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INEQUALITY IN PAKISTAN: A Sectoral Welfare Approach

Geoffrey A. JEHLE*

This paper presents new results on income distribution in Pakistan from an analysis of the full *HIES* data tapes for 1984-85, 1985-86, 1986-87, and 1987-88. An explicit social welfare methodology is adopted to measure changes in inequality at the national level, and within and across provinces, over the period. Estimated Lorenz curves for Pakistan and each of its provinces, in each of the four years, are provided, and cardinally significant Atkinson-Kolm-Sen ethical relative indices of inequality are computed. Strong evidence is found that income distribution in Pakistan improved from 1984-85 to 1987-88, and this conclusion is robust to a wide range of distributional values. Judgements on the trend in inequality over time within and across Pakistan's four provinces are shown to depend more heavily on the investigator's choice of distributional values.

I. Introduction

This paper presents new results from an ongoing study of income distribution in Pakistan over the period 1984 to 1988. There is strong evidence that income distribution in Pakistan improved from 1984-85 to 1987-88, and this conclusion is robust to a wide range of distributional values the investigator may choose to adopt. Judgements on the trend in inequality over time within and across Pakistan's four provinces are shown to depend more heavily on the investigator's choice of distributional values.

Income distribution in Pakistan has been the subject of a good deal of research in the past, with contributions by Kruijk and De Leeuwe (1985), Kruijk (1986), Ahmad and Ludlow (1989), and Havinga, Van den Andel, Haanappel, and Louter (1990) among the most recent. The present work extends that to date in two important respects. First, unlike most previous research on the subject, it proceeds from the belief that judgements about the distribution of income must be grounded in well-defined, and explicit, criteria of social welfare. The welfare-theoretic

*Professor and Chairman, Department of Economics, Vassar College, Poughkeepsie, New York 12601, USA.

methodology of inequality measurement was pioneered by Atkinson (1970), and developed further by Sen (1973), Dasgupta Sen and Starrett (1973), and Blackorby and Donaldson [(1978), (1980)], among others. The methodology of this paper is firmly rooted in that tradition. Second, all reported measurements have been computed from the full data sets collected in the *Household Income and Expenditure Survey (HIES)* for the years 1984-85, 1985-86, 1986-87, and 1987-88. Most previous work has had to rely on published summaries of this data, with the exception of Havinga et al., (1990), who report results of an analysis of the full 1984-85 *HIES* data, alone, and Ahmad and Ludlow (1989) who report for the early years 1976-77, 1979, and 1984-85. Working from the complete data sets, it becomes unnecessary to impose any *ad hoc* assumptions about distribution within published summary ranges.

Section 2 describes the data. Section 3 contains an introduction to the welfare approach to inequality measurement, and presents estimates of Lorenz curves for all of Pakistan, and each of its four provinces, over the period. A specific social evaluation function is proposed, and its associated index of inequality presented and discussed. In Section 4, cardinally significant Atkinson-Kolm-Sen ethical relative indices of inequality for all of Pakistan and for each of its four provinces, for each of the four years, are presented and discussed. Concluding comments are offered in Section 5.

II. Data

All computations were performed on the complete set of data collected in the annual *Household Income and Expenditure Survey (HIES)* of Pakistan for the years 1984-85, 1985-86, 1986-87, and 1987-88, made available to the author by the *Federal Bureau of Statistics*. 1987-88 is the most recent year for which the complete data set is available at present. The *HIES* is the most complete and representative survey of income and expenditure items in Pakistan, with all income and expenditure reported as monthly figures. The sample size is quite large, ranging from 16,581 households in 1984-85 to 18,145 households in 1987-88. In 1984-85, 7,461 urban households and 9,120 rural households were surveyed. By 1987-88, those numbers had both increased to 8,384 and 9,761 respectively.

Our principal interest is in the distribution of income, insofar as income most clearly determines both relative and absolute economic status. However, it is widely accepted that income items in the *HIES* are less reliably reported to surveyors than are expenditure items [Ahmad and Ludlow (1989); Havinga et al., (1990)]. Since income and expenditure are clearly correlated, this paper follows recent convention and adopts reported *expenditure* as a proxy for income. This is not without some drawbacks of its own, however. If, as seems reasonable to expect, expenditure exceeds income in the lower tail of the income distribution through incurring of indebtedness, and falls below income in the upper tail through savings, we can expect the distribution of expenditure to be more equal than the distribution of income. Thus, inferences drawn regarding the distribution of income on the basis of computations performed on expenditure data must be correspondingly qualified.

The counting unit in the *HIES* survey is the household, yet differences among households in the number, age, and earning status of household members make cross-household comparisons difficult to interpret. Consequently, this paper follows Havinga et al., (1990) and corrects for household size and composition using an equivalence scale proposed by Wasay (1977). The resulting reference unit is a single-earner household or single "adult equivalent." The number of adult equivalents in each household was determined as follows:

$$AE = x_1 + 0.8 * x_2 + 0.7 * x_3, \quad (1)$$

where x_1 is the number of earners in the household, x_2 is the number of other adults in the household, and x_3 is the number of children less than ten years old.

Havinga et al., (1990) note some possible deficiencies in these estimates. For one, there is no "economies of scale" factor included. Second, they believe a coefficient of 0.7 on the number of children under 10 may be high, considering the average age of that group in the 1984-85 *HIES* data they examined was less than five years old. Nonetheless, these figures are accepted for the present study since, imperfect as they may be, they represent the current state of knowledge for Pakistan. Moreover, adopting the same transformation scheme as Havinga et al., (1990) facilitates comparison between the results of this study and those of their analysis of the 1984-85 data by giving a common point of departure.[1]

To summarize, monthly expenditure is used as a proxy for monthly income, and the statistical unit in all subsequent computations is the single "adult equivalent". A caveat to the reader is also warranted. As good as it may be, the *HIES* survey is probably far from perfect, and this should be borne in mind in interpreting results presented in this or any other paper based on this data. There is, for example, a rather widely held belief among researchers who have worked with this data that both "tails" in the income distribution tend to be under-sampled for a variety of cultural, administrative and, perhaps, political reasons as well. The reader is therefore warned in advance to view all results critically.

III. The Social Welfare Approach to Inequality Measurement

A wide variety of statistical measures and index numbers have been used to measure income inequality, and Chakravarty (1990) catalogues and explores the

[1] Coulter et al., (1992) have shown how deficient equivalence scales can lead to problems in measuring inequality. Their cations should be borne in mind here.

properties of a great many of them. Sen (1973) divides them all into two broad classes. One he describes as *objective*, or purely statistical measures of dispersion, such as the variance, the coefficient of variation, the Lorenz curve and the Gini coefficient. The other he describes as *normative*. Normative measures of income inequality, "... try to measure inequality in terms of some *normative* notion of social welfare so that a higher degree of inequality corresponds to a lower level of social welfare for a given total of income" [Sen (1973), p.2]. An early example of this approach is Dalton (1920). More recent development has been given by Kolm (1969), Atkinson (1970), Sen (1973), Blackorby and Donaldson (1978), and Pyatt (1987), among others.

