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USE OF INEQUALITY MEASURES IN CALCULATING INCOME ELASTICITIES

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Kakwani (1980) has suggested the use of Gini Coefficient to estimate income elasticities. This paper shows that co-efficient of variation, another measure of inequality, can also be used efficiently to find income elasticities. Derivation of both the estimators are based on the use of Extended Linear Expenditure System.

I. Introduction

Studies on consumption pattern are usually based on estimation and analysis of elasticity coefficients for various consumption items. Income elasticities of demand have become more important in explaining the consumption pattern owing to Engle's law. These are also useful for a comparative study of the consumption pattern of any two or more samples. Such a comparative study on the basis of income elasticities may provide a fair picture of relative poverty in the two samples. Moreover, the differences in consumption patterns between any two samples may be related in a way to the differences in their income distributions. Pasha (1982) indicated that income elasticities of demand for various consumer goods are affected by the distribution of income. This relationship of distribution of income among households in a society and the resulting distribution of consumption expenditure on various items is effectively utilized by Kakwani (1980).

Kakwani has presented his formula for computation of expenditure (income) elasticity, where the elasticity coefficient is expressed as a function

of the Gini Index. This formula is based on a mathematical manipulation of expenditure elasticity formula derived from the Linear Expenditure System (LES). In this paper we have shown that the elasticity formula of Linear Expenditure System can be used to derive an alternative formula, for the computation of expenditure elasticity. It is suggested that another important measure of income inequality, namely the "coefficient of variation" can be used effectively to estimate the expenditure (income) elasticities. The formula derived here is conceptually identical to the LES formula. A simple rearrangement of terms in the LES formula has made it possible to use coefficient of variation in the computation of income elasticities.

Kakwani's formulation of expenditure elasticity is presented in Section II in which he has used the "Gini Index" to find elasticity of consumption expenditure on a commodity with respect to total expenditure. Derivation of an alternative formula for computation of expenditure elasticity in terms of "coefficient of variation" is also presented in the same section. A brief description of the data is presented in Section III and the results of the two formulae are discussed and compared in Section IV. Conclusions and some implications of our approach are then presented in Section V.

II.a Kakwani's Approach

LES equations are given by

$$v_i = a_i + b_i (v - a) \qquad (1)$$

where v_i is the expenditure[1] (per capita or household) on the ith commodity; $v = \Sigma v_i$; a_i = subsistence expenditure on the ith commodity; $a = \Sigma a_i$; b_i = marginal budget share of the ith commodity.

Equation (1) can be re-written in the following form

$$v_i = c_i + b_i (v) \qquad (1a)$$

where $c_i = (a_i - b_i \bullet a)$

This system of demand equations is based on the well known Stone-Geary form of utility function, which imposes the following restrictions on b_i.

[1] In our empirical exercise the unit of analysis is an average household.

$$0 < b_i < 1 \qquad (i = 1,2,3, \ldots n) \tag{2}$$

$$\Sigma\ b_i = 1 \tag{3}$$

Let $\tilde{n}_i$ be the elasticity of demand for the ith good with respect to total expenditure given by

$$\tilde{n}_i = b_i / w_i \tag{4}$$

where w_i is the average budget share of the ith commodity given by $w_i = v_{i*}/v_i$.

Further, let "G_i" be the Gini index for the distribution of expenditure on the ith commodity, and "G_t" the Gini index of total expenditure. Kakwani arrived at the following relationship by manipulating the elasticity formula of equation (4) which is derived from LES.

$$G_i = b_i \bullet v^* \bullet G_t / v_{i*} \tag{5}$$

where v_{i*} is the mean expenditure on the ith commodity and is given by

$$v_{i*} = a_i + b_i(v^* - a)$$

and v^* is the mean of total expenditure. Equations (4) and (5) together imply that,

$$\tilde{n}_i = G_i / G_t \tag{6}$$

where $\tilde{n}_i$ is the expenditure elasticity of the ith commodity at the mean expenditures. Thus Kakwani proves that the expenditure elasticity of the ith commodity at the mean expenditures is equal to the ratio of the Gini Indexes of the distributions of the ith commodity expenditure and the total expenditure, respectively [Kakwani (1980)].

To compute the Gini index, Kakwani has suggested that any of the standard formula can be used. He has not made any specific recommendation for the purpose of computation of the elasticity. We have used the following formula to compute the Gini index.[2]

[2] Household income and expenditure surveys present data by income groups. True Gini index of expenditures can be calculated if data were arranged (ranked) by the household expenditure, therefore using HIES data in our exercise we will in fact compute a concentration index of expenditure not a true Gini index.

$$\text{Gini index} = 1 - \Sigma S_p(Y_{j-1} + Y_{j+1}) \quad (7)$$

where S_p = cumulative population share of the jth income group; Y_{j-1} = cumulative income share of the j−1th income group; Y_{j+1} = cumulative income share of the j+1th income group.

