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INVESTMENT AND INCOME DISTRIBUTION PATTERN UNDER MUSHARKA FINANCE:
The Uncertainty Case

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In the present study we extend the model presented in Siddiqui and Zaman [The Certainty Case (1989)] in order to include the element of uncertainty. We find total investment higher when the mode of finance is close to Musharka finance. The income distribution pattern is favourable to providers of funds when output (profit) is higher and vice versa. These results have profound implications for Islamic banking, for they reveal that profits of banks and depositors would be higher during booms at the expense of producers. On the other hand, any shock would be shared by a larger group of economic agents.

Siddiqui and Zaman [The Certainty Case (1989)] analyzed basic characteristics of an economy based on Musharka finance in a deterministic framework. However, since we live in an uncertain world, any meaningful economic analysis of the subject must address this issue. Incorporating uncertainty in any economic model has always been a difficult task for economists (for a discussion on this issue in the context of Islamic economics, see Zarqa, (1983). Very often economic agents have to make decisions requiring future information which is usually incomplete and imperfect. In some cases we have a subjective or objective probability distribution of future outcomes. Uncertainty in this case does not take the form of vague ignorance, but of quite precise beliefs. In contrast to cases of choice with certianty, here we assign non-zero probabilities to more than one conceivable state of the world. Hirshleifer (1970) calls this "precise uncertainty," as it lacks psychological similarity to the ignorance, confusion and doubt that is associated with a feeling of "vague uncertainty" about the nature of the world and our knowledge of it.[1] "In order to analyze consumer behaviour, we must simplify and *idealize*. Individuals need not have identical beliefs as to the probability of occurrence of the various states. But each individual must know in advance the nature of his own entitlement i.e., his state distributed endowment. Also, he must know the state distributed consumption vectors attainable through his productive and market opportunities" [(Hirshleifer, (1970) p. 273].

In our present analysis of Musharka finance under uncertainty, we have assumed that each individual has the same "precise beliefs" about state contingent production possibilities (and hence consumption possibilities).[2] Like the certainty case [Siddiqui and Zaman, (1989)], he lives for two periods. In the first period he starts with a given endowment of corn, the only good available in the economy. He is also allowed to borrow or lend in a perfect market. His problem in the first period is to choose levels of current consumption and plantation based on his beliefs about production functions for different states, good or bad. Following the certainty model, we analyze and compare some basic characteristics of the two simple economies; one based on the traditional fixed interest system and the other on sharing system similar to the Islamic principle of "Musharka".

In Musharka finance individuals pool their resources and lend them to producers/entrepreneurs. Whether one (or more) of the producer is from amongst the lenders is immaterial for the analysis of Musharka finance, if one gets a separate reward for the labour input. In our model both types of farmers, lenders and borrowers, produce and consume a single commodity, corn. Profit (or loss) of the borrower is shared by them but the lender does not get profits exactly in the same proportion as his contribution. His share is less than that because the borrower gets a fraction of the profit as remuneration for his managerial input which is determined in the market. So far we have not drifted too much from the principle of Musharka. The only significant difference is that, in our model the lender does not suffer a loss in exactly the same proportion as his initial contribution, a major principle of Musharka finance. However, as we will see later, this only strengthens some of our key conclusions.

I. The Fixed Interest Model

The utility maximization problem (for the two period case), in the traditional system, with an element of risk, can be formulated as follows:

$$\text{Max } U(C_{1i}) + h\,[E\{U(C_{2i})\}]$$

[1] Hirshleifer (1970), p.273

[2] Non-identical subjective probability distribution would make the analysis too complicated without improving significantly the main results.

where $U(C_{2i}) = U\{q\, C2_{gi} + (1-q)\, C_{2bi}\}$.

C_{2gi} is consumption in the second period if conditions are good in the planting period, a probability denoted by q. The production function for this period is $f(P_i)$. C_{2bi} is consumption during bad conditions, for which the relevant production function is $b(P_i)$. C_{1i}, the first period consumption of the ith borrower, is equal to $Y1_i+B_i-P_i$, where Y_{1i} is the initial endowment, B_i is the amount of borrowing or lending and P_i is the planting. The discount rate for the second period consumption is h. Using the VonNeuman-Morgenstem utility function, the maximization problem for the ith farmer can be written:

$$\text{Max } U(Y_{1i}+B_i-P_i)+h[qU\{g(P_i)-(1+r)B_i\}+(1-q)U\{b(P_i)-P_i,B_i\,(1+r)B_i\}] \quad (1.1)$$

where $Y_{1i} + B_i - P_i = C_{1i}$.

$$g(P_i) - (1+r)B_i = C_{2gi},$$

$$b(P_i) - (1+r)B_i = C_{2bi}.$$

The first order conditions for the ith farmer with respect to P_i and B_i respectively, can be written as:

$$U'(C_{1i}) = hq\, U'(C_{2gi})\, g'(P_i) + h(l-q)\, U'(C_{2bi})\, b'(P_i) \quad (1.2a)$$

$$U'(C_{1i}) = hq\, U'(C_{2gi})\, (l+r) + h(l-q)\, U'(C_{2bi})\, (l+r) \quad (1.2b)$$

Equations (1.2a) can be expressed as the rate of commodity substitution

$$U'(C_i) / [qU'(C_{2gi}) + (l-q)\, U'(C_{2bi})] = h(l+r) \quad (1.3)$$

The two first order conditions can be simplified and rearranged to get

$$qU'(C_{2gi})\{g'(P_i) - (l+r)\} = -(l-q)U'(C_{2bi})\{b'(P_i) - (i+r)\}$$

$$\frac{U'(C_{2gi})}{U'(C_{2bi})} = \frac{-(1-q)\{b'(P_i) - (1+r)\}}{q\{g'(P_i) - (1+r)\}} \quad (1.4)$$

With positive marginal utilities, this implies that the marginal product in one period must be less than l+r.[3] There is no reason to believe that P_i will be identical for all the farmers as we saw in the certainty case. According to equation (1.4), for P_i to be

[3] For $f(P_i^k) = (A+e)P_i^k$ and $g(P_i) = (A-e)P_i^k$, this will be true for the bad period.

the same for all the farmers, borrowers or lenders, the ratio of marginal utilities of the two future states (good or bad) must be identical for all the individuals. As second period consumption depends on the initial endowments, which are different for the borrower and the lender, there is no reason why the left hand side of equation (1.4) will be the same for all the farmers.

Again these first order conditions cannot be used to get explicit solutions of the two choice variables in a general form. Furthermore, no analytical solutions for P_i and B_i exist for any combinations of appropriate utility and production functions. The only option left, therefore, is to use a numerical technique:

Assigning $U(C_{ti}) = Ln(C_{ti})$, $g(P_i) = (A+e)P_i^k$ and $b(P_i) = (A-e)P_i^k$, first order conditions with respect to P_i and B_i respectively are:

$$\frac{1}{(Y_{1i}+B_i-P_i)} = \frac{hqk(A+e)_i^{k-1}}{(A+e)P_i^k-(1+r)B_i} + \frac{hk(1-q)(A-e)_i^{k-1}}{(A-e)P_i^k-(1+r)B_i} \qquad (1.5)$$

$$\frac{1}{(Y_{1i})+B_i-P_i)} = \frac{hq(1+r)}{(A+e)P_i^k-(1+r)B_i} + \frac{h(1-q)(1+r)}{(A-e)P_i^k-(1+r)B_i} \qquad (1.6)$$

Dividing (1.5) by (1.6) we get:

$$\frac{khq(A+e)P_i^{k-1} - hq(1+r)}{(A+e)P_i^k - (1+r)B_i} = \frac{-kh(1-q)(A-e)P_i^{k-1} + h(1-q)(1+r)}{(A-e)P_i^k - (1+r)B_i} \qquad (1.7)$$

Equation (1.7) can be used to express B_i as a function of P_i and other parameters (see Appendix for the derivation).

$$B_i = \frac{(1+r)(A+e-2qe)P_i^k - k(A+e)(A-e)P_i^{2k-1}}{(1+r)^2 - k(1+r)(A-e+2qe)P_i^{k-1}} \qquad (1.8)$$

Equation (1.7) can be simplified into a quadratic equation in B_i (see Appendix for the derivation) to get the following expression for B_i.[4]

$$B_i = \frac{-X1 + \sqrt{X1^2 - 4X2\ X3}}{2X2} \qquad (1.9a)$$

$$B_i = \frac{-X1 + \sqrt{X1^2 - 4X2\ X3}}{2X2} \qquad (1.9b)$$

[4]Gain B_i is a function of P_i and other parameters.

where $X1 = -2A(1+r)P_i^k + h(Y_{1i}-P_i)(1+r)^2 - H_{pi}^k(1+r)(A+e-2qe),$

$X2 = (1+h)(1+r)^2,$

$X3 = (A+e)(A-e)P_i^{2k} - (Y_{1i}-P_i)\{h(1+r)P_i^k - (A+e-2qe)\}.$

Equations (1.8), (1.9a) and (1.9b) are shown in Figure 1. As we cannot get analytical solution for both B_i and P_i, we use these two equations to get numerical solutions for our choice variables. The methodology is similar to that contained in section II of Siddiqui and Zaman (1989). The three equations are used for both the borrower and the lender. For each value of r there are two sets of solutions: one for the lender and other for the borrower. At equilibrium the borrowing and lending values are equal.

II. The Sharing Model

2.1 Borrower's Case:

Introducing the element of risk in the sharing model, the maximization problem for the borrower is:

$$U(C_{1i}) + h\,EU(C_{2i})$$

where $EU(C_{2i}) = U\{q(C_{2gi}) + (1-q)(C_{2bi})\};$

$C_{1i} = Y_{1i}+B_i-P_i;$

$C_{2gi} = g(P_i) - m(B_i/P_i)\{g(P_i)-P_i\}-B_i;$

$C_{2bi} = b(P_i) - m(B_i/P_i)\{b(P_i)-P_i\}-Bi.$

B_i is the amount of borrowing, and $m(B_i/P_i)$ is the fraction of profit (or loss) accruing to the lender.

The first order condition with respect to P_i and B_i are:[5]

$$-U'(C_{1i})+hqU'(C_{2gi})[g'(P_i)-mB_i/P_i\{P_ig'(P_i)-g(P_i)$$
$$+h(1-q)U'(C_{2bi})[b'(P_i)-mb_i/P_i\{P_ib'(P_i)-b(P_i)\}] = 0$$

$$U'(C_{1i}) - hqu'(C_{2gi})[mg(P_i)/P_i-m\}+1]$$
$$- h(1-q)U'(C_{2bi})[mb(P_i)/P_i-m\}+1] = 0$$

[5] Using the definition of VonNeumanMorgenstern Utility function.

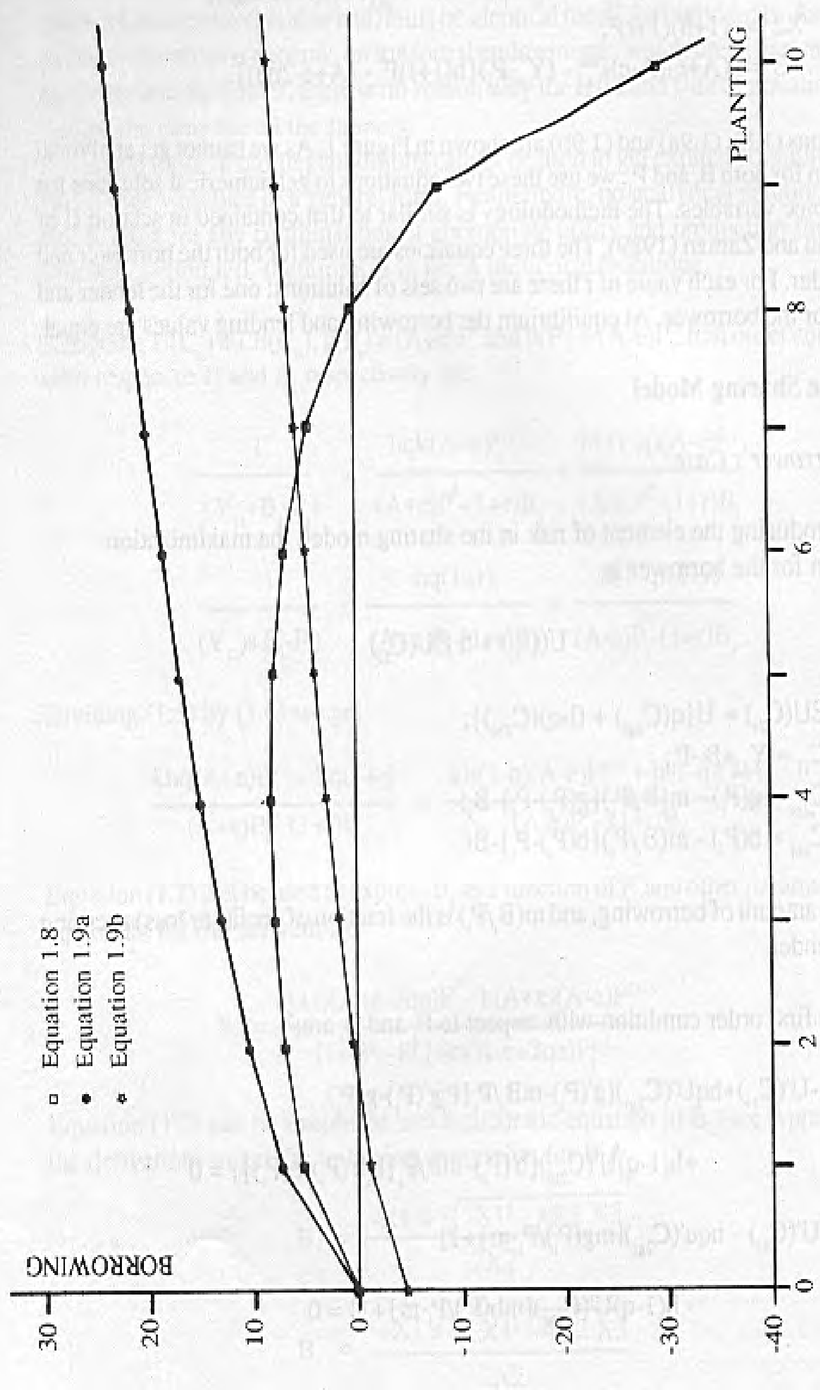

FIGURE 1

Interest Case

A=10, h=0.9, k=0.5, q=0.5, e=4

These two first order conditions are too complex to derive meaningful economic interpretation or a general solution for the choice variables. We, therefore, consider the specific case of a log utility function and the power production function,[6]