While Sen's distinction may be helpful in some respects, it is also potentially misleading, as Sen himself recognizes. One is rarely interested in "pure description" of the income distribution – indeed any such exercise would be rather sterile and uninteresting. Instead, one usually seeks to compare and *rank* alternative distributions as "better" or "worse" than one another. Of course, all such attempts are value-laden, whether the investigator explicitly intends it or not, since notions of "better" and "worse", or "improved" and "worsened", are themselves inherently value-dependent. This is now well-recognized in the literature. Blackorby and Donaldson (1978), for example, have shown how to "recover" from any scale-invariant summary statistic of income distribution the particular class of social evaluation function which would yield the same ranking of distributions by relative index.[2] In the wake of such work, many investigators have begun to appreciate how inappropriate it may be to rank distributions by such familiar "objective" measures as the Gini coefficient. The view adopted in this paper is that discussion should *begin* with the underlying criterion of social welfare that shall be at play.

Consider a society of N individuals, each having income $y_i > 0$, $i = 1 \ldots N$. We can represent the income distribution by the vector $y \in \mathbb{R}^N_{++}$ where $y = (y_1, \ldots, y_N)$. A social evaluation function is a real valued mapping $W : \mathbb{R}^N_{++} \rightarrow \mathbb{R}$ such that, for any y^1 and y^2 in $\mathbb{R}^N_{++}$, $W(y^1)$ is greater than, equal to, or less than $W(y^2)$, if and only if the distribution y^1 is socially preferred to, socially indifferent to, or socially worse than the distri-bution y^2, respectively.

The "social values" of the investigator are reflected in the properties with which $W(\bullet)$ is endowed. At a minimum, there is general agreement that any social evaluation function should satisfy a *Pareto* condition, an *anonymity* condition, and reflect no explicit bias in favour of *inequality* in the distribution of income. Together, these conditions require that the social evaluation function $W(\bullet)$ be *non-*

2 We preserve here the linguistic distinction drawn in the literature between social *welfare* functions, defined over individuals' utility functions, and social *evaluation* functions, defined directly over individuals' incomes. Though the distinction is often purely semantic, it seems worth preserving to avoid confusion.

decreasing, symmetric, and *quasiconcave.* Often, *strict* quasiconcavity replaces quasiconcavity.[3]

The level-sets, or social indifference curves, for three such functions are superimposed on Figure 1. That W(•) be non-decreasing simply requires that social indifference curves not be positively sloped, and not increase southeasterly. Symmetry requires that they be mirror-images of each other across the 45°-line. Quasi-concavity is a "curvature" requirement stipulating that the social indifference curves not be convex-toward-the-origin. Clearly these requirements, together, are sufficiently mild to encompass a wide range of distributional values – from the linear, or utilitarian, which reflect complete social indifference to inequality, to the right-angled, or "Rawlsian" which reflect complete social intolerance of inequality and rank distributions solely according to the income of the least well-off member of society. The third case depicted in Figure1 reflects an intermediate set of attitudes toward inequality, and is generated by a strictly quasi-concave social evaluation function. In effect, strict quasiconcavity rules out only the extreme utilitarian and Rawlsian possibilities, and requires at least some negative slope and outward curvature to the social indifference curves. Heuristically, the greater the degree of curvature in the social indifference curves, the greater the bias in favour of *equality* that is reflected.

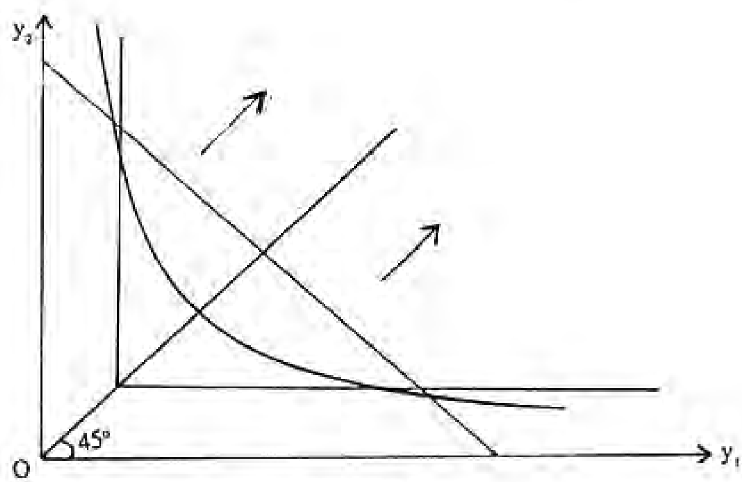

FIGURE 1

Social Indifference Curves for three Non-decreasing, Symmetric and Quasiconcave Social Evaluation Functions

[3] W(•) is quasiconcave if, for all y^1 and y^2 in $\mathbb{R}^N_{++}$, it satisfies $W(y^t) \geq \min[W(y^1), W(y^2)]$, where $y^t = [ty^1 + (1-t)y^2]$, for $0 \leq t \leq 1$. W(•) is *strictly* quasiconcave if the inequality holds strictly for $0 < t < 0$.

While there may be broad agreement on the *general* properties of social evaluation functions over income distributions, disagreements can be expected as soon as any *specific* choice of function is made from the broad class we have so far delineated. In our previous example, for instance, to choose any particular function is to choose a particular "curvature" to the social indifference curves, which in turn is to embrace a specific bias toward inequality in distribution. This is where values can clash, and where reasonable people can, and will, disagree. Before that battle is joined, it seems best to determine whether it needs to be joined at all in the present study of income distribution in Pakistan over the period 1984-88.

Some guidance in this respect can be obtained from Sen's (1973) work on the relationship between social evaluation functions and Lorenz curves. The Lorenz curve is one of the oldest tools for studying income distribution. It is obtained by ordering all individuals by income, from the poorest to richest, and then plotting the cumulative per cent of income held on the ordinate, against the cumulative per cent of the (ordered) population which holds it on the abscissa. Every Lorenz curve must pass through the points (0, 0) and (1, 1), and lie on or below the diagonal – the line of perfect equality – in the unit square. Hypothetical Lorenz curves for three different income distributions are illustrated in Figure 2.

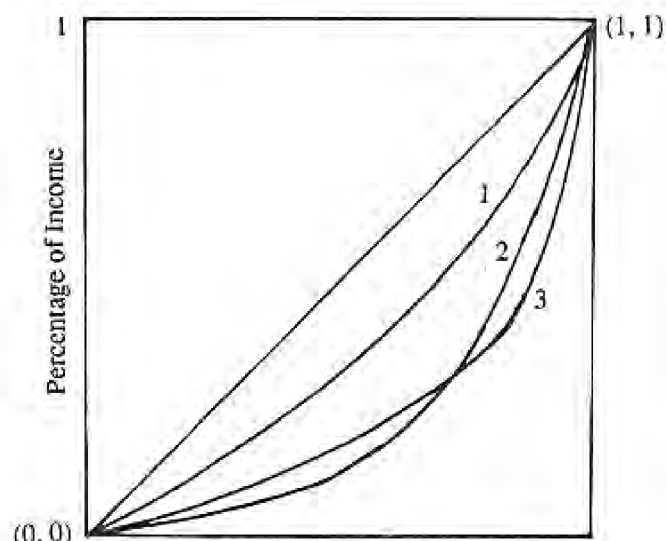

FIGURE 2

Hypothetical Lorenz Curve for three Income Distributions

Sen classifies the Lorenz curve as an "objective" measure of income distribution. However, if less inequality is socially preferred to more, the Lorenz curve does induce a social ordering over alternative distributions. That ordering, though, is not complete. For example, the Lorenz quasi-ordering is capable of ranking the uniformly "more equal" distribution 1 in Figure 2 as socially better than the uniformly "less equal" distribution 2. Yet it is incapable of ranking say, distribution 2 relative to distribution 3, whose Lorenz curves cross each other. By extending a result of Atkinson (1970), Sen (1973) is able to forge the following important link between the "objective" Lorenz curve measure and "normative" measures of income inequality built from the broad class of social evaluation functions we have considered so far.

