems to depress the estimates because of the impact of world recession in overall world trade. The exceptions are perhaps, Japan and some newly industrialized countries such as Hong Kong, Singapore, Korea, and Taiwan. The other

[5] Italinier at the Catholic University of Louvain provided, direct to the author, the following estimates of substitution elasticities for the periods of estimation. Belgium 1.78 contrasted with 2.049, Germany 0.239 against 3.176, France 0.4441 against 0.452, Italy 2.115 against 0.995, Netherlands 0.460 against 0, U.K. 1.045 against 1.831, and United States 1.133 against 3.554. For Japan, no estimate was obtained because of the incorrect sign of the substitution elasticity.

TABLE 4

Substitution Elasticities for Different Countries/Regions for (5) to (8), Manufactures Group

Name of Importing Country	Short Run	Static Model	Long Run	Expectations Coefficient	Comments
Canada	0.000260 0.000640			0.37277 0.05900	Incorrect sign
USA	−0.008520 0.000030		−1.5020	0.43271 0.03350	Low compared to Gana et. al., estimate on 1960-70 data
Japan		−0.003430 0.000130			Very low
Australia		0.002870 0.000130			Positive and significant, all noncompetitive imports
New Zealand		0.000260 0.000290			Positive and not significant
EEC	−0.009610 0.000350		−1.3783	0.30276 0.03357	
Other European	0.001110 0.000800			0.41686 0.08886	Not significant and incorrect sign
Other Developed	−0.004380 0.000860		−0.5870	0.25469 0.07995	
Latin American	−0.003740 0.000890		−1.2690	0.70521 0.06403	
African Countries	−0.005900 0.000170		−1.1510	0.13087 0.02940	
Oil Producing Middle East	−0.003720 0.000020		−0.8510	0.56267 0.00024	
Indonesia	−0.011950 0.001470		−1.6500	0.30397 0.07882	
Taiwan		0.002620 0.000580			Positive and significant, perhaps noncompetitive imports

TABLE 4
Contd.

Name of Importing Country	Short Run	Static Model	Long Run	Expectations Coefficient	Comments
Hong Kong		−0.008700 0.000170			Constant used
India	0.004710 0.000003		0.9420	0.50147 0.00068	
Korea	−0.001470 0.000990		−0.1680	0.12850 0.09539	
Malaysia	−0.007810 0.000030		2.6030	0.70007 0.00030	
Pakistan	−0.022490 0.000140			−0.60016 0.00761	Unstable model
Philippines	−0.005040 0.000460		−0.5040	0.00601 0.05847	Expectations coefficient insignificant
Singapore	−0.012430 0.000690		1.2430	0.07913 0.05271	Expectations coefficient insignificant
Thailand	−0.018680 0.002480		−3.1533	0.40761 0.07671	
Other S.E. Asia	−0.002470* 0.000780		−0.2470	0.05557 0.69730	Expectations coefficient not significant
S.E. Asia		−0.002190			Not constant
CMEA + Centrally Planned 1%	−0.001720 0.000690		−0.2760	0.37771	
China		−0.012400 0.001330			Constant used

Note: The short-run coefficients are derived directly from the coefficients on $\sigma_j(1-\delta_j)$ in the general model based on equation (3.3). This model also provides the expectations coefficient (for the δ_j). The long run coefficients are obtained by dividing the short run coefficients by (1 − expectations coefficient). The static model coefficients derive from equation (3.1).
The figures in the second line are standard errors.
*Marks a coefficient of at least twice the standard error.
**In the estimation $y = a' + b'(100x)$ equation was used instead of $y = a + bx$ and hence $b = 100b'$ when b is derived from b'

plausible reason for the low magnitude of the substitution elasticity is that the period of flexible exchange rate after 1972 may have induced economic agents to interpret changes in the prices of tradeable goods caused by exchange rate variation as purely transitory and they do not now react to changes in prices by adjusting their quantities. Third, when the general price level is stable, most changes in prices may be perceived to be changes in relative prices, and agents' responses are therefore greater as compared to the situation when the general price level is very variable, since under the latter situation agents view changes in their prices as changes in the general price level and consequently their response is muted.[6] The analysis of means and standard deviations of export and import prices of U.S.A., Japan and EEC reveal that the standard deviations and means of both export and import prices are much higher for 1974–80 as compared to 1965–73 (Table 5). The coefficients of variation in prices for EEC and U.S.A. are considerably higher for the second period than for the first period.