II.b Alternative Approach

The alternative formula is derived here through a simple transformation of expenditure elasticity formula based on LES in equation (4) above. This transformation is based on the definition of "b_i" in the ordinary least squares method. "b_i" is defined by equation (1a) as the slope coefficient of regression equation of v_i on v. The formula for "b_i" therefore can be written as

$$b_i = r_i \bullet sd(v_i) / sd(v) \quad (8)$$

where "r_i" is the coefficient of correlation between v_i and v; $sd(v_i)$ = standard deviation of v_i; and sd(v) = standard deviation of v.

Substituting equation (8) in equation (4) yields

$$\begin{aligned} \tilde{n}_i &= (r_i \bullet [sd(v_i)/sd(v)] / w_i \\ &= [r_i \bullet (sd(v_i)/v_{i*})/(sd(v)/v^*] \\ &= r_i \bullet cv(v_i)/cv(v) \end{aligned} \quad (9)$$

where "$cv(v_i)$" is the coefficient of variation of v_i and "cv(v)" is the coefficient of variation of v. "r_i" is the coefficient of correlation between "v_i" the expenditure on the ith commodity and "v" the total expenditure. Equation (9) gives an alternative formula for the computation of expenditure or income elasticity, where elasticity coefficient is expressed as a function of coefficient of variation rather than the ratio of Gini indices as in equation (6).

III. Data

The data used here is taken from the latest "Household Income and Expenditure Survey of Pakistan" for the year 1985-86. The elasticities are computed only for Pakistani rural areas. This data in the survey is presented in the grouped form, where grouping is done on the basis of household income. Our analysis, therefore, has been carried out at house-

hold level using average household expenditures and not the household's per capita expenditures.

We have computed expenditure elasticities for nine broad commodity groups as presented in the survey. These commodity groups are namely Food, Beverage and Tobacco; Apparel Textile and Footwear (Clothing); Transport and Communication; Cleaning Laundry and Personal Appearances; Recreation, Entertainment and Education; Rent; Fuel and Lighting; Household Furniture and Equipment; and Miscellaneous Consumption Expenditures.

IV. Results

The results of estimation of expenditure elasticities are reported in Table 1, for all the nine groups of commodities. Average expenditure share of each commodity group in the household budget for rural Pakistan is also presented in the same table. This provides an idea about the relative

TABLE 1

A Comparison of Elasticity Estimates by Alternative Formulas

Commodity Groups	Average Budget Share (%)	$\tilde{n}_i$(G)	$\tilde{n}_i$(C.V)	$\tilde{n}_i$(C.V. Crude)
Food	51.5	0.876	0.803	0.814
Clothing	7.9	0.899	0.844	0.851
Transport	3.8	1.599	2.159	2.409
Cleaning	4.7	0.946	0.953	0.962
Education	1.1	1.746	1.988	2.064
Rent	7.9	0.911	0.946	0.952
Fuel	6.3	0.695	0.617	0.635
Furniture	1.9	1.194	1.263	1.273
Miscellaneous	14.8	1.445	1.565	1.581

importance of a commodity group in the consumer's basket. In Table 1 $\tilde{n}_i$(C.V) represents expenditure elasticities calculated using equation (9) and $\tilde{n}_i$(G) using equation (6).

Looking at the average budget shares in Table 1, it shows that Food and Clothing take away the major chunk of household's budget in the rural areas of Pakistan. The elasticity coefficients have a value of less than unity in case of five commodity groups. These commodity groups are Food, Cloth, Cleaning, Rent, and Fuel. These commodity groups may be considered as necessities for consumption in rural Pakistan. It is interesting to note that Fuel has elasticity coefficient equal to 0.617 which is even less than the value of the coefficient for Food which is 0.803. One possible reason for it may be the inclusion of Beverage and Tobacco in the Food group which are expected to be relatively responsive to changes in income. Further, in the Pakistani rural society a large portion of food consumption is home produced (unpurchased) and there may be substantial under reporting of expenditure on food items. Probably the tendency of under reporting is higher in the lower income groups. Another explanation lies in the fact that the use of Fuel and Lighting in rural Pakistan is fairly similar in terms of kind and quality, looking across the households in various income groups. In addition to that being basic consumption items, expenditure on Fuel and Lighting does not respond much to the changes in income level.