Theorem 1: (Sen) Lorenz Curves and Social Evaluation Functions

Let $W(\bullet)$ be a symmetric and strictly quasiconcave social evaluation function. Let y^1 and y^2, each in IR^N_{++}, be two different distributions such that $\sum^N_{i=1} y^1_i = \sum^N_{i=1} y^2_i$. If y^1 (strictly) Lorenz-dominates y^2, then $W(y^1) > W(y^2)$. If, however, y^1 does not (strictly) Lorenz-dominates y^2, then there exists some $W(\bullet)$ such that $W(y^1) \leq W(y^2)$.

This result is interesting – both for what it does tell us, and for what it does not. On the encouraging side, the theorem says we may never need to have those arguments over which *specific* social evaluation function to adopt. As long as the Lorenz curves in question do not cross each other, and as long as certain conditions obtain on the income vectors in question, *every* symmetric and quasiconcave social evaluation function will rank the distributions in the same way as every other, and in the same way as the Lorenz-ranking ranks them. Unfortunately, the devil is in the details of the theorem. On the less encouraging side, note what is required of the income distributions in question. First, they must have the same *number* of individuals. Second, the *total income,* (or mean income) must be the same. If we hope to make meaningful comparisons of change in the national distribution of income in Pakistan over time, or in the distribution of income within and across provinces, it is too much to expect that *either one* of these conditions will ever be satisfied in the data.

That this is in fact the case is evident from Table 1, which reports the population of adult equivalents and mean real (monthly) expenditure per adult equivalent in Pakistan and its four provinces over the sample period. Part (A) looks across Pakistan as a whole and each of its four provinces separately, by year. That both mean expenditure and populations vary in the sample cannot be disputed. However, when looking across the years, neither the variation in mean expenditure, nor the variation in population, seem "too great." The maximum percentage change in mean expenditure in each of the categories hovers around 7 per cent; the maximum

TABLE 1

Mean Monthly Real Expenditure and Population of Adult Equivalents in Pakistan and Population of Adult Equivalents

Year	Mean Expenditure	Chng. from previous year	Adult Equivalents	Chng. from previous year
Pakistan				
1984-85	325.0		61038748	
1985-86	318.6	– 2.0	61749581	1.16
1986-87	343.8	7.9	60565793	– 1.92
1987-88	341.0	– 0.8	62132253	2.59
% Change first to last		4.9		1.79
% Change max. to min.		7.9		2.59
Punjab Province				
1984-85	309.8		36524236	
1985-86	315.5	1.8	35565082	– 2.63
1986-87	329.8	4.5	36293334	2.05
1987-88	332.5	0.8	36936856	1.77
% Change first to last		7.3		1.13
% Change max. to min.		7.3		3.86
Sindh Province				
1984-85	366.3		13197644	
1985-86	361.9	– 1.2	14474846	9.68
1986-87	388.4	7.3	14150094	– 2.24
1987-88	372.3	– 4.2	14060799	– 0.63
% Change first to last		1.6		6.54
% Change max. to min.		7.3		9.68
Baluchistan Province				
1984-85	332.7		8132849	
1985-86	265.4	– 20.2	9266843	13.94
1986-87	332.8	25.4	7808320	– 15.74
1987-88	329.3	– 1.1	8598322	10.12
%Change first to last		– 1.0		5.72
% Change max. to min.		25.4		18.68
NWFP Province				
1984-85	308.6		3184019	
1985-86	310.4	0.6	2442811	– 23.28
1986-87	327.2	5.4	2314045	– 5.27
1987-88	330.2	0.9	2536276	9.60
%Change first to last		7.0		-20.34
% Change max. to min.		7.0		37.60

PANEL-B

Mean Monthly Real Expenditure Across Provinces, By Year

Province	1984-85	1985-86	1986-87	1987-88
Punjab	309.8	315.5	329.8	332.5
Sindh	366.3	361.9	388.4	372.3
Baluchistan	332.7	265.4	332.8	329.3
NWFP	308.6	310.4	327.2	330.2
% Difference, Max : min.	18.7	36.4	18.7	13.1

PANEL-C

Population of Adults Equivalents Across Provinces, By Year

Province	1984-85	1985-86	1986-87	1987-88
Punjab	36524236	35565082	36293334	36936856
Sindh	13197644	14474846	14150094	14060799
Baluchistan	8132849	9266843	7808320	8598322
NWFP	3184019	2442811	2314045	2536276
% Difference, Max : min	1047.1	1355.9	1468.4	1356.3

percentage change in population ranges from 2.59 per cent to 9.68 per cent. Only the faint-hearted will balk at these. However, there are some troubling exceptions. In Baluchistan, the percentage change in mean expenditure per adult equivalent from 1985-86 to 1987-88 reaches 25.4 per cent. Over the same period, the reported population of adult equivalents increases by 18.68 per cent. The picture is even worse in the NWFP. From 1985-86 to 1987-88 the reported population increased by 37.60 per cent. The picture becomes even more discouraging in Parts (B) and (C), where we look across provinces by year. In Part (B) maximum percentage difference in mean expenditure range from 13.1 per cent in 1978-88 to 36.4 per cent in 1985-86. These, in turn, are dwarfed by the reported differences in population across provinces in Part (C), which range from a minimum of 1047 per cent to a maximum of 1468 per cent.

The problem of variable populations can, to some extent, be handled. Dasgupta, Sen and Starrett (1973) prove a result very similar in spirit to Theorem 1, but which faces up to the reality of variable populations. They show that if one is willing to accept some further restriction on the class of admissible social evaluation functions, the Lorenz quasi-ordering and all social evaluation functions with the required properties will again rank distributions in the same way, whenever the Lorenz quasi-ordering is capable of providing a ranking at all. They call the required restriction on admissible social evaluation functions, the *symmetry axiom for population*, (SAP).

Axiom: *Symmetry Axiom for Population*

Let $W^m(\bullet)$ denote the social evaluation function of a population of size m, and for any income distribution $(x_1, \ldots, x_n)$ and any positive integer r, consider the distribution y over a population of size nr defined by $y_i = y_{2i} = \cdots = y_{ri} = x_i$, $1 \leq i \leq n$. We say the social evaluation function satisfies the symmetry axiom of population if $W^{nr}(\bullet) = rW^n(\bullet)$.

Most would agree that SAP is a very mild additional restriction on the social evaluation function. In effect, it simply says that if the population is exactly replicated r times, welfare *per capita* must remain unchanged. Under SAP, we have the following theorem.

Theorem 2: *(Dasgupta, Sen and Starrett) Lorenz Curves and Social Evaluation Functions Satisfying SAP*

Let y^1 and y^2 be any two income distributions with the same mean income over population sizes n^1 and n^2, respectively, and let y^1 Lorenz dominate y^2. Let $W^1(\bullet)$ and $W^2(\bullet)$ be the respective social welfare levels. If for each n, $W^n(\bullet)$ is strictly quasiconcave and satisfies SAP, then $W^1(\bullet)/n^1 > W^2(\bullet)/n^2$.