$$\text{Max } Ln(C_{1i}) + h\,E\,[Ln(C_{2i})]$$

where

$$C_{1i} = Y_{1i} + B_i - P_i \tag{2.1}$$

and

$$C_{2i} = [(A+e)P_i^k - m\frac{B_i}{P_i}\{(A+e)P_i^k - P_i\} - B_i]q$$

$$+ [(A-e)P_i^k - m\frac{B_i}{P_i}\{(A+e)P_i^k - P_i\} - B_i]q$$

B_i is the total borrowing of the ith borrower and q is the probability of a good outcome. First order conditions with respect to P_i and B_i can be written as follows:

$$\frac{1}{Y_{1i}+B_i-P_i} = \frac{hq[k(A+e)P_i^{k-1} - mB_i(k-1)(A+e)P_i^{k-2}]}{(A+e)P_i^k - mB_i(A+e)P_i^{k-1} + (m-1)B_i}$$

$$+ \frac{h(1-q)[k(A-e)P_i^{k-1} - mB_i(k-1)(A-e)P_i^{k-2}}{(A-e)P_i^k - mB_i(A-e)P_i^{k-1} + (m-1)B_i} \tag{2.2}$$

$$\frac{1}{Y_{1i}+B_i-P_i} = \frac{hq[m(A+e)P_i^{k-1} - m+1]}{(A+e)P_i^k - mB_i(A+e)P_i^{k-1} + (m-1)B_i}$$

$$+ \frac{h(1-q)\,[m(A-e)P_i^{k-1} - m+1]}{(A-e)P_i^k - mB_i(A-e)P_i^{k-1} + (m-1)B_i} \tag{2.3}$$

Dividing (2.2) by (2.3) we get (2.4)

$$\frac{kq(A+e)P_i^{k-1} - qmB_i(A+e)(k-1)P_i^{k-2} - qm(A+e)P_i^{k-1} - q}{(A+e)_i^k - mb_i(A+e)P_i^{k-1} - B_i} =$$

$$\frac{-k(1-q)(A-e)_i^{k-1} + mb_i(1-q(A-e)(k-1)P_i^{k-2} + m(1-q)(A-e)P_i^{k-1} + (1-q)}{(A-e)P_i^k - mb_i(A-e)P_i^{k-1} - B_i} \tag{2.4}$$

[6]Production functions for the good and the bad cases are the same as those of the interest case.

Equation (2.4) can be simplified into a quadratic equation in B_i (see Appendix) to get:[7]

$$B_i = \frac{-X4 + \sqrt{X4 - 4X5\,X6}}{2\,X5} \qquad (2.5a)$$

or

$$B_i = \frac{-X4 + \sqrt{X4 - 4X5\,X6}}{2\,X5} \qquad (2.5b)$$

where

$X4 = m(A+e)(A-e)(k-1)P_i^{2k-2} - km(A+e)(A-e)P_i^{2k-2} + MP_i^{k-1}(A+e-2qe)$
$-kP_i^{k-1}(A-e+2qe)+mP_i^{k-1}(A-e+2qe)+1,$

$X5 = m^2(A+e)(A-e)(k-1)P_i^{2k-3} + m(k-1)P_i^{k-2}(A-e+2qe)$, and

$X6 = (k-m)(A+e)(A-e)P_i^{2k-1} - P_i(A+e-2qe).$

Equation (2.3) can also be simplified to get another quadratic equation in B_i (see Appendix):

$$B_i = \frac{-X7 + \sqrt{X7 - 4X8\,X9}}{2X8} \qquad (2.6a)$$

$$B_i = \frac{-X7 + \sqrt{X7 - 4X8\,X9}}{2\,X8} \qquad (2.6b)$$

where

$X7 = (Y_{1i}-P_i)[hm^2(A+e)(A-e)P_i^{2k-2} + 2hmAP_i^{k-1}+h]-2AP_i^k-$
$m(2+h)(A+e)(A-e)P_i^{2k-1} - h(A+e-2qe)P_i^k,$

$X8 = m^2(1+h)(A+e)(A-e)P_i^{2k-2} + mP_i^{k-1}[2A+h(A-e)+h(A+e)]+(1+h)$, and

$X9 = A(A+e)(A-e)P_i^{2k} - (Y_{1i}-P_i)[hm(A+e)(A-e)P_i^{2k-1} + hP_i(A+e-2qe)].$

As we are unable to get analytical solutions, equations (2.5a), (2.5b), (2.6a) and (2.6b) are used to get numerical solutions for P_i and B_i. All these equations are shown graphically in Figure 2.

[7] B_i is a function of P_i and other parameters.

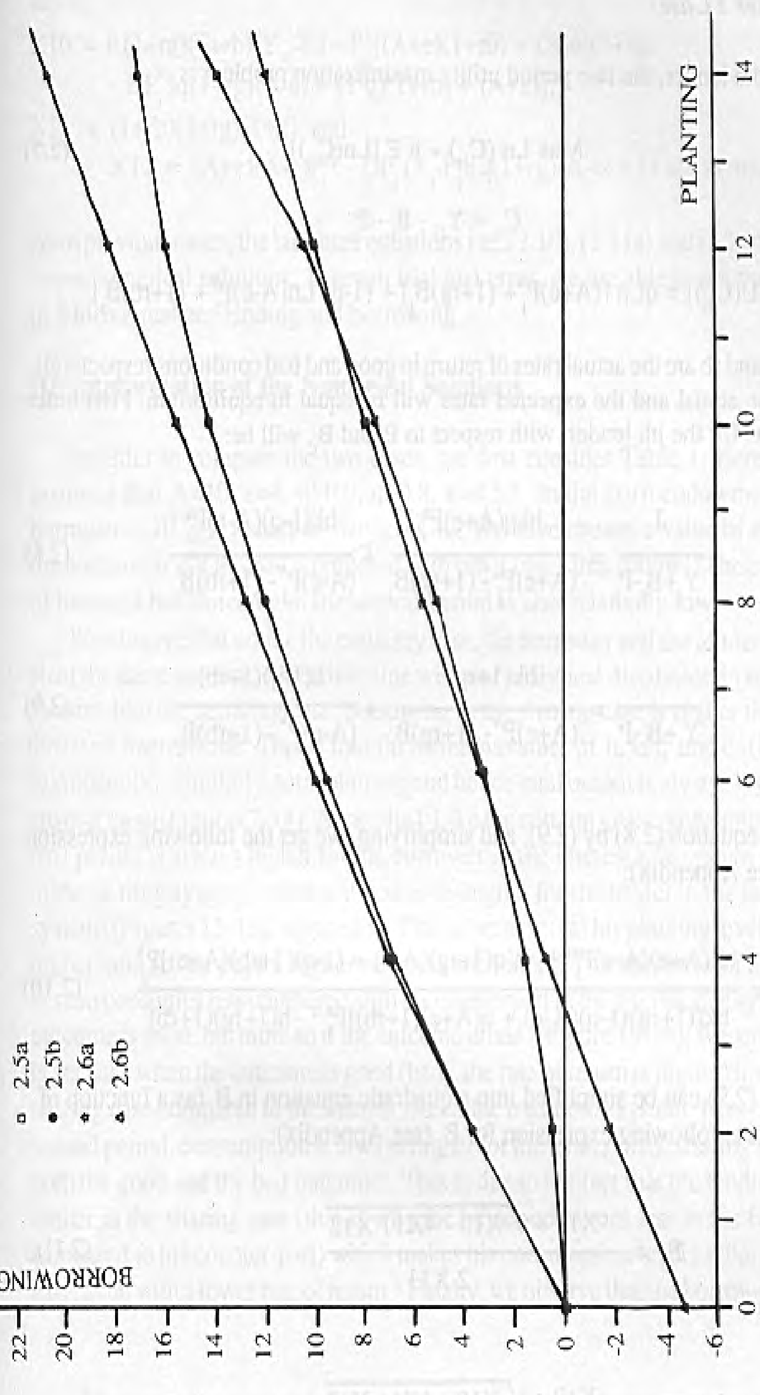

FIGURE 2

Sharing; Borrower's Case

A=10, h=0.9, q=0.5, k=0.5, e=4

2.2 Lender's Case:

For the lender, the two period utility maximization problem is

$$\text{Max Ln }(C_{1j}) + h\,E\,[\text{Ln}(C_{2j})] \tag{2.7}$$

where $$C_{1j} = Y_{1j} - B_j - P_j$$

and $$E[U(C_{2j})] = q\text{Ln}\,[(A+e)P_j^k + (1+rg)B_j] + (1-q)\,\text{Ln}[A-e)P_j^k + (1+rb)B_j]$$

where rg and rb are the actual rates of return in good and bad conditions respectively. Again, the actual and the expected rates will be equal in equilibrium. First order conditions for the jth lender, with respect to P_j and B_j, will be:

$$\frac{1}{Y_j+B_j-P_j} = \frac{hkq(A+e)P_j^{k-1}}{(A+e)P_j^k - (1+rg)B_j} + \frac{hk(1-q)(A-e)P_j^{k-1}}{(A-e)P_j^k - (1+rb)B_j} \tag{2.8}$$

$$\frac{1}{Y_j+B_j-P_j} = \frac{hk(1+rg)}{(A+e)P_j^k - (1+rg)B_j} + \frac{h(1-q)(1+rb)}{(A-e)P_j^k - (1+rb)B_j} \tag{2.9}$$

Dividing equation (2.8) by (2.9), and simplifying, we get the following expression for Bj (see Appendix):

$$B_i = \frac{hk(A+e)(A-e)P_j^{2k-1} - h[q(1+rg)(A-e) + (1-q)(1+rb)(A+e)]P_j^k}{hk[(1+rg)(1-q)(A-e) + q(A+e)(1+rb)]P_j^{k-1} - h(1+rg)(1+rb)} \tag{2.10}$$

Equation (2.9) can be simplified into a quadratic equation in B_j (as a function of P_j) to get the following expression for B_j (see Appendix):

$$B_j = \frac{-X10 + \sqrt{X10 - 4X11\ X12}}{2\ X11} \tag{2.11a}$$

or $$Bj = \frac{-X10 + \sqrt{X10 - 4X11\ X12}}{2\ X11} \tag{2.11b}$$

where

$$X10 = h(1+rg)(1+rb)(Y_{1j}-P_j) - P_i^k[(A+e)(1+rb) + (A-e)(1+rg) - DP_j^k[q(1+rg)(A-e) + (1-q)(1+rb) + (A+e)],$$

$$X11 = (1+rb)(1+rg)(1+h), \text{ and}$$

$$X12 = (A+e)(A-e)P_i^{2k} - DP_j^k(Y_j-P_j)[q(1+rg)(A-e) + (1-q)(1+rb)(A+e)].$$

As in previous cases, the last three equations i.e., (2.10), (2.11a) and (2.11b) are used to get numerical solutions. Through trial and error, we are able to get the value of m which equalizes lending and borrowing.

III. Interpretation of the Numerical Solutions

In order to compare the two cases, we first consider Table 1. Here we have assumed that A=10, e=4, h=0.9, q=0.8, k=0.5.[8] Initial corn endowment for the borrower is 10, and that of the lender is 40. We have chosen a value of e such that the bad case is not too bad (compared with other cases that follow). The probability of having a bad outcome in the second period is also relatively low.

We observe that unlike the certainty case, the borrower and the lender no longer plant the same amount, which is in line with our analytical discussion in section 3.1. Second, like the certainty case, borrowing in the sharing case is higher than that of the fixed interest case. This is true for different values of h, k, q and e [(Figures 3-6) Appendix]. Similarly, total planting and hence total output is always higher in the sharing case[(Figures 7-14), Appendix]. Like the certainty case, consumption in the first period is always higher for the borrower in the interest rate system compared to the sharing system, whereas it is usually higher for the lender in the interest rate system [(Figures 15-18), Appendix]. This is because: (a) his planting level is always higher, and (b) he pays a higher return to the lender. The borrower in the sharing system consumes less compared with his counterpart in the interest rate system if the outcome is good, but more so if the outcome is bad [(Figure 19-26), Appendix]. This is because when the outcome is good (bad), the rate of return is higher (lower) in the sharing case compared to the interest rate of the traditional system. However, in the second period, consumption is always higher for the lender in the sharing system for both the good and the bad outcomes. This is due to the fact that the lending level is higher in the sharing case (that is why the lender consumes less in the first period compared to his counter-part) which makes his consumption level higher in the bad case, even with a lower rate of return.[9] Finally, we observe that the borrower is worse

[8] In choosing these values we had same criteria in mind as discussed in the certainty case [Siddiqui and Zaman (1989)].