TABLE 5

Means and Standard Deviations of Export and Import Price Index, SITC (5) to (8)

Name of the Country	Price	1965–1973 Mean	1965–1973 Standard Deviation	1974–1980 Mean	1974–1980 Standard Deviation
U.S.A.	Export	55.53	6.64	113.13	19.17
	Import	47.56	8.01	121.49	30.37
E.E.C.	Export	52.06	8.39	119.35	28.59
	Import	45.12	8.00	120.08	28.78
Japan	Export	56.47	9.73	118.00	21.14
	Import	39.72	7.48	123.77	34.83

[6] This argument can be presented in the context of the Phillips Curve. The slope of the short-run Philips curve depends on the previous variability of the general price level.

V. Predictive Performance of the Bilateral Trade Model

Other approaches in addition to the Hickman-Lau model have been used in a detailed study by Parikh (1986). The validity of various models was examined by considering the parameter values and values of certain test statistics, and the usefulness of these import share models was ascertained by assessing the predictive ability of various models over the sample period, and preferably for a new body of data. However, the extension of the series on trade shares is not yet available for the years 1981, 1982 and 1983 and for this reason comparison of the models has to be made on their ability to fit the sample data.

The CES approach predicts the bilateral flows in absolute form at constant prices. This paper converts flows to import shares.

Criteria for Evaluation

Various criteria for the evaluation of predictive performance are available in the literature but the four used here are the square of the correlation coefficient between actual and predicted shares, the root mean square error, the regression coefficient of actual on predicted shares, and Theil's inequality coefficient. The predicted values are converted to shares and compared with actual shares. The comparisons are made for the United States as an importing country, concentrating on the main exporters to the United States. In Tables 6, 7 and 8, are presented results of predictions using the Hickman-Lau model. It is clearly seen that many partner countries were included in the estimation of flows from a supplying country to an importing country. These results were compared with the other direct approach namely the Almost Ideal Demand System (AIDS), [Deaton and Muellbauer, (1980)]. The AIDS was used for estimating trade shares on the same data and CES and AIDS predictions were compared. The AIDS was the model developed by Deaton-Muellbauer in which current value shares depend upon prices of exporting countries and the real volume of imports. A full derivation and results are available in Parikh (1988). The general conclusion was that prediction errors were lower in AIDS as compared to CES but when degrees of freedom adjustments were made (since AIDS uses more variables in the estimating equations) the slight advantage of AIDS was reversed in favour of the CES model.

VI. Conclusions

The Hickman-Lau model on bilateral shares obviously requires the use of bilateral price deflators and consequently the results are not encouraging when data on bilateral prices is not available. The other problem with the model is the heteroscedasticity issue and although three different assumptions have been tried and convergence is achieved with one, it is not guaranteed that other forms of the heteroscedasticity assumption may not yield better results. The issue of heteroscedastic disturbances has not been taken any further.