Theorem 2 is encouraging in several respects. Provided we are willing to accept the SAP, and provided mean incomes are the same, it assures us that the ranking of distributions obtained on the basis of Lorenz dominance will exactly parallel the ranking obtained in a comparison of welfare per head – regardless of which *particular* social evaluation function we adopt, and regardless of how great the differences in population may be. This raises the possibility that we may still be able to forestall serious disagreement over the appropriate social evaluation function to adopt. Where the relevant Lorenz curves do not "cross", we can make intertemporal and cross-province comparisons of income distribution in Pakistan with confidence that the same rankings would be duplicated had we based those comparisons on *any one* a very broad class of social evaluation functions.

To explore that possibility, Lorenz curves for all of Pakistan and for each of its four provinces, in each of the four years 1984-85, 1985-86, 1986-87, and 1987-88 were estimated from expenditure data extracted from the full *HIES* data sets. Following Kakawani and Podder (1973), the logarithmic form of the following equation for the Lorenz curve was fitted to the data using OLS:

$$\eta = \pi^{\alpha} e^{-\beta(1-\pi)}, \qquad (2)$$

where η, ranging from zero to unity, is the cumulative percentage of total expenditure by all adult equivalents, π is the percentage of adult equivalents accounting for η, e is the transcendental constant, and α and β are parameters to be estimated. In every case, the fit was very good. Complete estimated equations are reported in Appendix.

As we have remarked, difficulties in drawing meaningful welfare conclusions from the data arise only when Lorenz curves cross. Therefore, a search was made to determine if, and where, that may occur. A summary of those findings is presented in Figures 3 and 4, and those figures require some description. In each case, the cumulative per cent of the population is depicted along the bottom axis. Where the estimated Lorenz curves do not cross, the relative ranking of the respective distributions by the Lorenz dominance criterion is indicated in the ordering of the horizontal lines from top (best distribution, or at least unequal), to bottom (worst or most unequal). The π value at which Lorenz curves cross, when they do, corresponds to the cross-over point in the horizontal lines. Thus, for example in Figure 3, the national distribution of expenditure by adult equivalents for 1986-87 uniformly Lorenz-dominates the distribution in 1984-85. In comparing 1986-87 with 1985-86, however, a cross-over occurs around 0.74. This tells us that the Lorenz curve for 1986-87 lies everywhere *above* the Lorenz curve for 1985-86 up to a π value of 0.74, and everywhere *below* it thereafter.