[9] Compared to the interest rate.

TABLE 1(a)

A=10, e=4, h=0.9, q=0.8, k=0.5

Sharing Case (m = 0.6206, rg = 1.8222, rb = 0.4263)

	Y1	B	P	Y2g[a]	Y2b[b]	PBG*	PBB**	C1	C2g	C2b	U
Bor	10	10.10	12.650	49.79	21.34	28.65	14.48	7.50	21.15	6.863	4.55
Len	40	(-)10.10	5.758	33.59	14.40	(-)28.65	(-)14.48	24.09	62.24	28.870	6.76

TABLE 1(b)

Interest Case

(r = 0.9035)

	Y1	B	P	Y2g[a]	Y2b[b]	PBG*	PBB**	C1	C2g	C2b	U
Bor	10	5.13	6.462	35.59	15.25	9.755	9.755	8.663	25.83	5.497	4.80
Len	40	(-)5.13	9.055	42.13	18.05	(-) 9.755	(-)9.755	25.820	51.88	27.810	6.69

[a]Output in good case.
[b]Output in bad case.

*Pay back (Principal and interest or profit or loss) in good case.
**Pay back in bad case.

off in the sharing system compared to the fixed interest rate system (true for different values of h, k, q and e [Figures 27-30), Appendix]. The rate of interest r in the fixed interest rate model is between the rates of return rg and rb of the sharing model.

Keeping all other things the same while increasing the value of e to 6 (Table 2) does not make any qualitative difference. However, once we increase the value of e to 8 (Table3), the bad case becomes very bad, some interesting changes take place. First the rate of return for the bad case in the sharing model becomes negataive. Second, the utility level for the borrower is relatively higher in the sharing case compared to the fixed interest rate case, and the difference of utility levels for the lender becomes negligible (utility of the lender in the sharing case is slightly higher compared to the fixed interest rate case.)

Third, the gain in the utility level for the borrower in the sharing case is not only due to the rate of return for the bad outcome being negative (as a matter of fact, borrower's consumption for the bad case in the sharing model is now lower than that of his counterpart in the fixed interest rate case), but also due to the fact that borrowing and planting are still higher in the sharing case. This makes the borrower consume more in the first period compared to the interest rate case, and get a relatively higher level of output in case of a good outcome. Even after paying a higher rate of return for the good case, he enjoys better consumption. This is because in the sharing case the lender agrees to share the expected loss which allows the borrower to keep a high level of planting.

It is important to remind the reader that our sharing rule does not exactly correspond to the principle of Musharka finance. Specially, in our model, in case of loss the lender's share is less than proportionate to his capital contribution. However, this only strengthens our result that under Musharka Finance the burden of the loss will be even higher on the lender.

IV. Implications for Islamic Banking and Income Distribution

In this paper we have analyzed investment and income distribution pattern under a sharing system similar to Musharka finance (compared to the fixed interest rate system) with an element of uncertainty. We assumed that all agents (the borrower and the lender) assign identical probabilities to all possible future outcomes. In this context our certainty model [(Siddiqui and Zaman, (1989)] becomes a special case of the present model.[10]

In the certainty model we concluded that: (1) investment is always higher in the sharing case, and (2) lenders are better off as compared with their counterparts in the interest rate system. The first result holds for all different cases we have considered in the present model involving uncertainty. But lenders under the sharing case

[10] With only one possible future outcome.

TABLE 2(a)

A=10, c =6, h =0.9, q=0.8, k=0.5

Sharing Case (m=0.4750, rg=1.8249, rb=0.0999)

	Y1	B	P	Y2g	Y2b	PBG	PBB	C1	C2g	C2b	U
Bor	10	10.10	12.650	49.79	21.34	28.65	14.48	7.50	21.15	6.863	4.55
Len	40	(-)10.10	5.758	33.59	14.40	(-)28.65	(-)14.48	24.09	62.24	28.870	6.76

TABLE (2(b)

Interest Case

(r = 6850)

	Y1	B	P	Y2g	Y2b	PBG	PBB	C1	C2g	C2b	U
Bor	10	3.35	5.30	36.83	9.209	5.645	5.645	8.050	31.19	3.564	4.79
Len	40	(-)3.35	10.22	51.16	12.790	(-) 5.645	(-)5.645	26.440	56.79	18.420	6.70

TABLE 3(a)

A=10, e=8, h=0.9, q=0.8, k=0.5

Sharing Case (m=0.0830, rg=0.5841, rb=0.009)

	Y1	B	P	Y2g[1]	Y2b[2]	PBG[3]	PBB[4]	C1	C2g	C2b	U[5]
Bor	10	2.69	5.015	40.31	4.479	4.261	2.666	7.675	36.05	1.812	4.72
Len	40	(-)2.69	10.650	58.75	6.528	(-)4.261	(-)2.666	26.650	63.02	9.194	6.66

TABLE 3(b)

Interest Case

(r = 0.1700)

	Y1	B	P	Y2g[1]	Y2b[2]	PBG[3]	PBB[4]	C1	C2g	C2b	U[5]
Bor	10	1.91	4.350	37.54	4.171	2.235	2.235	7.56	35.31	1.937	4.72
Len	40	(-)1.91	11.16	60.14	6.683	(-)2.235	(-)2.235	26.93	62.38	8.918	6.66

[1]Output in good case. [2]Output in bad case. [3]Pay back in good case. [4]Pay back in bad case. [5]Utility.

(compared to the interest rate system) are not always better off in an uncertain scenario. If one of the two possible outcomes is bad, then the borrowers are better off compared with their counterparts in the interest rate case (if the future outcome happens to be bad).

It is important to point out that our analysis and the results depend on the rather strong assumption that all economic agents assign identical probabilities to all possible future outcomes. However, this would only strengthen our results because differences in beliefs or complete ignorance or vague uncertainty would increase the vulnerability of any financial system where the borrowers of funds make future commitments of fixed amounts against uncertain future income.

The results imply that the sharing system is inherently more stable and would help in preventing any prolonged business cycle. Furthermore, it will result in a more equitable income distribution pattern: favoring savers in good days and investors in bad days, just the opposite of what we observe in the contemporary capitalist system. The implication of these results for the contemporary banking system is important. In the current system depositors of commercial banks get a relatively small portion of banks' profits as interest payment. In real terms, if we account for the rate of inflation, these interest payments are even lower. A World Bank report states that during the last two decades, real interest rates paid on savings were generally negative in a sample of thirty developingcountries [Monthly South, (1989), page 37]. The report recommends that in order to boost up savings, it is imperative for the developing countries to keep the interest rate at a reasonably positive level.[11]

If banks in Islamic countries (almost all of them being developed or underdeveloped) do most of their business under Musharka and Mudarba[12] arrangements, it can affect the level and form of savings positively.[13] On the other hand, banks would receive a higher percentage of the profits earned by the producers/firms. In the light of these results, the reasoning is: (a) the level of investment will be higher, (b) by sharing the risks of businesses, banks will get higher returns during good years. Indeed, in case of an economic downturn, profits of banks will be lower and during extremely bad years they may even turn negative. But even then, the implications for the whole economy and consequently for the banks and the depositors will not be negative. This is because in the sharing system, in case of an economic shock, depositors and banks would settle for a lower rate of profit (or even for a loss in some

[11] The report pointed out that this has become more important with the new developments in Eastern Block which will attract more and more international capital in the coming years.

[12] Siddiqui (forthcoming), finds that the implication of Mudarba finane is similar to what we have found in case of Musharka. As a matter of fact, according to our results, under Mudarba finance, the level of investment is even higher.

[13] Due to higher inflation and low nominal rate of interest, many people save invest their savings in gold, real estate, etc.

extreme cases). This will solve the insolvency problem for both the banks and the firms/producers and significantly shorten the period of recession. During the inflationary period, the loosers are, generally, the depositors and the banks. By sharing the profits of the businesses, the firms and the depositors will be generally insured against inflation.[14]

Transition to Islamic Banking is not a trivial issue. Banking officials and workers will have to be trained to apply the new techniques of investment and portfolio management efficiently. Project appraisal departments will have to be opened in all the banks. But real success would depend on the government's ability to convince the industrialists (and to some extent the bankers) that the rules of the game have now changed. Our results clearly show that the Islamic banking system has serious distributional implications. If a significant portion of banks' assets are based on principles of Islamic finance, the profits of the industrialists will be curtailed. Take the case of Pakistan where industrial loans are granted at 15 to 18 per cent interest rate with the official rate of inflation at 10 per cent. One can only imagine the proportion of business profits going to the banks. Needless to mention, the common depositors usually receive negative interest payments. Iqbal and Mirakhor (1987) point out that when an attempt to implement Islamic Banking system was made in the Zia era in Pakistan, the industriaists had informed the government that they will not accept any arrangement in which they will receive a lower profit level. It is clear that the transition to the new system can be successfully completed if: (a) the government can convince all economic agents that the rules of the game have changed, and (b) it is supported by economists and bankers who are well versed in the Islamic banking system and are serious about implementing it.

The most important conclusions of this paper (along with the certainty case, Siddiqui and Zaman, (1989) is that: (a) under a sharing system similar to Musharka Finance, investment (and hence saving) level is higher compared to the fixed interest rate system, and (b) it generates an income distribution pattern which is more desirable. Unlike the socialist system it does not require a decline in efficiency to have a favourable distributional consequence. An understanding of the implications of Musharka and Mudarba finance, calls for further research aimed at investigating the role of the monetary and the fiscal authorities in an Islamic economic system. Further research aimed at studying problems related with inflation, budget deficit and the government's power to create money are particularly recommended.

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14 We intend to take up the problems of inflation and budget inflation in the Islamic economic system in the near future.

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Appendix

Derivation of Equation (1.8):

$$\frac{khq(A+e)_i^{k-1} - hq(1+r)}{(A+e)_i^k - (1+r)B_i} = \frac{-kh(1-q)(A-e)_i^{k-1} + h(1-q)(1+}{(A-e)_i^k - (1+r)B_i} \qquad (1.7)$$

Cross multiplying (1.5)

$$khq(A+e)(A-e)P_i^{2k-1} - hq(1+r)(A-e)P_i^k - khq(A+e)(1+r)B_iP_i^{k-1} + hq(1+r)^2 B_i$$

$$= -kh(1-q)(A+e)(A-e)P_i^{2k-1} + h(1-q)(1+r)(A+e)P_i^k$$

$$+ kh(1-q)(A-e)(1+r)B_iP_i^{k-1} - h(1-q)(1+r)^2 B_i.$$

Or

$$k(A+e)(A-e)P_i^{2k-1}(q+1-q) - (1+r)P_i^k [q(A-e) + (1-q)(A+e)]$$

$$- k(1+r)B_iP_i^{k-1}[q(A+e) + (1-q)(A-e)] + (1+r)^2 B_i(a+1-q) = 0.$$

Or

$$k(A+e)(A-e)P_i^{2k-1} - (1+r)Pk (1+r)P_i^k (A+e-2qe)$$

$$- k(1+r)B_iP_i^{k-1}(A-e+2qe) + (1+r)^2 B_i = 0.$$

Or

$$B_i[(1+r)^2 - k(1+r)(A-e) + 2qe)P_i^{k-1}] = (1+r)(A+e-2qe)P_i^k - k(A+e)(A-e)P_i^{2k-1}$$

Finally

$$B_i = \frac{(1+r)(A+e-2qe)P_i^k - k(A+e)(A-e)P_i^{2k-1}}{(1+r)^2 - k(1+r)((A-e+2qe)P_i^{k-1}} \qquad (1.8)$$

Derivation of Equations (1.9a) and (1.9b).

Starting from equation (1.6) we have:

$$\frac{1}{(Y_{1i}+B_i-P_i)} = \frac{hk(A+e)P_i^{k-1}}{(A+e)P_i-(1+r)B_i} + \frac{hk(1-q)(A-e)P_i^{k-1}}{(A-e)P_i-(1+r)P_i}$$

Or

$$\frac{1}{(Y_{1i}+B_i-P_i)} = \frac{[hq(1+r)(A-e)(P_i^k-hq(1+r)^2B_i+h(1-q)(1+r)(A+e)P_i^k-h(1-q)(1+r)^2B_i]}{[(A+e)(A-e)P_i^{2k}-(A+e)(1+r)B_iP_i^k-(A-e)(1+r)B_iP_i^k+(1+r)^2B_i^2]}$$

Or

$$(1+r)^2B_i^2-[(A+e)(1+r)P_i^k+(A-e)(1+r)P_i^k]B_i+(A+e)(A-e)P_i^{2k}$$

$$=(Y_{1i}-P_i)[hq(1+r)(A-e)P_i^k+h(1-q)(1+r)(A+e)P_i^k]$$

$$+(Y_{1i}-P_i)[-hq(1+r)^2-h(1-q)(1+r)^2]B_i+[h(1+r)P_i^k\,q(A-e)$$

$$+(1-q)(A+e)]B_i+[-h(1+r)^2(q+1-q)]B_i^2.$$

Or

$$[(1+r)^2(1+h)]B_i^2+[(1+r)P_i^k(-A-e-A+e)+(Y_{1i}-P_i)h(1+r)^2$$

$$-h(1+r)P_i^k\,q(A-e)-(1-q)(A+e)]B_i+(A+e)(A-e)P_i^{2k}$$

$$-(Y_{1i}-P_i)[h(1+r)P_i^k\,[q(A-e)+(1-q)(A+e)]]=0.$$

Or

$$[(1+r)^2(1+h)]B_i^2+[-2A(1+r)P_i^k+(Y_{1i}-P_i)(h(1+r)^2]-h(1+r)P_i^k(A+e-2qe)]$$

$$+(A+e)((A-e)P_i^{2k}-(Y_{1i}-P_i)\{h(1+r)P_i^k(A+e-2qe)]=0$$

The last expression is a quadratic equation which can be solved for B_i. So either:

$$B_i = \frac{-X1 + \sqrt{X1 - 4X2\,X3}}{2X2} \qquad (1.9a)$$

Or

$$B_i = \frac{-X1 - \sqrt{X1 - 4X2\,X3}}{2X2} \qquad (1.9b)$$

where $X1 = -2A(1+r)P^k_i + h(Y_{1i} - P_i)(1+r)^2 - hP^k_i(1+r)(A+e-2qe)$,
$X2 = (1+h)(1+r)^2$, and
$X3 = (A+e)(A-e)P^{2k}_i - (Y_{1i} - P_i)\{h(1+r)P^k_i(A+e-2qe)\}$.