The other two criticisms of the Hickman-Lau approach are the restrictive assumptions of separability and homotheticity and the need for a flexible model. A situation where these assumptions are not required

TABLE 6

Predictive Performance of the CES Approach for the United States, 1965–1980, Commodity Groups (5) to (8)

Name of Supplying Country	Square of Correlation Coefficient	Root Mean Square Error	Regression Coefficient of Actual on Predicted	Theil's Inequality Coefficient
Canada	0.23715	0.01750	0.56877	0.00632
Japan	0.54541	0.01265	1.15709	0.00312
EEC	0.97091	0.01279	0.97037	0.00174
Other European	0.92690	0.00308	0.97810	0.00251
Latin American	0.36217	0.00665	0.68998	0.02627
Taiwan	0.96935	0.00445	0.92637	0.01582
Hong Kong	0.37607	0.00356	1.16919	0.01074
Republic of Korea	0.93832	0.00265	0.93091	0.01305
Other Developed	0.45204	0.00349	0.58929	0.03416
Malaysia	0.22077	0.00104	0.72773	0.03349
Singapore	0.89160	0.00170	0.90528	0.03392
CMEA + Centrally Planned	0.44591	0.00126	0.79150	0.03347

and where hypotheses can be tested is preferable to the Hickman-Lau model.

The estimates of substitution elasticities by the importing countries of the world have been derived and it is needless to state that all the estimates are not in agreement with *a priori* signs and magnitudes. Most of the substitution elasticities are lower than unity in each of the import markets and this is probably due to the inclusion of periods after 1974 in the data set. The growth in the volume of trade was not very significant after 1974 and relative price differences played a less important role in the post 1973 period. When the general price level is very variable, agents will perhaps view changes in prices as changes in the general price level and their output response will be absent. Conversely, in an environment with a stable price level, most changes in prices would be perceived to be changes in relative prices and agents responses would be greater.

TABLE 7

Predictive Performance of the CES Approach for the EEC, 1965–1980, Commodity Groups (5) to (8)

Name of Supplying Country	Square of Correlation Coefficient	Root Mean Square Error	Regression Coefficient of Actual on Predicted	Theil's Inequality Coefficient
United States	0.70232	0.00459	0.97489	0.00269
Japan	0.94170	0.00202	0.98522	0.00361
Other European	0.85867	0.00192	1.07022	0.00022
CMEA + Centrally Planned	0.89776	0.00127	1.00942	0.00288
Canada	0.61940	0.00142	0.65771	0.02113
Other Developed	0.38899	0.00134	1.13416	0.00864
Latin American	0.28765	0.00122	1.81105	0.01134
African Countries	0.86423	0.00242	1.06181	0.01860
Hong Kong	0.33367	0.00086	0.89625	0.00822

TABLE 8

Predictive Performance of the CES Approach for the Japan, 1965–1980, Commodity Groups (5) to (8)

Name of Supplying Country	Square of Correlation Coefficient	Root Mean Square Error	Regression Coefficient of Actual on Predicted	Theil's Inequality Coefficient
United States	0.57882	0.02792	0.65432	0.00622
EEC	0.59703	0.02516	0.58935	0.01023
Other European	0.54917	0.00375	0.63425	0.00335
Republic of Korea	0.65916	0.02702	0.61358	0.21139
Other Developed	0.35121	0.00517	0.82772	0.03674
Latin American	0.60890	0.00803	0.69312	0.08293
African Countries	0.40147	0.01936	0.48412	0.20837
Taiwan	0.70592	0.01541	0.65026	0.24393
Hong Kong	0.67851	0.00361	0.92970	0.06212
CMEA + Centrally Planned	0.49653	0.00677	0.64247	0.00278
China	0.14902	0.00875	0.31590	0.16275

The restrictive nature of the Hickman-Lau model both on theoretical and empirical considerations may also be responsible for the somewhat poor estimates on substitution elasticities. However, the advantage of the model in terms of simplicity and its economy in the use of parameters[7] prevails over other models in the literature and it may remain one of the more useful models in the World Link study. The imports of a country can be allocated among exporting countries in such a way that total exports from trading partners to a given country are equal to imports

[7] The model used in this paper is a linearized version but non-linear estimation is possible. It has not been attempted because of the primary interest of this study in Asian developing countries and the rest of the world and, as there is a convergence problem on the linearized model, no attempt has been made at non-linear estimation.

which means that adding-up condition is automatically satisfied by the estimated model. The other important quantitative magnitude is the substitution elasticity which can be compared among different import markets and, for the same market, between two time periods.