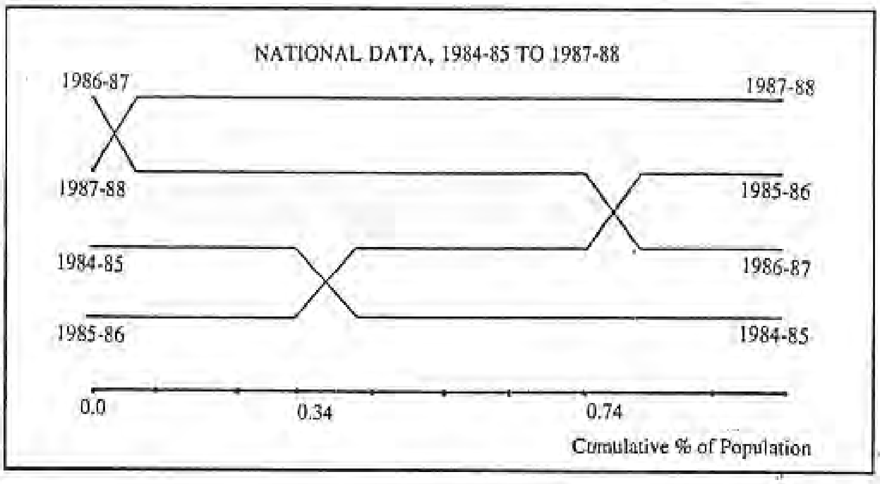

FIGURE 3

Lorenz Curve Rankings, National Data,1984-85 to 1987-88

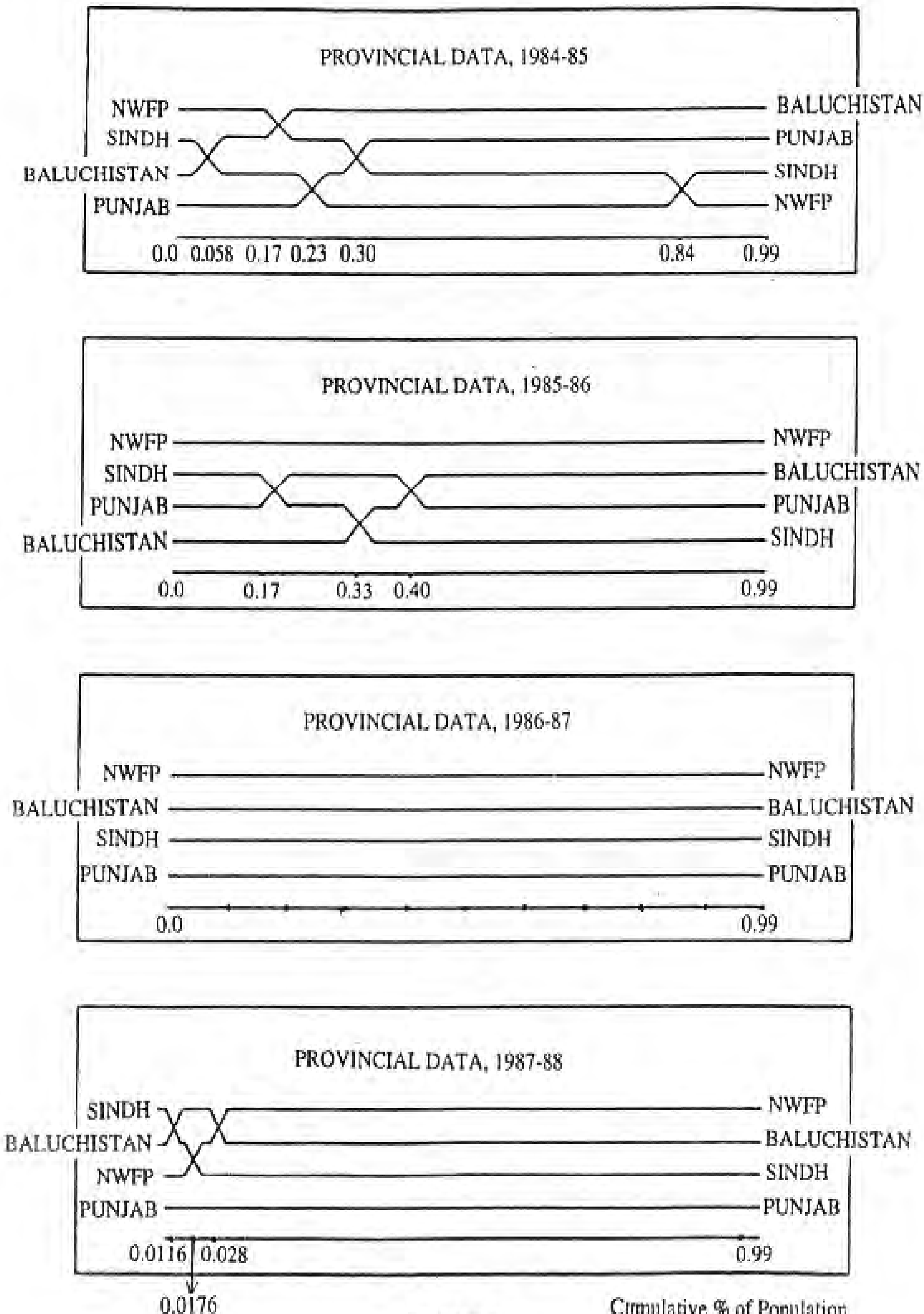

FIGURE 4

Lorenz Curve Rankings, the Provinces, by Year

Since mean expenditure by adult equivalent given in Table 1, Panel A, does not show tremendous variation across the years at the national level, Figure 3 allows us to draw several conclusions, each of which, under Theorem 2, we know to be quite robust across a broad range of social evaluation functions or distributional values. We can, for example, feel quite confident that by 1987-88, the end of the period in question, the distribution of income had improved over the situation prevailing in 1984-85, the beginning of the period concerned. Similarly, income distribution had improved by 1987-88 over what it had been in 1985-86. A clear comparison of 1987-88 and 1986-87 is somewhat less easy to make, since the Lorenz curves for those two years do cross. However, the cross-over occurs quite low down in the distribution. Since the Lorenz curve in 1987-88 lies strictly above that in 1986-87 for *virtually* the entire range of π values, it seems reasonable to conclude with some degree of confidence that the distribution by 1987-88 was less unequal than in 1986-87, as well, and that conclusion would prove to be robust to a still very wide range of social welfare criteria.

As we attempt to trace the evolution in the national distribution over the sequence of individual years between the beginning and the end of the period, however, the picture becomes less clear and more equivocal. Since the cross-overs occur fairly high up in the range of π values, it is difficult to compare the distributions in 1984-85 and 1985-86, and 1985-86 and 1986-87. In the former case, Figure 3 tells us that from 1984-85 to 1985-86 the distribution of income became "less equal" at the low end of the income range, and "more equal" in the middle and higher ends. From 1985-86 to 1986-87, income appears to have become more equally distributed in the low and middle income ranges, but less equally distributed in the higher income range. Very little more can be said.

Figure 4 looks across the four provinces of Pakistan in each of the four years examined. There, truly robust conclusions on relative equality across the provinces are even more difficult to draw for at least two reasons. First, recalling Table 1, there are much greater differences in mean real expenditure across the provinces within any given year than across the years when the nation is viewed as a whole. Second, there is a large number of cross-overs that occur in the Lorenz curves, especially in the first two years, 1984-85 and 1985-86.

By 1986-87 and 1987-88, however, the picture becomes a little bit clearer. In those two years inter-province differences in mean real expenditure per adult equivalent, while perhaps still high for some readers' liking, are relatively low compared to earlier years, as Table 1 shows. Moreover, no cross-overs in the Lorenz curves for 1986-87 occur at all, and those that occur in 1987-88 do so only at the very low end of the income range. Over both of the last two years, the relative ranking of provinces remains essentially the same. The NWFP ranks highest, followed in order by Baluchistan, Sindh and Punjab.

This preliminary examination of the *HIES* data enables us to draw several broad conclusions about the income distribution in Pakistan, and to have a fair degree of confidence that the conclusions are robust to a very broad range of distributional values. On a national level, the income distribution seems to have improved from 1984-85 to 1987-88, and from 1986-87 to 1987-88. Thus, a favourable trend of sorts appears to be present. Looking across the provinces, income appears to be most equally distributed in the NWFP, followed by Baluchistan and Sindh, with the most unequal distribution found in Punjab. This ranking of the provinces also appears to be fairly stable and persistent.

However, at the very high degree of generality we have so far been able to preserve, little else can be said with great confidence. In order to refine our assessments, we must be willing to sacrifice some generality by committing further with respect to the choice of social welfare criteria. We can, however, continue to forestall *bitter* debate on that choice by adopting a very flexible functional form of social evaluation function.

Consider the *constant elasticity of substitution (CES)* class of social evalua-tion functions:

$$W^N_r(y) = \begin{cases} N\left[\frac{1}{N}\sum^N_{i=1} y_i^r\right]^{1/r}, & r \neq 0,\ r \leq 1; \\ N\prod^N_{i=1} y_i^{1/N}, & r = 0. \end{cases} \tag{3}$$

This is the symmetric mean of order r, scaled by population, N. For any choice of parameter $r \leq 1$, the resulting social evaluation function is symmetric, increasing, and quasiconcave. For $r = 1$, the social evaluation, function is the utilitarian form, corresponding to the case of "linear" social indifference curves illustrated in Figure 1. As $r \to -\infty$, W^N_r, converges to a Rawlsian social evaluation function, corresponding to the case of "right-angled" social indifference curves in Figure 1. For finite $r < 1$, (3) is strictly quasiconcave and expresses distributional values between these two extremes, generating social indifference curves like the intermediate case depicted in Figure 1. As r decreases away from unity, a greater bias in favour of *equality* in the distribution of income is imposed. In Figure 1, this would correspond to social indifference curves with greater and greater degrees of curvature.

The form (3) is also homothetic and additively separable in individual incomes. Together, these two properties imply certain (heretofore unrequired) distributional values which should be made clear. Specifically, they imply a marginal rate of social substitution (MRSS) between any two individuals i and j which is both *invariant* to scaler multiplication of the income vector and *independent* of the incomes of any other individuals. To see this, note that

$$\mathrm{MRSS}_{ij} \equiv \left| \frac{\partial W^N_r / \partial y_i}{\partial W^N_r / \partial y_j} \right| = \left| \left(\frac{y_i}{y_j}\right)^{r-1} \right| \tag{4}$$

Moreover, the elasticity of (social) substitution between any pair of incomes y_i and y_j, given by

$$\sigma_{ij} \equiv \frac{d \log (y_j / y_i)}{d \log (-\mathrm{MRSS}_{ij})} \tag{5}$$

is everywhere *constant* and related to the parameter r by $\sigma = 1/(1-r)$. Heuristically, σ can be thought of as the percentage decline in the relative incomes of any two individuals which is required in order to increase the rate at which we are prepared, with social indifference, to transfer income from the richer to the poorer individual by 1 per cent.[4]

Finally, note that (3) satisfies the symmetry axiom of population, SAP, of Dasgupta, Sen and Starrett (1973). To see this, simply note that $W_r^{kN}(\bullet) = kW_r^N(\bullet)$, as required.

Any homothetic social evaluation function, like (3), can be used to construct *relative* indices of inequality. A relative index is one which depends only on inequality in income *shares*, and not on (absolute) income differences. An important class of *ethical* relative indices of inequality – indices which can be viewed as implied by (and implying) explicit social evaluation functions – has developed out of the work of Atkinson (1970), Kolm (1969) and Sen [(1973) (AKS)]. AKS indices depend upon the notion of the *equally distributed equivalent income,* y^e. This is defined as that income which, if given to each individual in society, would result in a distribution of income ethically indifferent to the existing one, according to the underlying criterion of social welfare. If $W(\bullet)$ is any social evaluation function, $y = (y_1, \dots y_N)$ the income distribution in question, and $e = (1, \dots, 1)$ an N-vector of ones, then y^e is defined implicitly by

$$W(y) = W(y^e e). \tag{6}$$

Letting $\mu(y)$ denote mean income, the AKS index corresponding to W is defined as

$$I(y) = 1 - \frac{y^e}{\mu(y)} \tag{7}$$

[4] See Blackorby and Donaldson (1978) or Jehle (1991), for more discussion of *CES* social evaluation functions.

for $\mu(y) \neq 0$. I(y) ranges continuously between zero and 1. It takes the value zero when there is complete equality, and larger values indicate greater inequality.