Derivation of equation (2.5a) and (2.5b):

In equation (2.4), we have

$$\frac{kq(A+e)P^{k-1}_i - qmB_i(A+e)(k-1)P^{k-2}_i - qm(A+e)P^{k-1}_i - q}{(A+e)P_i - mB_i(A+e)P_i - B_i}$$

$$\frac{-k(1-q)(A-e)P^{k-1}_i + mB_i(1-q)(A-e)(k-1)P^{k-2}_i + m(1-q)(A-e)P^{k-1}_i + 1-q}{(A-e)P^k_i - mB_i(A-e)^{k-1}_i - B_i}$$

Or

$$kq(A+e)(A-e)P^{2k-1}_i - qmB_i(A+e)(A-e)(k-1)P^{2k-2}_i - qm(A+e)(A-e)P^{2k-1}_i$$

$$-q(A-e)P^k_i - kqmB_i(A+e)(A-e)P^{2k-2}_i + qm^2B^2_i(A+e)(A-e)(k-1)P^{2k-3}_i$$

$$+ qm^2B_i(A+e)(A-e)P^{2k-2}_i + qmB_i(A-e)P^{k-1}_i - kqB_i(A+e)P^{k-1}_i$$

$$+ qmB^2_i(A+e)(k-1)P^{k-2}_i + qmB_i(A+e)P^{k-1}_i + qB_i$$

$$= -k(1-q)(A+e)(A-e)P^{2k-1}_i + mB_i(1-q)(A+e)(A-e)(k-1)P^{2k-2}_i$$

$$+ m(1-q)(A+e)(A-e)P^{2k-1}_i + (A+e)(1-q)P^k_i + kmB_i(1-q)(A+e)(A-e)P^{2k-2}_i$$

$$- m^2B^2_i(1-q)(k-1)(A+e)(A-e)P^{2k-3}_i - m^2B_i(1-q)(A+e)(A-e)P^{2k-2}_i$$

$$- mB_i(1-q)(A+e)P_i^{k-1} + kB_i(1-q)(A-e)P_i^{k-1} - mB_i^2(1-q)(A-e)(k-1)P_i^{k-2}$$

$$- mB_i(1-q)(A-e)P_i^{k-1} - B_i(1-q).$$

Or

$$k(A+e)(A-e)P_i^{2k-1}(q+1-q) - mB_i(A+e)(A-e)(k-1)P_i^{2k-2}$$

$$(q+1-q)m(A+e)(A-e)P_i^{2k-1}(q+1-q) - P_i^k\{q(A-e)+(1-q)(A+e)$$

$$- kmB_i(A+e)P_i^{2k-2}(q+1-q) + m^2B_i^2(A+e)(A-e)(k-1)P_i^{2k-3}(q+1-q)$$

$$+ m^2B_i(A+e)(A-e)P_i^{2k-2}(q+1-q) + mB_iP_i^{k-1}\{q(A-e)+(1-q)(A+e)\}$$

$$+ kB_iP_i^{k-1}\{q(A+e) + (1-q)(A-e)\} + mB_i^2(k-1)P_i^{k-2}\{q(A+e)$$

$$+ mB_iP_i^{k-1}\{q(A+e)+(1-q)(A-e)\} + B_i(q+1-q) = 0.$$

Or

$$k(A+e)(A-e)P_i^{2k-1} - mB_i(A+e)(A-e)(k-1)P_i^{2k-2} - m(A+e)(A-e)P_i^{2k-1}$$

$$- P_i^k(A+e-2qe) - kmB_i(A+e)(A-e)P_i^{2k-2} + m^2B_i^2(A+e)(A-e)(k-1)P_i^{2k-3}$$

$$+ m^2B_i(A+e)(A-e)P_i^{2k-2} + mB_iP_i^{k-1}(A+e-2qe) - kB_iP_i^{k-1}(A-e+2qe)$$

$$+ mB_i^2(k-1)P_i^{k-2}(A-e+2qe) + mB_iP_i^{k-1}(A-e+2qe) + B_i = 0.$$

Or

$$[m^2(A+e)(A-e)(k-1)P_i^{2k-3} + m(k-1)P_i^{k-2}(A-e+2qe)]B_i^2$$

$$+ [-m(A+e)(A-e)(k-1)P_i^{2k-2} - km(A+e)(A-e)P_i^{2k-2} + m^2(A+e)(A-e)P_i^{2k-2}$$

$$+ mP_i^{k-1}(A=e-2qe) - kP_i^{k-1}(A+e-2qe) + mP_i^{k-1}(A-e+2qe) + 1]B_i$$

$$+ k(A+e)(A-e)P_i^{2k-1} - m(A+e)(A-e)P_i^{2k-1} - P_i^k(A+e-2qe).$$

The last expression is a quadratic equation which can be solved for B_i, i.e.,

$$B_i = \frac{-X4 + \sqrt{X4^2 - 4X5\,X6}}{2\,X5} \qquad (2.5a)$$

Or

$$B_i = \frac{-X4 - X4\sqrt{X4^2 - 4X5\,X6}}{2\,X5} \tag{2.5b}$$

where $X4 = -m(A+e(A-e(k-1)P_i^{2k-2}-km(A+e)(A-e)P_i^{2k-2}+m^2(A+e)(A-e)P_i^{2k-2}$
$+mP_i^{k-1}(A+e-2qe)-kP_i^{k-1}(A+e-2qe)+mP_i^{k-1}(A-e+2qe)+1$,
$X5 = m^2(A+e(A-e)(k-1)P_i^{2k-3}+m(k-1)P_i^{k-2}(A-e+2qe)$, and
$X6 = k(A+e)(A-e)P_i^{2k-1}-m(A+e)(A-e)P_i^{2k-1}-P_i^k(A+e-2qe)$.

Derivation of E1quation (2.6a) and (2.6b):

Starting from equation (2.3) we

$$\frac{1}{(Y_{1i}+B_i+P_i)} = \frac{hg[m(A+e)P_i^{k-1}+1]}{(A+e)P_i^k-mB_i(A+e)P_i^{k-1}-B_i} + \frac{h(1-g)[m(A-e)P_i^{k-1}+1}{(A-e)P_i^k-mB_i(A-e)P_i^{k-1}-B_i}$$

Or

$$\frac{1}{(Y_{1i}+B_i-P_i)} =$$

$$\frac{[hqm(A+e)(A-e)P_i^{2k-1}+hq(A-e)P_i^k-hqm^2B_i(A+e)(A-e)P_i^{2k-2}-hqmB_i(A-e)P_i^{k-1}}{[(A+e)(A-e)P_i^{2k}-mB_i(A+e)(A-e)P_i^{2k-1}-B_i(A+e)P_i^k-mB_i(A+e)}$$

$$\frac{-hqmB_i(A+e)P_i^{k-1}-hqB_i+hm(1-q)A+e)(A-e)P_i^{2k-1}+h(1-q)m^2B_i(A+e)}{(A+e)(A-e)P_i^{2k-1}+m^2B_i^2(A+e)(A-e)P_i^{2k-2}}$$

$$\frac{(A-e)P_i^{2k-1}-h(1-q)mB_i(A+e)P_i^{k-1}-h(1-q)mB_i(A-e)P_i^{k-1}-hB_i(1-q)]}{+mB_i^2(A+e)P_i^{k-1}-(A-e)B_iP_i^k+mB_i^2(A-e)P_i^{k-1}+B_i^2]}$$

Or

$$\frac{1}{(Y_{1i}+B_i-P_i)} =$$

$$\frac{[hm(A+e)(A-e)P_i^{2k-1}+hP_i^k\{q(A-e)+(1-q)(A+e)\}-hm^2B_i}{[(A+e)(A-e)P_i^{2k-2}mB_i(A+e)(A-e)P_i^{2k-1}+m^2B_i^2(A+e)(A-e)P_i^{2k-2}}$$

$$\frac{(A+e)(A-e)P_i^{2k-2}-hmB_i(A-e)P_i^{k-1}-hmB_i(A+e)P_i^{k-1}-hB_i]}{-B_iP_i^k(A+e+A-e)+mB_i^2\,P_i^{k-1}(A+e+A-e)+B_i^2]}$$

Cross multiplication gives

$$(A+e)(A-e)P_i^{2k}-2mB_i(A+e)(A-e)P_i^{2k-1}+(mB_i)^2(A+e)$$
$$(A-e)P_i^{2k-2}-2AB_iP_i^k+B_i^2+2AmB_i^2P_i^{k-1}$$
$$=(Y_{1i}-P_i)[hm(A+e)(A-e)P_i^{2k-1}+hP_i^k(A+e-2qe)]+(Y_{1i}-P_i)B_i[-hm^2(A+e)$$
$$(A-e)P_i^{2k-2}-hm(A-e)P_i^{k-1}-hm(A+e)P_i^{k-1}-h]+B_i[hm(A+e)(A-e)P_i^{2k-1}+hP_i^k$$
$$(A+e-2qe)]+B_i^2[-hm^2(A+e)(A-e)P_i^{2k-2}-hm(A-e)P_i^{k-1}-hm(A+e)P_i^{k-1}-h]$$

Or

$$[m^2(A+e)(A-e)P_i^{2k-2}+mP_i^{k-1}[2A+h(A-e)+h(A-e)]+hm^2(A+e)(A-e)P_i^{2k-2}$$
$$+h+1]\,B_i^2+[(Y_{1i}-P_i)\{hm^2(A+e)(A-e)P_i^{2k-2}+2AhmP_i^{k-1}+h\}-2AP_i^k-2m(A+e)$$
$$(A-e)P_i^{2k-1}-hm(A+e)(A-e)P_i^{2k-1}-hP_i^k(A+e-2qe)]B_i+(A+e)(A-e)P_i^{2k}-Y_{1i}-P_i)$$
$$\{hm(A+e)(A-e)P_i^{2k-1}+hP_i^k(A+e-2qe)\}=0.$$

Again we get a quadratic equation which can be solved for B_i, i.e.,

$$B_i=\frac{-X7+\sqrt{X^27-4\,X8\,X9}}{2X8} \qquad (2.6a)$$

Or

$$B_i=\frac{-X7+\sqrt{X^27-4\,X8\,X9}}{2X8} \qquad (2.6b)$$

where $X8 = m^2(A+e)(A-e)_i^{2k-2}+mP_i^{k-1}\{2A+h(A-e)+h(A-e)\}$
$+hm^2(A+e)(A-e)P_i^{2k-2}+h+1,$

$X7 = (Y_{1i}-P_i)\{hm^2(A+e)(A-e)P_i^{2k-2}+2AhmP_i^{k-1}+h\}-2AP_i^k$
$-2m(A+e)(A-e)P_i2^{k-1}-hm(A+e)(A-e)P_i2^{k-1}-hP_i^k(A+e-2qe)$, and

$X9 = (A+e)(A-e)P_j^{2k}-(Y_{1i}-P_j)\{hm(A+e)(A-e)P_i^{2k-1}+hP_j^k(A+e-2qe)\}.$

Derivation of Equation (2.10):

Dividing (2.8) by (2.9) we get:

$$\frac{qk(A+e)P_j^{k-1}-q(1+rg)}{(A+e)P_j^k-(1+rg)B_j} = \frac{-k(1-g)(A-e)P_j^{k-1}+(1-g)(1+rb)}{(A-e)P_j^k-(1+rb)B_j}$$

$$qk(A+e)(A-e)P_j^{2k-1}-q(1+rg)(A-e)P_j^k-qk(A+e)(1+rb)B_jP_j^{k-1}+q(1+rg)(1+rb)B_j$$

$$= -k(1-q)(A+e)(A-e)P_j^{2k-1}+(1-q)(1+rb)(A+e)P_j^k$$

$$+k(1+rg)(1-q)(A-e)B_jP_j^{k-1}-(1+rg)(1+rb)(1-q)B_j$$

Or

$$k(A+e)(A-e)P_j^{2k-1}-P_j^k\{q(1+rg)(A-e)+(1-q)(1+rb)(A+e)\} =$$

$$kB_jP_j^{k-1}\{1+rg)(1-q)(A-e)+q(A+e)(1+rb)\} - B_j(1+rg)(1+rb)$$

Or

$$B_j = \frac{k(A+e)(A-e)P_j^{2k-1} - P_j^k\{q(1+rg)(A-e) + 1-g)(1+rb)(A+e)\}}{kP_j^{k-1}\{(1+rg)((1-q)(A-e) + q(A+e)(1+rb) - (1+rg)(1+rb)} \qquad (2.10)$$