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Appendix A

Data

This appendix presents, the nature of available data, their weaknesses and limitations.

The information of trade by origin/destination for sixty seven countries prepared by the United Nations [UN Secretariat, (1983)] is published at one digit level for Standard International Trade Classification (SITC) categories in the United Nations Trade Statistics. The United Nations publishes such information in current value terms for SITC groups (0+1), (2+4), (3), (5) to (8) and (0) to (9) total exports. Data are also available for the above SITC groups in constant 1975 prices. The description of main groups are (0) + (1) = Food and live animals, Beverages and tobacco, (2) + (4) = Crude materials, edible oil except fuels, and animal and vegetable oils and fats, (3) = Mineral fuels, lubricants and related materials, (5) + (6) + (7) + (8) = Chemicals, manufactured goods chiefly by material, miscellaneous manufactured articles, commodities and transactions not classified according to kind, and machinery and transport equipment, and (9) commodities and transactions not classified according to kind and residual errors.

In order to obtain trade flows at constant prices, the United Nations developed the information on price deflators. The export price deflators were intended to represent international price movements. For the group (5) to (8), manufactures and chemicals, the prices are built up country by country from unit values implicit in the detailed trade data. Quantity and value data were collected for SITC (5) to (8) inclusive as published in the Yearbook of International Trade Statistics. The data were selected at the 4-digit level and unit value prices were computed for each commodity. From these, a Paasche unit value index was computed for each year. As quantities are given in weight, it is not certain whether what is included under a four digit code is comparable from year to year.

The export price deflators were used to obtain trade flows at constant prices. No data on bilateral prices were available. The derived constant price trade flows matrix was for 67 countries. As our interest was not in a large number of small developed, Latin American and African countries individually, the aggregation leading to 25 countries/regions was used. New export price deflators and import price deflators were derived using aggregated current price and constant price trade flows matrix for 25 countries/regions.

In this study, all prices are taken in U.S. dollars and the prevailing exchange rates were historically used when export prices of countries were converted to U.S. dollars. In the World Link Model, exchange rates do not play any significant role at present. However, it is through the conversion of export prices and import demands into U.S. dollars from country models where exchange rates are required in the Link Model.

Appendix B

Derivation of Hickman-Lau Models

This appendix presents derived relationships on the constant dollar quantity of exports from the ith to jth country, (x_{ij}).

Assume there exists a quantity index of imports for each country or region of the CES type

$$m_j^* = [\sum_{j=1}^{n} a_{ij} x_{ij}^{-\rho_j}]^{-1/\rho_j} = [\sum_{i=1}^{n} a_{ij} x_{ij}^{(\sigma_j-1)/\sigma_j}]^{\sigma_j/(\sigma_j-1)} \qquad (A.1)$$

where $\sigma_j = (1/1+\rho_j)$ is the elasticity of substitution between the imports of any two countries in the jth market. Set

$$\sum_{i=1}^{n} a_{ij}^{\sigma_j} = 1$$

since we are permitted the arbitrary scalar transformation of the quantity index. The cost minimizing quantities of imports demanded for attaining a specified level of m_j^* are given by

$$x_{ij} = a_{ij}^{\sigma_j} m_j^* \left(\frac{P_{ij}}{P_j^{m*}}\right)^{-\sigma_j} \quad (A.2)$$

where
$$P_j^{m*} = \left[\sum_{i=1}^{n} a_{ij}^{\sigma_j} P_{ij}^{(1-\sigma_j)}\right]^{1/(1-\sigma_j)} \quad (A.3)$$

is the price index of imports with the property that

$$P_j^{m*} m_j^* = \sum_{i=1}^{n} P_{ij} x_{ij} = M_j \quad (A.4)$$

i.e., the product of the CES price and quantity indices is equal to the actual total value of the imports. The individual import demand functions may be written alternatively as

$$x_{ij} = a_{ij}^{\sigma_j} P_j^{m*} m_j^* P_{ij}^{-\sigma_j} P_j^{m*(\sigma_j-1)} = a_{ij}^{\sigma_j} M_j P_{ij}^{\sigma_j} P_j^{m*(\sigma_j-1)} \quad (A.5)$$