AKS indices have several important advantages over other alternatives, such as the Gini coefficient or Theil index. For one, the AKS index is always *cardinally* significant. Specifically, I(y) always measures the percentage of total income that can be saved by moving from the existing distribution to one of complete equality with social indifference. In addition, for fixed population size and constant mean income, the AKS index is always *normatively* significant. To see this, note from (7) that, for any two income vectors y^1 and y^2 in IR^N_{++} where $\mu(y^1) = \mu(y^2)$, we will have $I(y^1)$ greater than, equal to, or less than $I(y^2)$ if and only if $W(y^1)$ is less than, equal to, or greater than $W(y^2)$, respectively.

For the social evaluation function (3), y^e is uniquely determined for each income distribution and given by

$$y^e = \begin{cases} \left(\frac{1}{N}\sum_{i=1}^{N} y_i^r\right)^{1/r}, & r \neq 0,\ r \leq 1; \\ \prod_{i=1}^{N} y_i^{1/N}, & r = 0. \end{cases} \tag{8}$$

Note that, from (3), this equally distributed income measures *per capita* welfare, as well. The corresponding AKS index, then, reduces to

$$I_r(y) = \begin{cases} 1 - \left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{y_i}{\mu(y)}\right)^r\right]^{1/r}, & r \neq 0,\ r \leq 1; \\ 1 - \prod_{i=1}^{N}\left(\frac{y_i}{\mu(y)}\right)^{1/N}, & r = 0. \end{cases} \tag{9}$$

$I_r(y)$, derived from the flexible social evaluation function in (3), of course shares all general properties of AKS indices mentioned earlier. It is always cardinally significant, and it is normatively significant over fixed populations with equal mean incomes. Also, for finite $r < 1$, (3) is strictly quasiconcave and so, by Theorem 1, $I_r(y)$ is consistent with the Lorenz quasi-ordering for distributions with given populations and mean incomes. If one distribution (strictly) Lorenz-dominates another, the former will have a lower index value. Since (3) also satisfies SAP, it fulfills the requirements of Theorem 2 as well, so $I_r(y)$ will also reflect the Lorenz quasi-ordering of distributions across populations of different size, provided mean incomes are equal.

Of course, one principal reason for committing ourselves to a particular class of social evaluation function, and associated AKS index, as we have, is that the Pakistani data here do not always offer us nice clear rankings based on (strict) Lorenz dominance. We have seen that Lorenz curves do, in fact, cross quite often in the data, making it impossible to render truly general judgements in every instance. Also, mean real expenditure in the sample varies sometimes considerably, and populations do not differ from one another simply by integer replications.

What *are* the welfare properties of ethical indices of inequality in the absence of Lorenz-dominance, and where mean incomes and populations differ? Here, we are truly on unchartered ground. Of course, it is quite common in the literature to observe (potentially spurious) normative inferences being drawn in similar situations.[5] The fact remains, however, that we really know next to nothing at high levels of generality about how these indices behave in such circumstances. In general, we simply have no reason to be confident that the welfare connotations that inevitably will attach themselves to intertemporal, cross-province, or cross-country comparisons of the computed index numbers will be meaningful and in accord with the set of distributional values to which we intended to commit ourselves by our choice of underlying social evaluation function.[6] These are questions that can not be fully addressed here. However, we can, and should, explore what biases are introduced by our particular choice of social evaluation function and associated AKS index.

On the question of variable mean incomes, consider the general definition of any AKS index given in equation (7). There it is clear that, other things equal, a higher mean income will generate a higher index of inequality for a given equally distributed income. Thus, in a sense, this index implies the view that inequality is socially worse in "richer" societies than it is in "poorer" ones. This is, of course, a view that one could adopt and even defend. However, it is by no means the only one, as Sen (1973) has argued. Moreover, the way in which different means affect these indices *qualitatively* is quite apart from how they do so *quantitatively*. One could, for example, be quite happy with the direction of the influence of higher means, but believe the magnitude of the effect should be larger or smaller. The reader is simply alerted to the effect that variable means will have on the index computations reported below, and is left free to accept or reject it.[7] Of course, in those instances in which means do not vary "too much" – such as national comparisons across years – we need not be "too concerned" about this effect at all.

5 See, for example, Atkinson (1973), himself, who makes cross-country comparisons even though his own welfare results are only established for the case of fixed populations.

6 Consider, for example, the three-person society with incomes (1, 2, 3) and the four-person society with incomes (1, 2, 3, 4). Which is the less unequal ("better") distribution? Income in the four-person society is much more concentrated at the top, yet the AKS index in (9) for $r = 0.5$ is lower in the four-person society than it is in the three-person one ($0.040 < 0.045$). If, however, we choose $r = -2.0$ then inequality is rated as higher in the four-person case than in the three-person case ($0.267 > 0.258$).

7 Sen (1973), for one, argues that "inequality" and "mean income level" should probably enter separately into one's special assessment of the income distribution. That, of course, still leaves open the questions of how each should enter qualitatively and quantitatively.

On the question of variable populations, we have already noted that the social evaluation function in (3) satisfies SAP. Thus, if the society were "replicated" k times, per capita welfare, and the index I_r in (7), would remain completely unchanged. This might be considered a distinct advantage of our choice of social evaluation function and index. But how does the index behave when populations differ in ways other than simple replication, as will inevitably be the case in any real world situation, including the case of Pakistan at hand? We can begin to get some insight, and perhaps some reassurance as well, by engaging in a simple "thought-experiment".

Consider a society with population size N. Let y^e_N denote the equally distributed income in that society, computed as in (8). Suppose the society "grows" by the addition of an arbitrary number $M > 0$ of new members. Let y^e_M denote the equally distributed income for the new members considered as a separate group, and computed as in (8). The larger society now contains $N + M$ total members. Let y^e_{N+M} be the equally distributed income again from (8), for the larger society considered as a whole. Next note that we can write

$$y^e_{N+M} = \left(\frac{N}{N+M}(y^e_N)^r + \frac{M}{N+M}(y^e_M)^r\right)^{1/r} \qquad (10)$$

when $r > 0$. Equation (10) gives some interesting insights into the behaviour of the index I_r – particularly when we recall that the equally distributed income can, in our case, be directly interpreted as per capita welfare in the relevant group. It tells us that per capita welfare in the larger population of size $N + M$ can be decomposed into a particular weighted average of per capita welfare in the base society of size N and per capita welfare in the M additional members considered separately.

First, suppose per capita welfare among the M new members is equal to per capita welfare in the base population of size N. From (10), if $y^e_M = y^e_N$, we have $y^e_{N+M} = y^e_N$, so per capita welfare remains unchanged by the addition of those new members. With equal means, the inequality index I_r will also be unchanged. Similarly, if per capita welfare among the additional members if larger (smaller) than per capita welfare in the base population, per capita welfare in the larger $N + M$ member society will be larger (smaller) than in the base population, and the index I_r will reflect a corresponding decrease (increase) in inequality.