Derivation of Equations (2.11a) and (2.11b):

Simplifying right hand side of equation (2.9) we have:

$$\frac{1}{(Y_{ij}-B_j-P_j)} =$$

$$\frac{[hq(1+rg)(A-e)P_j^k-hq(1+rb)(1+rg)B_j+h(1-q)(1+rb)(A+e)P_j^k-h(1-q)(1+rg)(1+rb)B_j}{[A+e)(A-e)P_j^{2k}-(A+e)(1+rb)P_j^kB_j-(A-e)(1+rg)P_j^kB_j+(1+rg)(1+rb)B_j^2]}$$

Or

$$\frac{1}{(Y_{ij}-B_j-P_j)} =$$

$$\frac{[hP_j^k\{q(1+rg)(A-e)+(1-q)(1+rb)(A+e)\}-h(1+rg)(1+rb)B_j}{[(A+e)(A-e)P_j^{2k}-B_jP_j^k\{(A+e)(1+rb)+(A-e)(1+rg)\}+(1+rg)(1+rb)B_j^2}$$

Cross multiplication gives:

$$B_j^2(1+rg)(1+rb)-B_jP_j^k\{A+e)(1+rb)+(A-e)(1+rg)+(A+e)(A-e)P_j^{2k}$$

$$= hP_j^k(Y_{ij}-P_j)\{q(1+rg)(A-e)+(1-q)(1+rb)(A+e)\}h(1+rg)(1+rb)(Y_{ij}-P_j)B_j$$

$$+hB_jP_j^k\{q(1+rg)(A-e)+(1-q)(1+rb)(A+e)\}-h(1+rg)(1+rb)B_j^2$$

Or

$$[(1+rg)(1+rb)(1+h)]B_j^2+[P_j^k[A+e)(1+rb)+(A-e)(1+rg)]$$

$$+h(1+rg)(1+rb)(Y_{ij}-P_j)-hP_j^k\{q(1+rg)(A-e)+(1-q)(1+rb)(A+e)\}B_j$$

$$+(A+e)(A-e)P_j^{2k}-hP_j^k(Y_{ij}-P_j)\{q(1+rg)(A-e)+(1-q)(1+rb)(A+e)\}.$$

This is a quadratic equation which will give values for bj as follows:

$$B_j = \frac{-X10+\sqrt{X10^2-f\,X11\,X12}}{2X11} \qquad (2.11a)$$

Or

$$B_i = \frac{-X10 - \sqrt{X10^2 - 4\,X11\,X12}}{2X11} \qquad (2.11b)$$

Where $X10 = P_j^k\{(A+e)(1+rb)+(A-e)(1+rg)\}+h(1+rg)(1+rb)$
$(Y_{1j}-P_j)-hP_j^k\{q(1+rg)(1+rb)(1+h),$

$X11 = (1+rg)(1+rb)(1+h)$, and

$X12 = (A+e)(A-e)P_j^{2k}-hP_j^k(Y_{1j}-P_j)\{q(1+rg)(A-e)+(1-q)(1+rb)(A+e)\}]$.