In the base period all export prices are set equal to unity and, given our normalization rule, all import prices are equal to unity as well. Hence

$$x_{ij}^{o} = a_{ij}^{\sigma_j} M_j^{o} \quad (A.6)$$

Define
$$m_j = \Sigma x_{ij} \quad (A.7)$$

as the total quantity of imports to the jth country or region. m_j is the arithmetic sum of imports in constant prices and not the CES quantity index of imports m_j^*. By using equations (A.4) and (A.7), equation (A.6) can be written as

$$x_{ij}^{o} = a_{ij}^{\sigma_j} \sum_{i=1}^{n} x_{ij}^{o} = a_{ij}^{\sigma_j} m_j^{o} \quad (A.8)$$

where m_j^{o} is the quantity of the base year imports to the jth country.

The base year share of the ith country's quantity of imports can be identified as

$$\alpha_{ij}^{o} = \frac{x_{ij}^{o}}{m_j^{o}} = a_{ij}^{\sigma_j} \qquad (A.9)$$

Thus one can write

$$x_{ij} = \alpha_{ij}^{o} \; M_j \; P_{ij}^{-\sigma_j} \; P_j^{m*(\sigma_j - 1)} \qquad (A.10)$$

$$P_j^{m*} = [\sum_{i=1}^{n} \alpha_{ij}^{o} \; P_{ij}^{(1-\sigma_j)} \;]^{1/(1-\sigma_j)} \qquad (A.11)$$

Now the total quantity of imports to the jth country m_j, is given by

$$m_j = \sum_{i=1}^{n} x_{ij} = [\sum_{i=1}^{n} \alpha_{ij}^{o} P_{ij}^{-\sigma_j} \;] \; [\sum_{i=1}^{n} \alpha_{ij}^{o} P_{ij}^{(1-\sigma_j)} \;]^{-1} M_j \qquad (A.12)$$

In other words

$$M_j = m_j \; [\sum_{i=1}^{n} \alpha_{ij}^{o} \; P_{ij}^{(1-\sigma_j)} \;] \; [\sum_{i=1}^{n} \alpha_{ij}^{o} \; P_{ij}^{-\sigma_j} \;]^{-1} \qquad (A.13)$$

Substituting equation (A.13) into equation (A.10), we obtain

$$x_{ij} = \alpha_{ij}^{o} \; P_{ij}^{-\sigma_j} \; [\sum_{k=1}^{n} \alpha_{kj} \; P_{kj}^{-\sigma_j} \;]^{-1} \; m_j \qquad (A.14)$$

(A.14) is an exact expression for any given level of imports. However (A.14) is non-linear and hence difficult to estimate and aggregate. We, therefore, linearize (A.14) in the P_{ij} by a Taylor's series expansion around $P_{ij} = 1$, all i and j. Choose a base period in which all export prices are set equal to unity. First order expression of (A.14) results in

$$x_{ij} = \alpha_{ij}^{o} m_j + \alpha_{ij}^{o} m_j (-\sigma_j) (P_{ij} - 1) - \alpha_{ij}^{o} m_j (-\sigma_j) [\sum_{k=1}^{n} \alpha_{kj}^{o} (P_{kj} - 1)] \qquad (A.15)$$