Finally, notice that when N is very large and M is relatively small, $N/(N+M) \approx 1$ and $M/(N + M) \approx 0$. Thus, if population differences are relatively small, per capita welfare differences – and so measured inequality differences – between the base population and the expanded one will also be small, regardless of the distribution of income among the additional group considered separately.[8]

[8] Similar constructions as (10), and similar conclusions on the behaviour of the AKS index, obtain for the case of $r = 0$, as well. Details are omitted to simplify exposition.

One is quite encouraged by all this to conclude that the ethical AKS index (9) "handles" the variable population problem fairly well. While completely compelling support may be lacking, we do have assurances, by virtue of its consistency with SAP, that replications of the population will leave measured inequality unchanged. The results of the "thought experiment" suggest that – regardless of the normative values we impose by our choice of parameter r – the index I_r will behave in quite sensible and desirable ways across populations that differ in other ways, as well. Of course, one can always agree that the qualitative influence is acceptable, but quibble with the quantitative extent.

IV. AKS Indices for Pakistan, 1984 to 1988

AKS indices have been widely used and computed for many countries to date. Surprisingly, this has not been the case for Pakistan. In this section, we present AKS indices computed for all of Pakistan and each of its four provinces over the period 1984 to 1988. As before, all computations were performed on expenditure data from the full *HIES* data tapes for the years 1984-85, 1985-86, 1986-87, and 1987-88, using adult equivalents as the individual unit.

The AKS index (7) was computed for a wide range of alternative values for the "ethical" parameter, r. However, to avoid unnecessary clutter, results are reported for only a representative subset here. Results obtained for many parameter values other than those reported are available from the author on request. Indices for values of the parameter r equal to 0.8, 0.5, 0.0, -0.33, -1.0, and -3.0 are reported in Table 2. To help motivate that choice, recall that the elasticity of social substitution between any two individuals is constant and given by $\sigma = 1/(1-r)$. As we have noted σ can be thought of as the percentage decline in the relative incomes between any two individuals which is required in order to increase by 1 per cent the rate at which we are prepared, with social indifference, to transfer income from the richer to the poorer individual. The preceding set of r-values corresponds to the set of σ values, 5, 2, 1, 0.75, 0.5, and 0.25, respectively. If, for example, $r = 0.5$, we have $\sigma = 2$, so a 200 per cent decline in relative incomes is required to increase the rate of social substitution in favour of the relatively poorer individual by 1 per cent. The marginal rate of social substitution (MRSS), itself, depends on relative incomes and is given in (4).

An examination of Table 2 shows that the calculated AKS indices behave as expected. Inequality is generally very low for large values of r, increasing steadily as r decreases and the underlying welfare criterion becomes more and more sensitive to the circumstances of the less well-off members of society. Correspondingly, when r is close to unity, reflecting a more "utilitarian" criterion, the per cent of total expenditure required to eliminate inequality without loss of social welfare is quite low, hovering around 2-3 per cent for the country as a whole and each of the

TABLE 2

AKS Indices for Pakistan and its Provinces, 1984-85 to 1987-88
(Expenditure per Adult Equivalent)

R Value (sigma)	0.8 (5)	0.5 (2)	0.0 (1)	− 0.33 (0.75)	− 1.0 (0.5)	− 3.0 (0.25)
Pakistan						
1987-88	0.03253	0.07418	0.13035	0.16147	0.21400	0.33043
1986-87	0.03383	0.07762	0.13702	0.16985	0.22470	0.34221
1985-86	0.03332	0.07739	0.13932	0.17497	0.23793	0.41027
1984-85	0.03453	0.07972	0.14228	0.17814	0.24611	0.76194
Punjab Province						
1987-88	0.03555	0.08059	0.14047	0.17322	0.22779	0.34562
1986-87	0.03501	0.08021	0.14121	0.17475	0.23053	0.34885
1985-86	0.03336	0.07697	0.13713	0.17095	0.22848	0.35543
1984-85	0.03406	0.07883	0.14122	0.17725	0.24696	0.77883
Sindh Province						
1987-88	0.03164	0.07221	0.12671	0.15665	0.20675	0.31528
1986-87	0.03516	0.08028	0.14064	0.17352	0.22770	0.34095
1985-86	0.03320	0.07718	0.13839	0.17274	0.23051	0.35044
1984-85	0.03571	0.08225	0.14587	0.18146	0.24621	0.73770
Baluchistan Province						
1987-88	0.02252	0.05290	0.09659	0.12207	0.16671	0.27121
1986-87	0.02483	0.05818	0.10553	0.13263	0.17905	0.28082
1985-86	0.02965	0.07066	0.13334	0.17300	0.25092	0.48973
1984-85	0.03335	0.07548	0.13190	0.16392	0.22439	0.58118
NWFP Province						
1987-88	0.02158	0.04951	0.08820	0.11039	0.14948	0.24701
1986-87	0.02329	0.05419	0.09754	0.12223	0.16438	0.25513
1985-86	0.02413	0.05663	0.10333	0.13062	0.17881	0.29214
1984-85	0.02840	0.06791	0.12652	0.16112	0.22102	0.34518

provinces separately. By the time r has declined to -3.0, reflecting a more "Rawlsian" criterion, the per cent of expenditure required to eliminate inequality with no decline in welfare is both larger and less uniform across Pakistan and its provinces – ranging from a low of 25 per cent in the NWFP during 1987-88, to a high of 78 per cent in Punjab during 1984-85. Looking across the years, Table 2 also shows a slightly irregular but generally downward trend in inequality in Pakistan and the provinces of Sindh, Baluchistan, and NWFP over the four-year period shown. Trends are much less discernable for Punjab.

To simplify these, and other comparisons, ranking diagrams similar to those used earlier have been constructed. In each, the r-value is indicated along the horizontal axis, with movement to the right corresponding to a *decrease* in the built-in bias against inequality of the index. Vertically, the ranking is always from best (most equal) at the top, to worst (most unequal) at the bottom.

Figure 5 at Pakistan as a whole across four years. The picture that emerges there is much clearer than when we considered rankings by the Lorenz-dominance criterion in Figure 3. Except for one shift in the relative ranking of inequality in the intermediate years that occurs at fairly high levels of r, there is evidence of a regular and sustained improvement in the national distribution from the beginning to the end of the period. Moreover, that ranking is quite robust to a very wide range of ethical values.