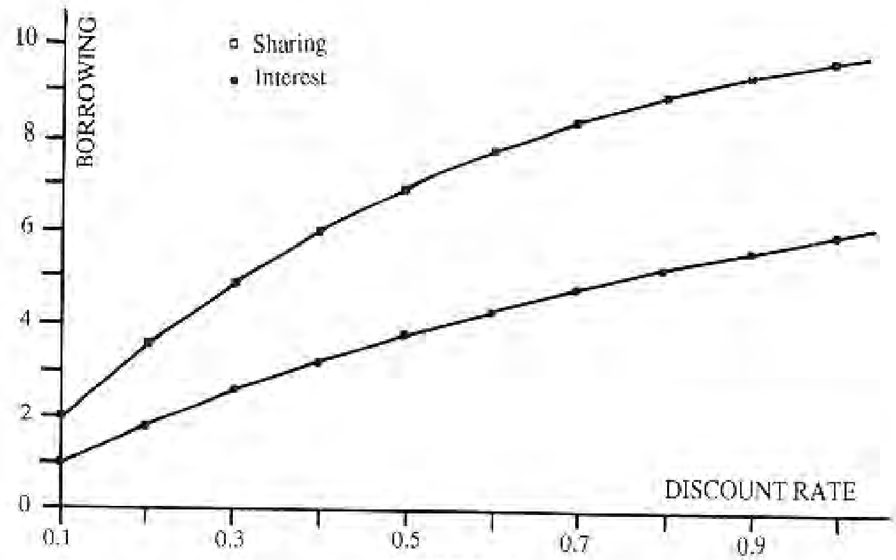

FIGURE 3

Borrowing with Uncertainty for different d
A=10, k=0.5, c=4, q=0.5 yi=10, yj=40

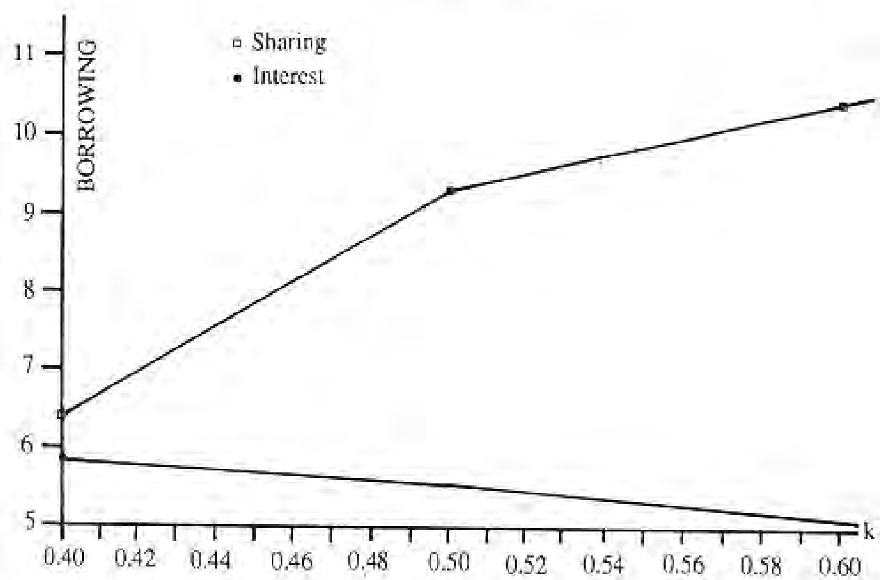

FIGURE 4

Borrowing with Uncertainty for different k
A=10, c=4, q=0.5, h=0.9

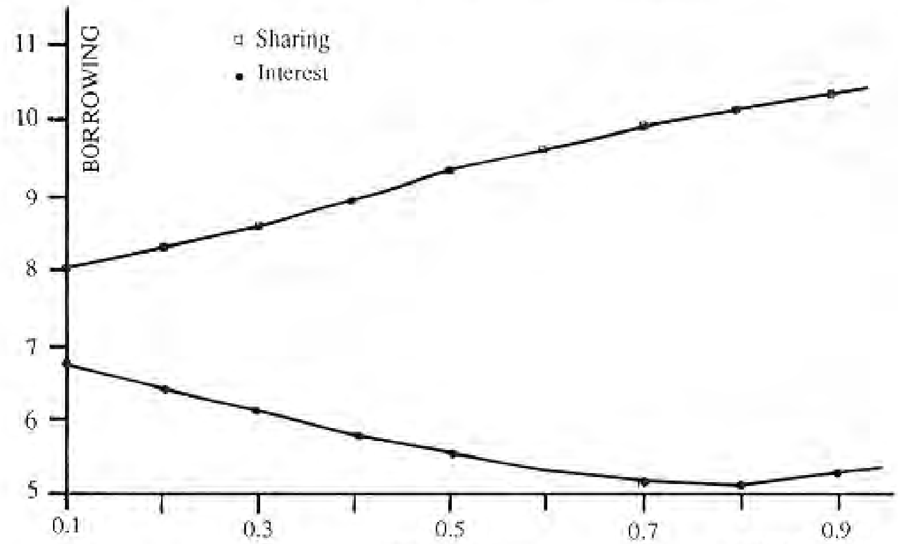

FIGURE 5

Borrowing with Uncertainty for different q
A=10, k=0.5, h=0.9, c=4, yi=10, yj=40

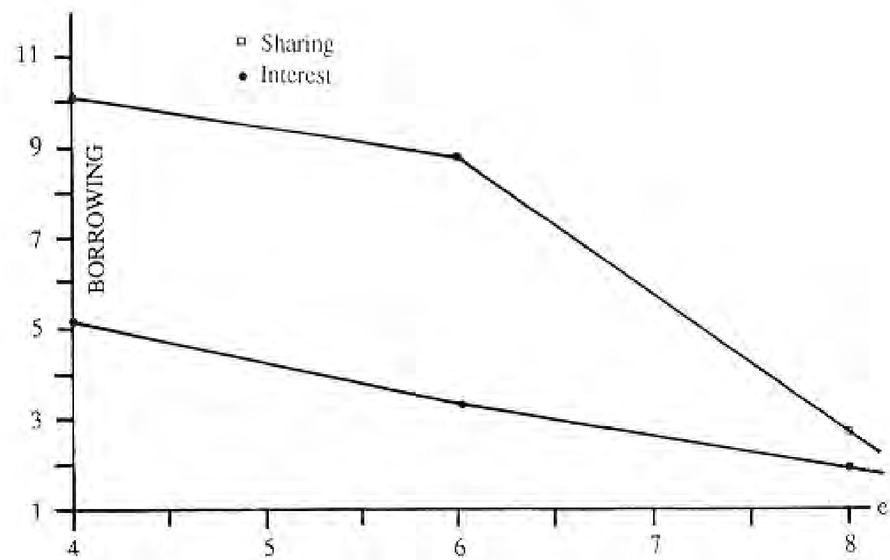

FIGURE 6

Borrowing with Uncertainty for different c
A=10, k=0.5, h=0.9, q=0.5

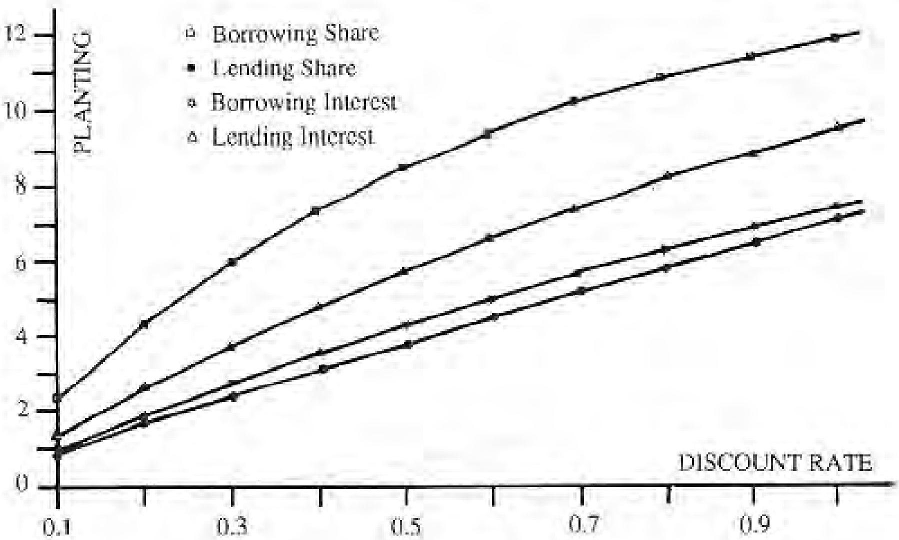

FIGURE 7

Planting with Uncertainty for different d
A=10, k=0.5, c=4, q=0.5, yi=10, yj=40

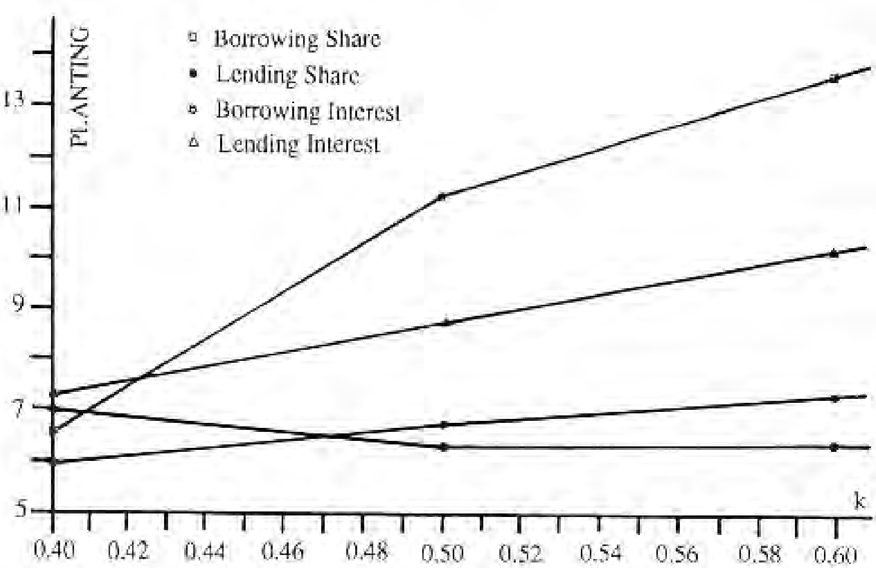

FIGURE 8

Planting with Uncertainty for different k
A=10, c=4, q=0.5, h=0.9

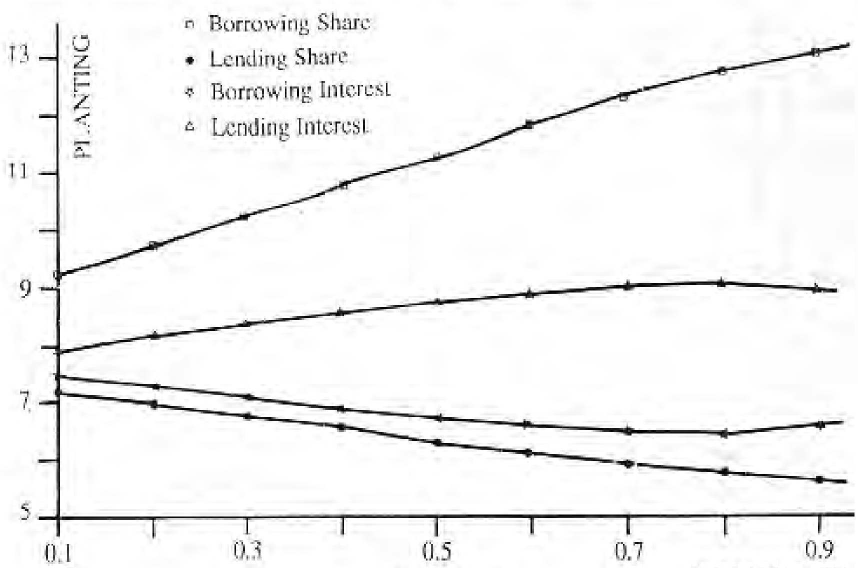

FIGURE 9

Planting with Uncertainty for different q
A=10, k=0.5, h=0.9, e=4, yi=10, yj=40

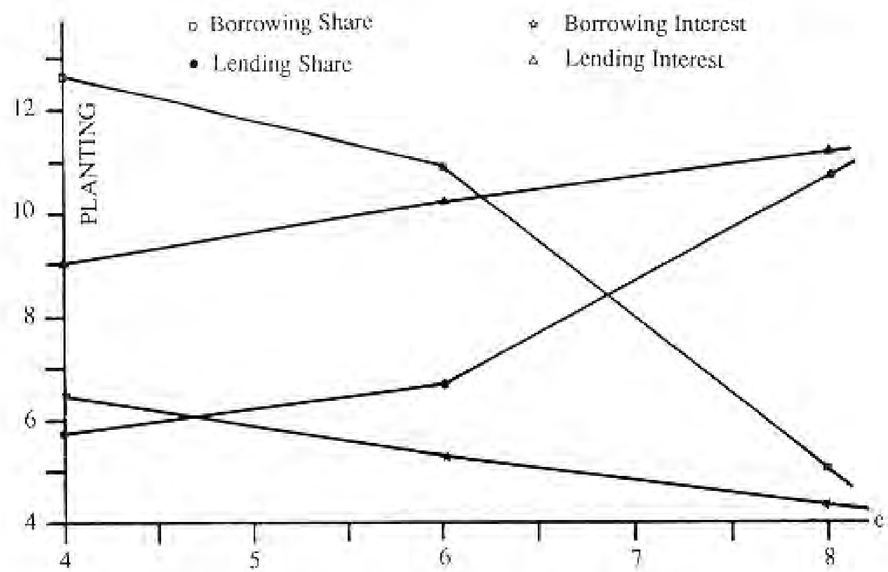

FIGURE 10

Planting with Uncertainty for different e
A=10, k=0.5, h=0.9, q=0.5

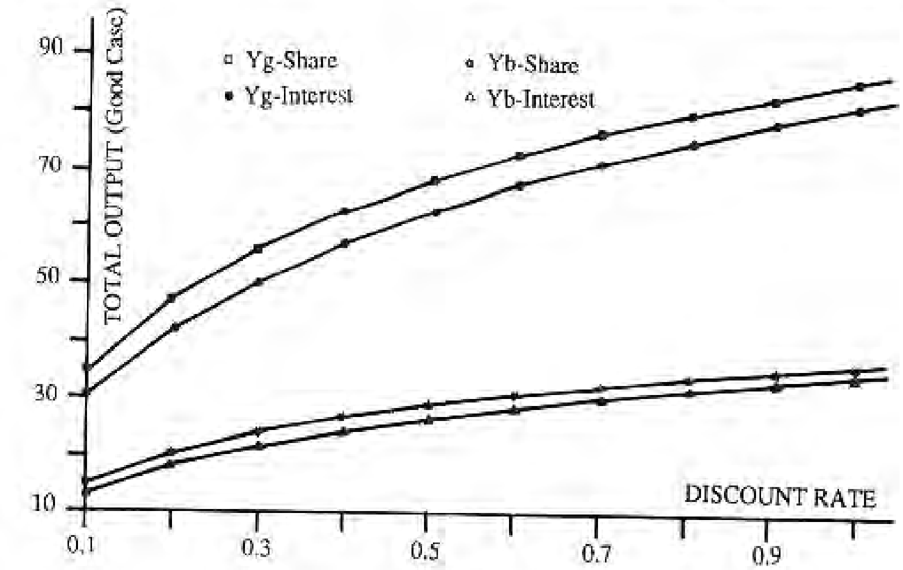

FIGURE 11

Total Output with Uncertainty for different d

A=10, k=0.5, e=4, q=0.5, yi=10, yj=40