Observing that $\sum_{k=1}^{n} \alpha_{kj}^{o} = 1$, equation (A.15) simplifies to

$$x_{ij} = \alpha^o_{ij}\ m_j - \sigma_j\ (P_{ij} - \sum_{k=1}^{n} \alpha^o_{kj}\ P_{kj})\ \alpha^o_{ij}\ m_j \qquad (A.16)$$

If we now define a new import price index

$$P^m_j = \sum_{k=1}^{n} \alpha^o_{kj}\ P_{kj} \qquad (A.17)$$

having the interpretation of a fixed weighted average of the export prices of all countries in the jth market, equation (A.16) may be rewritten as

$$x_{ij} = \alpha^o_{ij}\ m_j - \sigma_j\ (P_{ij} - P^m_j)\ \alpha^o_{ij}\ m_j \qquad (A.18)$$

(A.18) satisfies the adding-up property, i.e.,

$$\sum_{i=1}^{n} x_{ij} = m_j$$

as $\sum_{i=1}^{n} \alpha^o_{ij} = 1$ and $\sum_{i=1}^{n} \alpha^o_{ij}\ P_{ij} = P^m_j$

If we further linearize x_{ij} in terms of m_j around m^o_j we have

$$x_{ij} = \alpha^o_{ij}\ m_j - \sigma_j\ \alpha^o_{ij}\ m_j\ (P_{ij} - P^m_j) = \alpha^o_{ij}\ m_j - \sigma_j\ x^o_{ij}\ (P_{ij} - P^m_j) \qquad (A.19)$$

In the above model, the notations used have the following interpretation:

x_{ij} = constant dollar quantity of exports from the ith country to the jth country;

x^o_{ij} = base year quantity;

α^o_{ij} = base year ith country's share in jth country's imports;

m_j = constant dollar quantity of imports of the jth country;

σ_j = elasticity of substitution between imports from any two countries in the jth market;

P_{ij} = price index of exports of country i to country j;

P^m_j = $\sum_{i=1}^{n} \alpha^o_{ij}\ P_{ij}$ = price index of imports of country j;

t = time trend set at zero in base year;

r_{ij} = trend coefficient.

If the original CES quantity index of imports for each country or region is a function also of time i.e.,

$$m_j^* = [\sum_{i=1}^{n} a_{ij} \; e^{r_{ij}t} \; x_{ij}^{(\sigma_j - 1)/\sigma_j}]^{\sigma_j/(\sigma_j - 1)} \qquad (A.20)$$

where the r_{ij}'s may be considered to be parameters indicating taste changes over time. Using similar assumptions as before, we can derive

$$x_{ij} = \alpha_{ij}^o \; m_j - \sigma_j \; x_{ij}^o \; (P_{ij} - P_j^m) + \sigma_j \; x_{ij}^o \; r_{ij} t \qquad (A.21)$$

The assumption

$$\sum_{i=1}^{n} \alpha_{ij}^o \; r_{ij} = 0 \qquad (A.22)$$

will have to be taken into consideration in the estimation to satisfy the adding-up condition.

Now, we adopt the assumption of static price expectations but adopt the partial adjustment hypothesis.

$$x_{ij} = \delta_j \; x_{ij}^* + (1 - \delta_j) \; (x_{ij})_{-1} \quad 0 \leqslant \delta_j \leqslant 1 \qquad (A.23)$$

where x_{ij}^* is the quantity of imports desired by the jth country from the ith supplying country. If equation (A.19) is rewritten to refer to the desired quantity demanded from i by j, we have

$$x_{ij}^* = \alpha_{ij}^o \; m_j - \sigma_j \; x_{ij}^o \; (P_{ij} - P_j^m) \qquad (A.24)$$

Combining (A.24) with (A.23) yields

$$x_{ij} - (x_{ij})_{-1} = \delta_j \; [\alpha_{ij}^o \, m_j - (x_{ij})_{-1}] - \delta_j \sigma_j \; [x_{ij}^o \, (P_{ij} - P_j^m)] \qquad (A.25)$$