Since Pakistan is a developing country, its rural area is highly underdeveloped. This is also reflected in the consumption pattern, as the value of elasticity coefficient for the commodity groups like Transport and Communication, and Education and Recreation is found to be significantly greater than unity. This is just in accordance with the norms in a rural society such as of Pakistan. Further, the three commodity groups which can be classified as luxury goods in the case of rural Pakistan, have the lowest priority on the average in the consumer's budget (Table 1).

It may also be noted from the table that the results of the two formulas reported as $\tilde{n}_i$(G) and $\tilde{n}_i$(C.V) are slightly different although their origin is the same. Using the formula based on Gini index would not give exactly the same results as given by LES formula. In some cases it under-estimates while in others over-estimates the true LES elasticities. Results indicate that in case of the three commodity groups namely Food, Clothing, and Fuel, it slightly over-estimates the elasticity coefficient, while for the remaining six groups it has given an under estimate. Probably this inconsistency is for two reasons: firstly, it may be due to grouping and aggregation bias[3]

[3] On the data set of rural Pakistan, HIES 1979, Ercelawn (1988) computed aggregation errors in the measurement of income inequalities for various indices of inequalities. He finds an error

as grouped data are being used. Secondly, as explained in footnote (2), true Gini, G_t and G_i (concentration indexes in our numerical exercise) have been estimated lower than their true values.[4] Further, the degree of under-estimation in G_t and G_i may be different and in the case of G_i this under-estimation may vary for different commodity groups in a sample. It seems important to mention here that while Kakwani's formula may over or under estimate the values of elasticity coefficients as compared to those obtained by using LES formula in its original form, the alternative formulation suggested here will not do so.

It is also seen from Table 1 that the expenditure on all the nine commodity groups have a high and positive correlation with the total expenditure. That is why a simple ratio of coefficients of variation gives quite a good approximation of the values of elasticity coefficients. These values are reported in Table 1 as $\tilde{\pi}_i$(C.V. Crude), where $\tilde{\pi}_i$(C.V. Crude) = $cv(v_i)/cv(v)$.

To observe the closeness of the crude estimate to the actual values of elasticity coefficients we have calculated mean square error. The mean square error for the crude estimate based on coefficients of variation is 0.008. In this particular case the elasticity estimate obtained through the Gini index have a relatively higher mean square error which is 0.045. But the crude estimate based on coefficients of variation may not always give better results as compared to the estimate based on Gini index. However, for practical purposes a simple ratio of coefficients of variation can give an approximation (always an over-estimate of the magnitude) of the elasticity coefficient, given the knowledge about the correlation between the two variables.

V. Conclusions

The consumption pattern of rural Pakistan represents a picture of low incomes and provides indications of a backward and underdeveloped society. Expenditure on food items (including beverage and tobacco) accounts for more than fifty per cent of the household budget on the average. Many other items which are treated as necessity in the modern age are unable to justify their appropriate importance in the basket of a household in the rural Pakistan. The expenditure elasticity coefficients

of more than 25 per cent in the coefficient of variation, as against less than 2 per cent in the Gini coefficient. However, we find no evidence to conclude that coefficient of variation will always give a higher aggregation error as compared to Gini index for various data sets.

[4] For example, Pyatt (1980) has shown that the concentration index of a variable is always less than or equal to its true Gini index in magnitude.

for most of the commodity groups have a value of either close to unity or greater than unity.

For a given data set Kakwani's formula may not give the same results as the original elasticity formula of LES in the presence of various imperfections, such as, estimation of pseudo-Gini in place of true Gini, grouping and aggregation errors and if the underlying function is not exactly (and positively) linear.

Elasticity formula presented in this paper (elasticity coefficient as a function of coefficients of variation) gives the same results as LES formula in its original form, even for grouped data. It is shown in Section II that on a theoretical basis the formula in equation (9) is directly derived from LES formula of equation (4), without making any simplifying assumptions. The only difference is in the concepts involved in their computations. The LES formula of equation (4) requires the estimation of a set of regression equations to compute elasticities, whereas the relationship in equation (9) requires the coefficient of correlation between the two variables and their coefficients of variation.

Again, it may be useful to mention here that the LES method imposes certain restrictions on "b_i" as described in equations (2) and (3) of Section II. These restrictions in turn become the limitations of LES method in computing elasticities, since these restrictions imply that all commodities consumed (for which elasticity is being calculated) are normal goods. Because for an inferior good, income (expenditure) elasticity has to be negative which in turn requires a negative value of "b_i". For empirical purposes it is not necessary to impose these restrictions while computing income (expenditure) elasticity using the formula based on coefficients of variation, although it is an alternative form of the original LES formula. The sign of the correlation coefficient describes whether a commodity is an inferior good or a normal good.

The formula developed in this paper has a general applicability for the estimation of elasticity between any two variables. Income (consumption) elasticities have been used here only as an illustration.

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