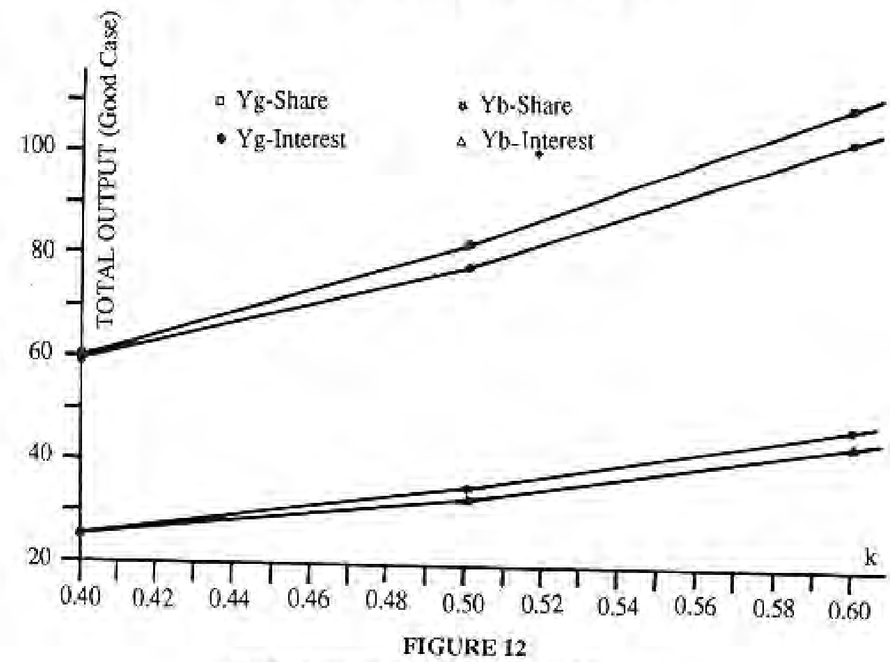

FIGURE 12

Total Output with Uncertainty for different k

A=10, e=4, q=0.5, h=0.9

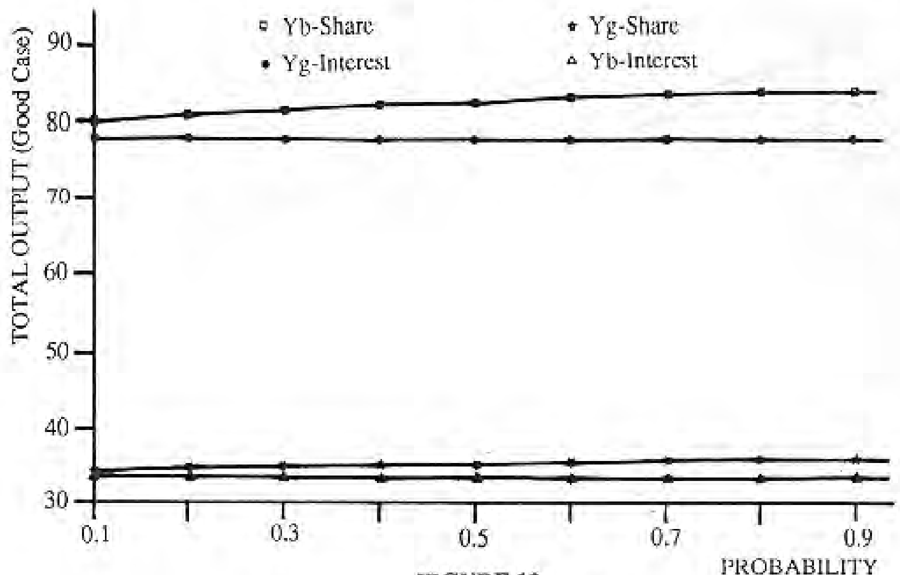

FIGURE 13

Total Output with different Probabilities
A=10, e=4, h=0.9, l=0.5

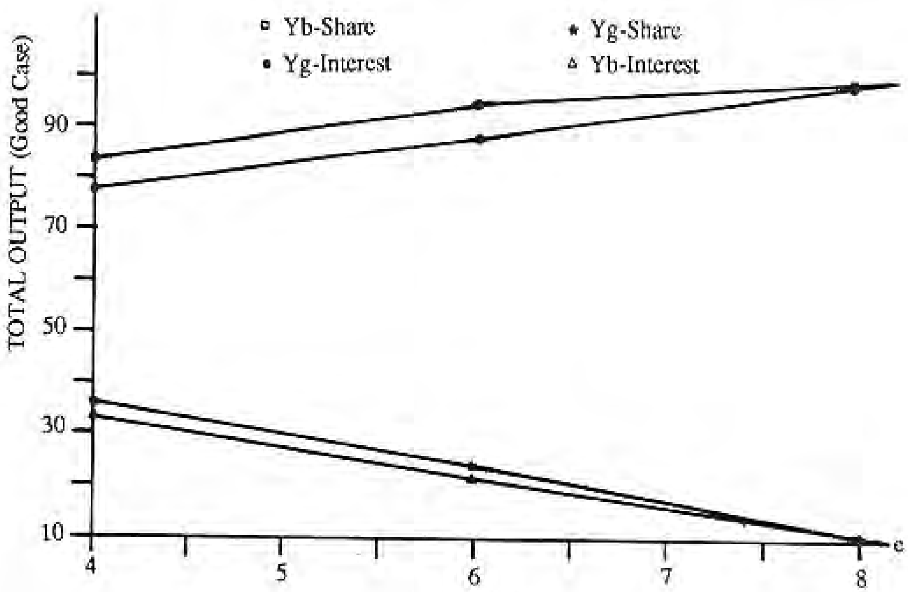

FIGURE 14

Total Output with Uncertainty for different e
A=10, k=0.5, h=0.9, q=0.5

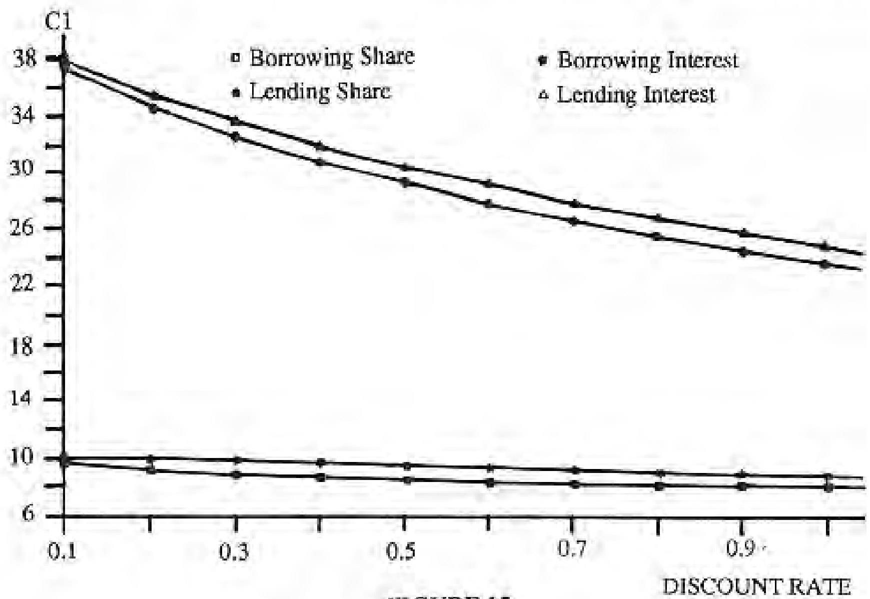

FIGURE 15

C1 with Uncertainty for different d
A=10, k=0.5, e=4,q=0.5, yi=10, yj=40

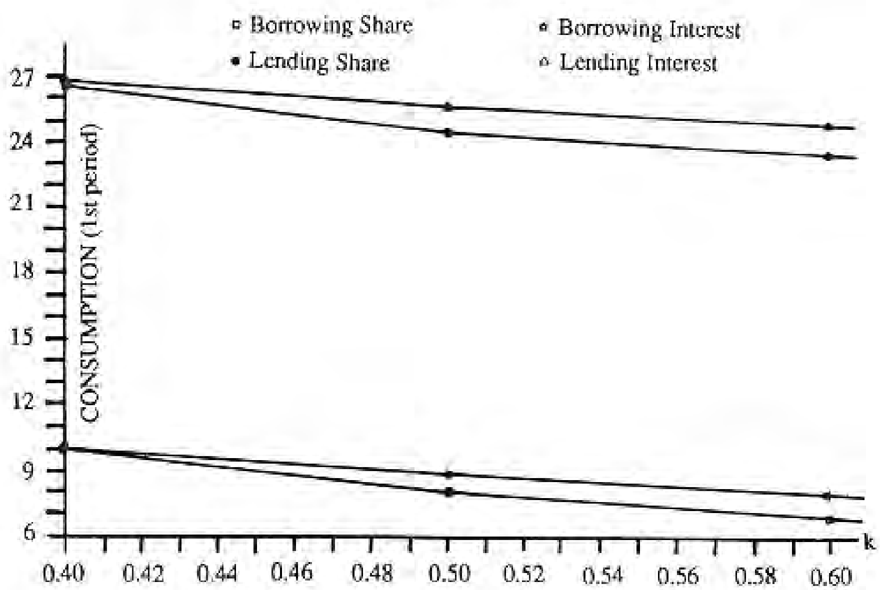

FIGURE 16

Consumption (1st period) for different k
A=10, h=0.9, e=4, q=0.5

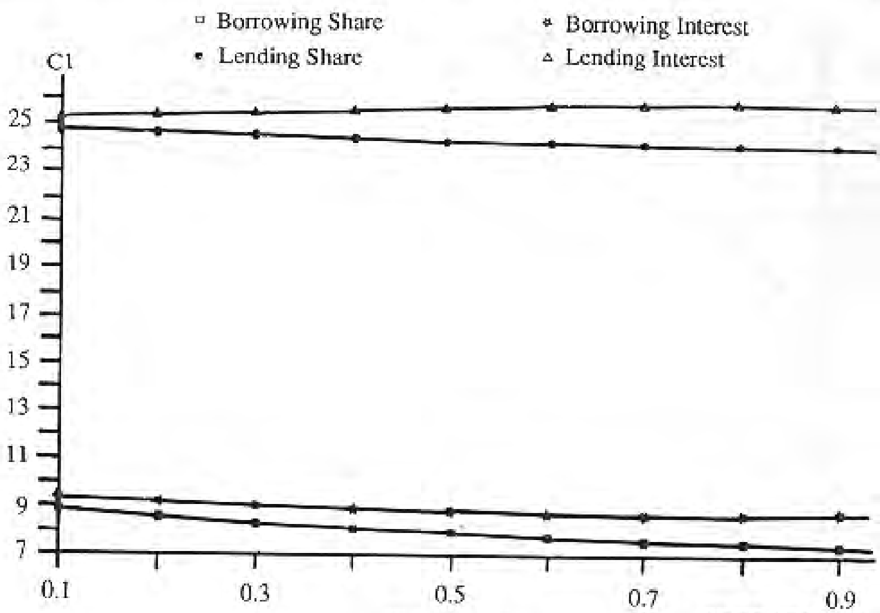

FIGURE 17

C1 with Uncertainty for different q
A=10, e=4, k=0.5, h=0.9

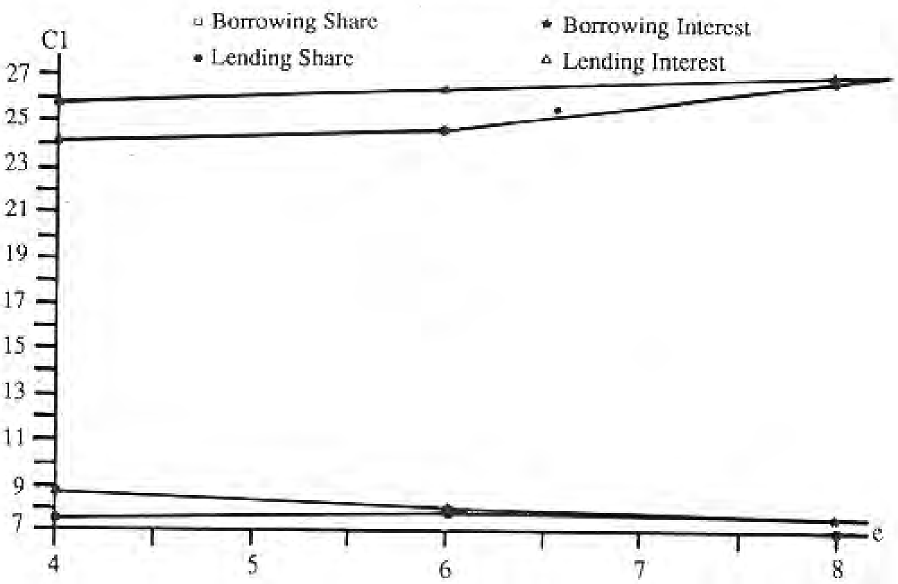

FIGURE 18

C1 with Uncertainty for different e
A=10, k=0.5, h=0.9, q=0.5

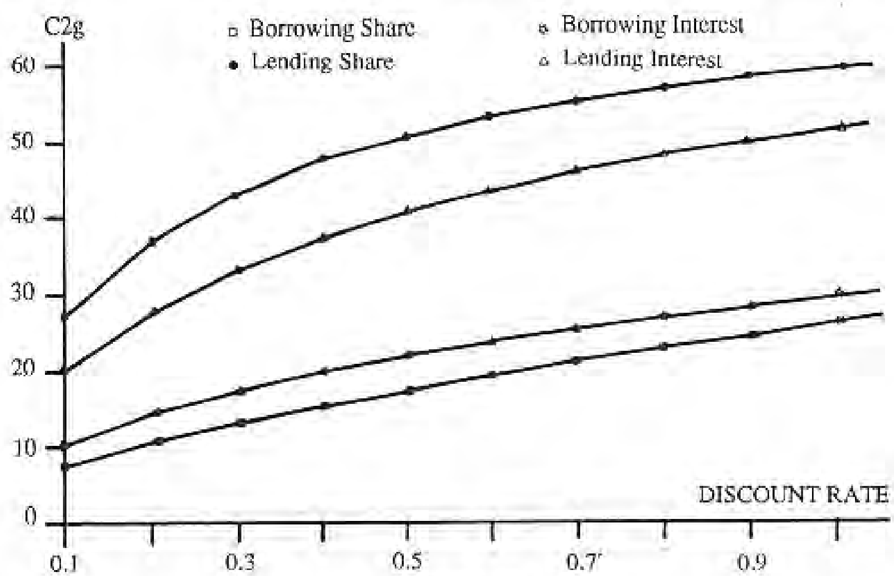

FIGURE 19

C2g with Uncertainty for different d
A=10, k=0.5, c=4, q=0.5, yi=10, yj=40

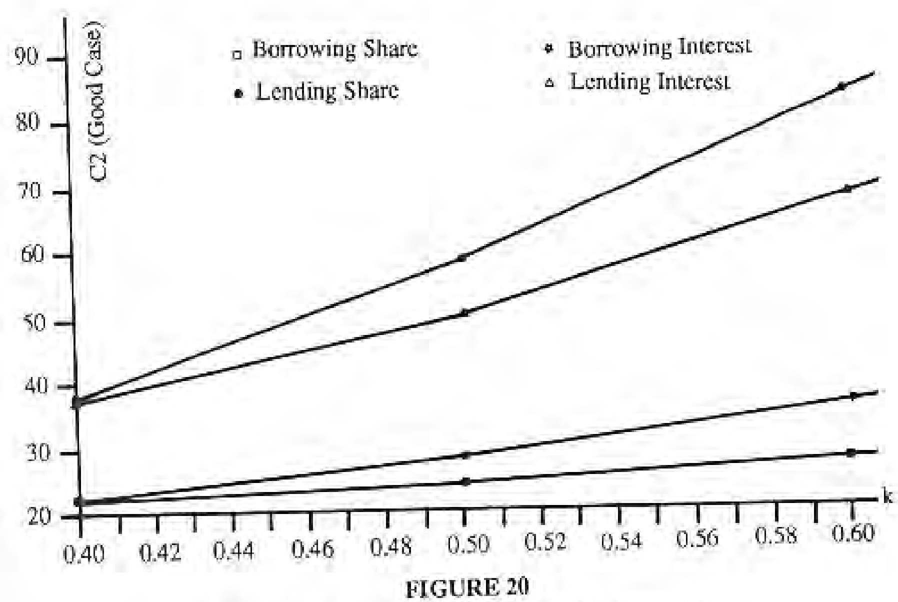

FIGURE 20

Consumption (2nd period, good case) for different k
A=10, h=0.9, c=4, q=0.5

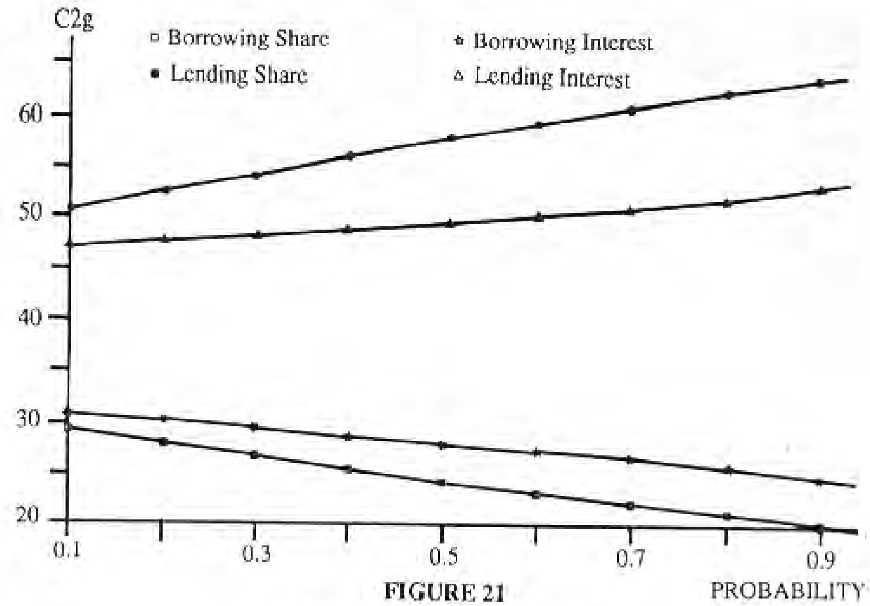

FIGURE 21