Summing equation (A.25) over i after substituting $x_{ij}^o = \alpha_{ij}^o \, m_j$, yields

$$\sum_{i=1}^{n} x_{ij} = \delta_j m_j + (1 - \delta_j) \; (m_j)_{-1} \qquad (A.26)$$

where use has been made of

$$\sum_{i=1}^{n} \alpha_{ij} = 1, \; \sum_{i=1}^{n} \alpha_{ij}^{o} P_{ij} = P_j^m \text{ and } \sum_i x_{ij} = m_j,$$

and the right hand side will be equal to m_j only if $\delta_j = 1$. This means that there is no adjustment lag and the model is a static version. Thus, the lagged adjustment specification is not consistent with the adding-up criterion.

An alternative, which yields an acceptable specification under plausible assumptions concerning the formation of price expectations, is the one where we assume immediate adjustment with an adaptive expectation lag. In its general form, the adaptive expectation hypothesis is

$$(P_{ij} - P_j^m) = \beta_{ij} + \delta_j (P^e_{ij(-1)} - P^{me}_{j(-1)}) + (1-\delta_j)(P_{ij} - P_j^m) + \gamma_{ij} t \quad \text{(A.27)}$$

where $0 < \delta_j < 1$.

Let us write (A.19) in terms of expected prices

$$x_{ij} = \alpha_{ij}^o m_j - \sigma_j x_{ij}^o (P_{ij}^e - P_j^{me}) + \sigma_j x_{ij}^o r_{ij} t \ldots \quad \text{(A.28)}$$

Substitute (A.27) into (A.28) and then use (A.28) lagged one period to eliminate the lagged expected price term, the result is

$$\begin{aligned} x_{ij} = \alpha_{ij}^o m_j + \sigma_j x_{ij}^o (\delta_j r_{ij} - \beta_{ij}) + \sigma_j x_{ij}^o [(1-\delta_j) r_{ij} - \gamma_{ij}] t \\ - \sigma_j (1-\delta_j) x_{ij}^o (P_{ij} - P_j^m) + \delta_j (x_{ij} - \alpha_{ij}^o m_j)_{-1} \end{aligned} \quad \text{(A.29)}$$

The parameter δ_j and σ_j are identifiable but the others are not. This is discussed in the text.

Appendix C

LIST OF COUNTRIES

1. Canada
2. United States
3. Japan
4. Australia
5. New Zealand
6. EEC[a]
7. Other European[b]
8. Other Developed[c]
9. Latin American[d]
10. African Countries[e]
11. Oil Producing Middle Eastern[f]
12. Indonesia
13. Taiwan
14. Hong Kong
15. India
16. The Republic of Korea
17. Malaysia
18. Pakistan
19. Philippines
20. Singapore
21. Thailand
22. Other South East Asia[g]
23. South East Asia[h]
24. CMEA & Centrally Planned[i]
25. China

[a] EEC: France, West Germany, Belgium-Luxembourg, Netherlands, Italy, U.K., Denmark, Ireland, Greece.

[b] Norway, Sweden, Austria, Finland, Switzerland, Iceland, Portugal, Spain.

[c] Malta, South Africa, Turkey, Israel, Yugoslavia.

[d] Argentina, Brazil, Chile, Colombia, Ecuador, Mexico, Peru, Venezuela, Caribbean and Central America and other Latin America.

[e] Algeria, Gabon, Libya, Nigeria, Egypt, Angola to Zambia and all other African Countries and West Asian Oil Importing Countries.

[f] Iran, Iraq, Kuwait, Saudi Arabia, Bahrain, Oman, Qatar, United Arab Emirates.

[g] Brunei, Burma, Fiji, Macau, Papua New Guinea, Sri Lanka.

[h] Afghanistan, Bangladesh, Laos, Nepal, Western Samoa.

[i] Bulgaria, Czechoslovakia, Germany DR, Hungary, Poland, Romania, USSR.

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