C2g with Uncertainty for different q
A=10, c=4, k=0.5, h=0.9

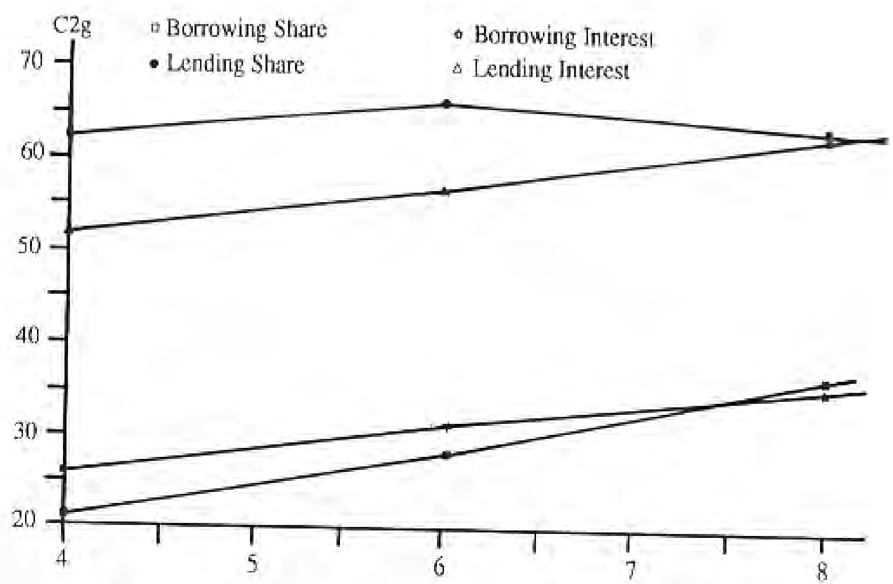

FIGURE 22

C2g with Uncertainty for different c
A=10, k=0.5, h=0.9, q=0.5

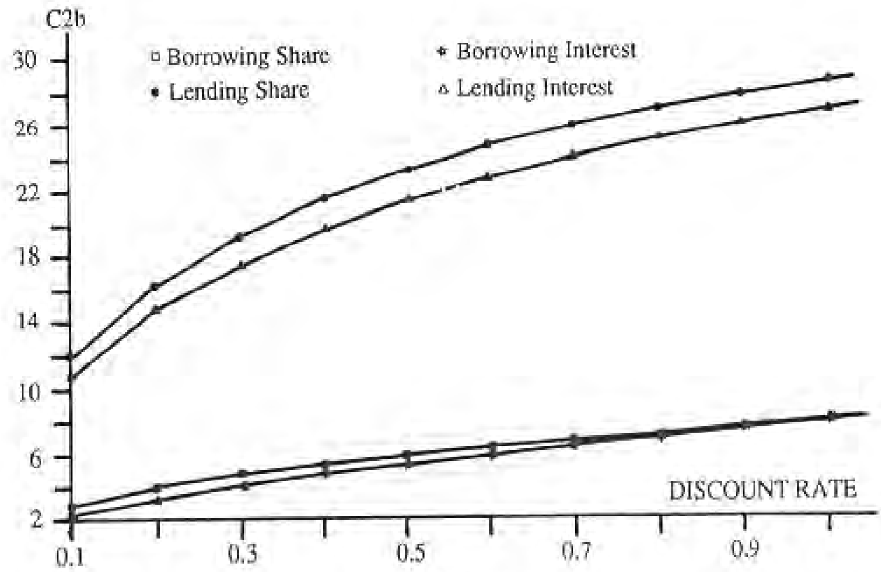

FIGURE 23

C2b with Uncertainty for different d
A=10, k=0.5, e=4, q=0.5, yi=10, yj=40

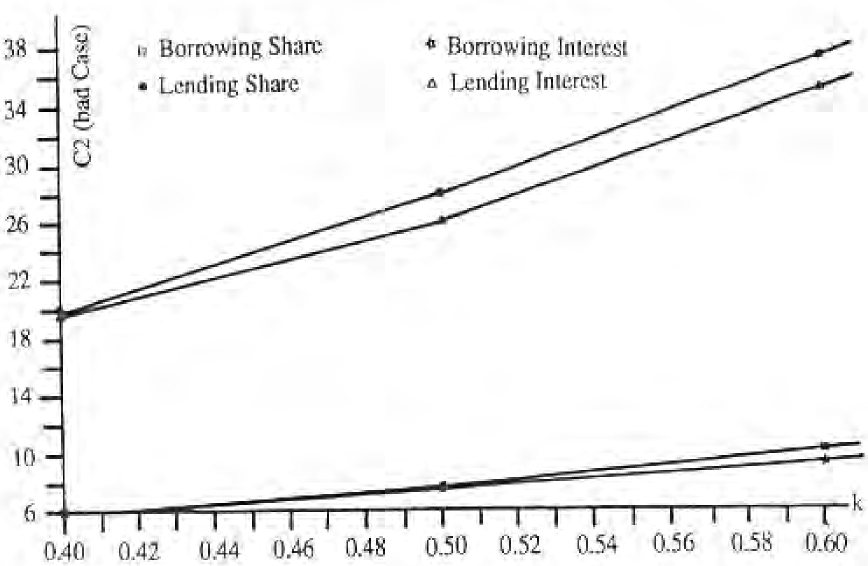

FIGURE 24

Consumption (2nd period, bad case) for different k
A=10, h=0.9, e=4, q=0.5

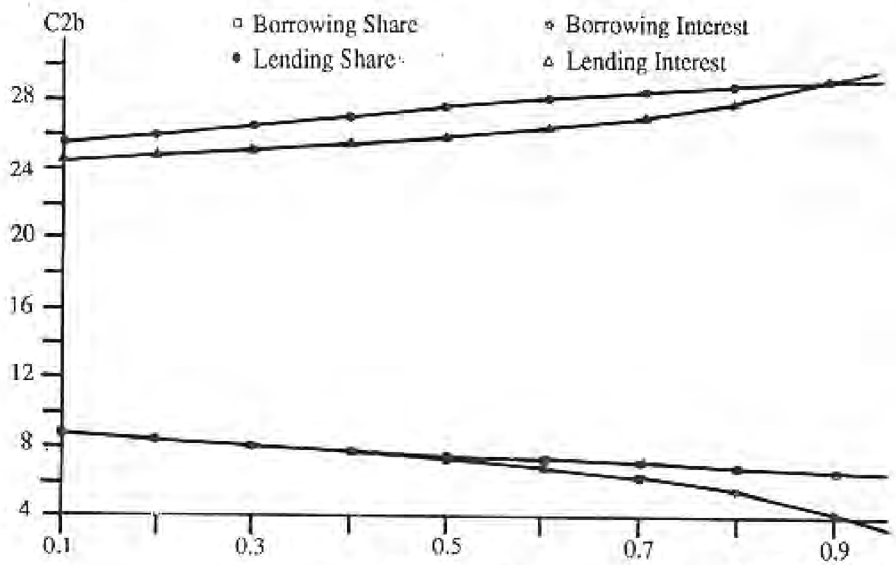

FIGURE 25

C2b with Uncertainty for different q
A=10, c=4, k=0.5, h=0.9

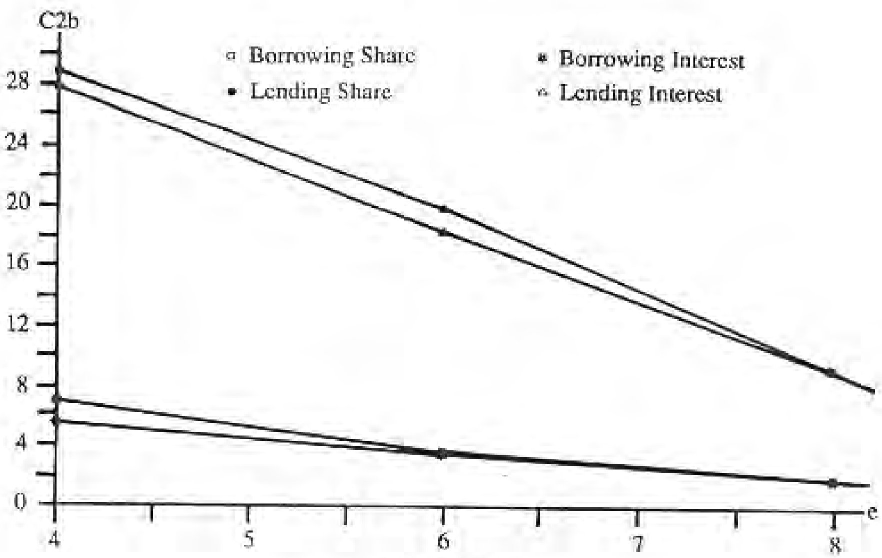

FIGURE 26

C2b with Uncertainty for different c
A=10, k=0.5, h=0.9, q=0.5

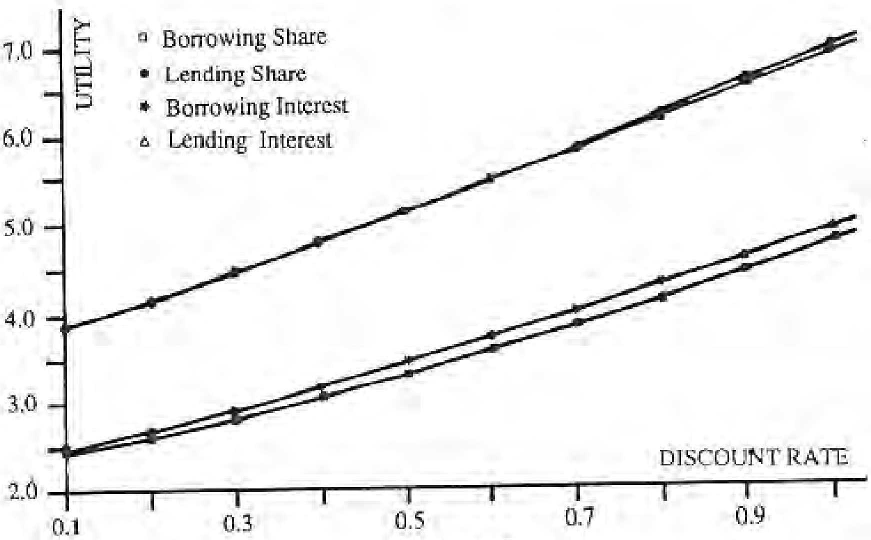

FIGURE 27

Utility with Uncertainty for different d
A=10, k=0.5, c=4, q=0.5, yi=10, yj=40

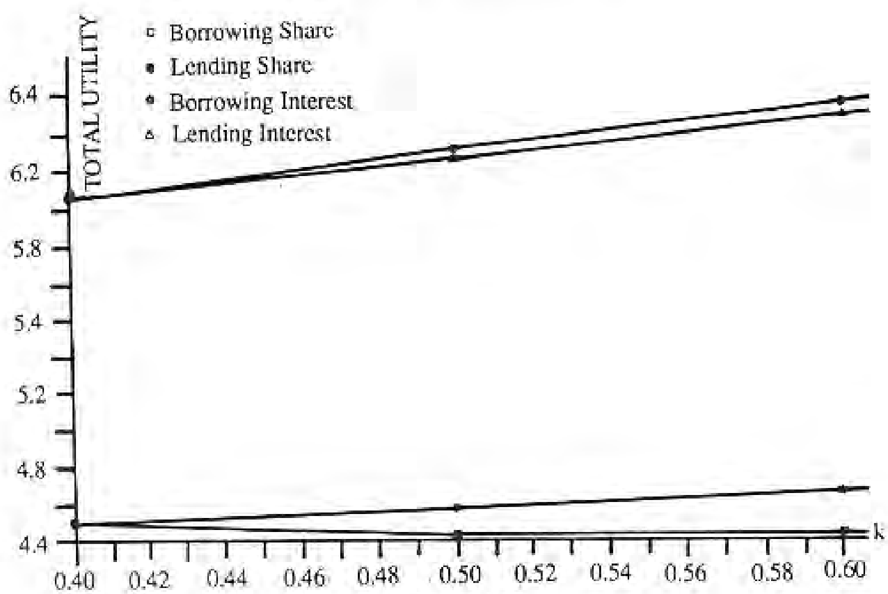

FIGURE 28

Total Utility with Uncertainty for different k
A=10, c=4, q=0.5, h=0.9

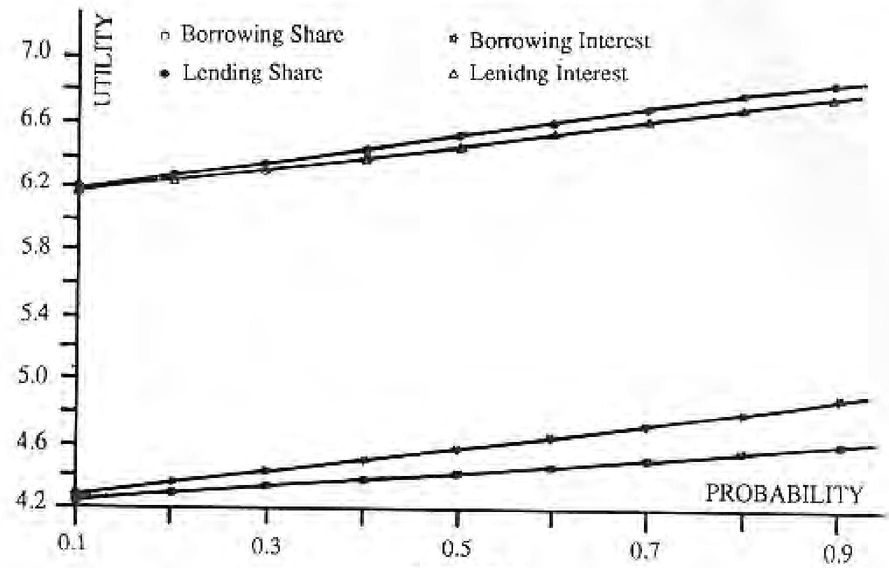

FIGURE 29

Utility with Uncertainty for different q
A=10, e=4, k=0.5, h=0.9

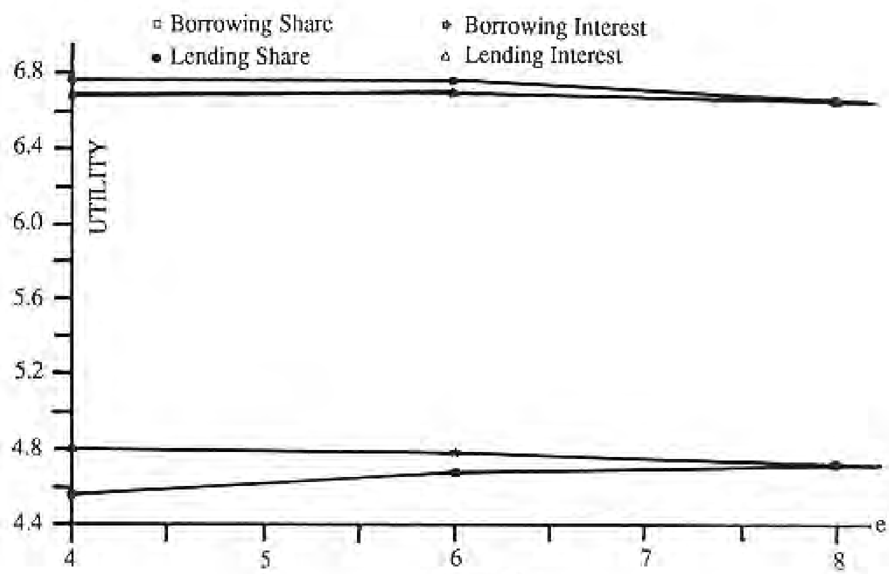

FIGURE 30

Utility with Uncertainty for different e
A=10, k=0.5, h=0.9, q=0.5