Similar (and generally robust) trends are also seen as we look across years at the distributions in Sindh, Baluchistan, and NWFP in Figure 6. In NWFP, there is

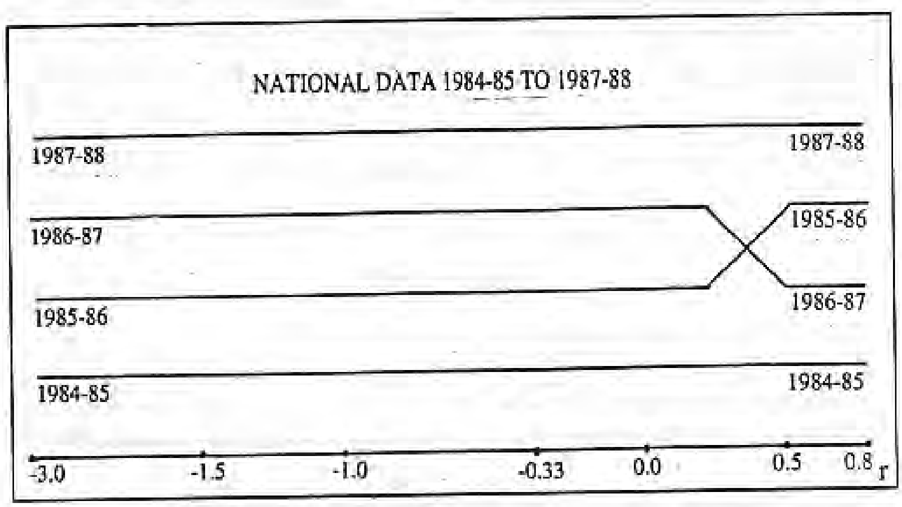

FIGURE 5

Rankings by AKS Index, Pakistan 1984-85 to 1987-88

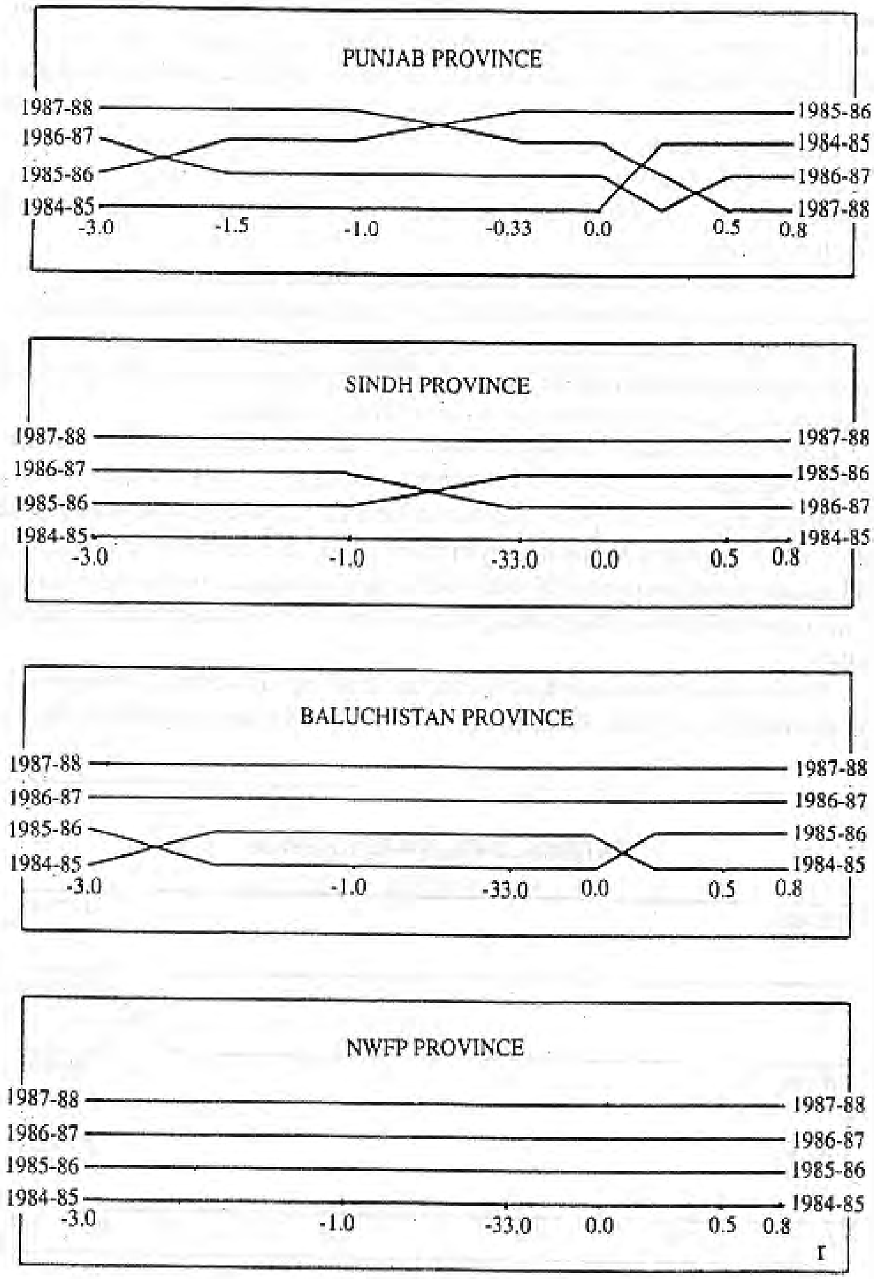

FIGURE 6

Rankings by AKS Index, Pakistan's Provinces, 1984-85 to 1987-88.

a steady reduction in inequality in each of the four years which is robust to the entire range of r-values. In Sindh and Baluchistan, we see a similarly robust and unequivocal improvement from the beginning of the period in 1984-85 to the end in 1987-88. A single cross-over occurs between 1985-86 and 1986-87 in the case of Sindh at the intermediate range of r-values. This may suggest the presence of a relatively greater proportion of the population in the low-income ranges (to which the index becomes increasingly sensitive as we move to the left) during 1985-86 compared to 1986-87. The double cross-over that occurs in the case of Baluchistan in the first two years of the period is difficult to intuit. When the index is *very* sensitive to the situation of the least well-off, we would say there was improvement in the income distribution from 1984-85 to 1985-86 – and similarly when the index is *very* insensitive to their plight!

The findings for Punjab offer us useful and robust inferences only very grudgingly. There is some fairly strong evidence of a reduction in inequality from the beginning of the period to the end, at least for r-values less than zero. However, even in that range, nothing like a regular trend emerges.

One final set of comparisons is provided in Figure 7, where we look at the relative ranking of provinces in each of the four years separately. The picture by 1987-88 is quite clear. The data unambiguously suggest that income distribution in NWFP was best, followed by Baluchistan and Sindh, with distribution in Punjab being the worst. This is *virtually* the conclusion one comes to looking at 1986-87. The only exception is in the shift of relative ranking which occurs between Sindh and the Punjab at very high levels of the parameter r. The significance of that cross-over, however, may not be too great for at least two reasons. First, the absolute size of the difference in the computed indices in that range of r-values across those two provinces is very small. Second, that shift in relative ranking contradicts the very clear and robust conclusion we came to when making that same comparison on the basis of the Lorenz-dominance criterion and the estimated Lorenz curves. To conclude, therefore, at very high levels of generality that the relative ranking of provinces remained stable between 1986-87 and 1987-88 is therefore probably warranted.

In the early years, 1984-85 and 1985-86, there is a certain regularity discernible, in spite of the superficial appearance of somewhat erratic behavior in the relative ranking of provinces. In those two years, as in every other, NWFP ranks best in terms of equality. In 1984-85, NWFP is followed by Balluchistan over the entire range of r-values, as it is in every other year except 1985-86. There, the relative ranking of Baluchistan shifts wildly with variation in the ethical parameter of the inequality index – from its usual place of second best, right down to worst. A possible explanation for this wide swing may lie in the unusual and sharp drop in mean real expenditure in Baluchistan between 1984-85 and 1986-87, seen in Table 1, Panel B. Mean real expenditure in the sample drops from Rs.332.7 in 1984-85 to

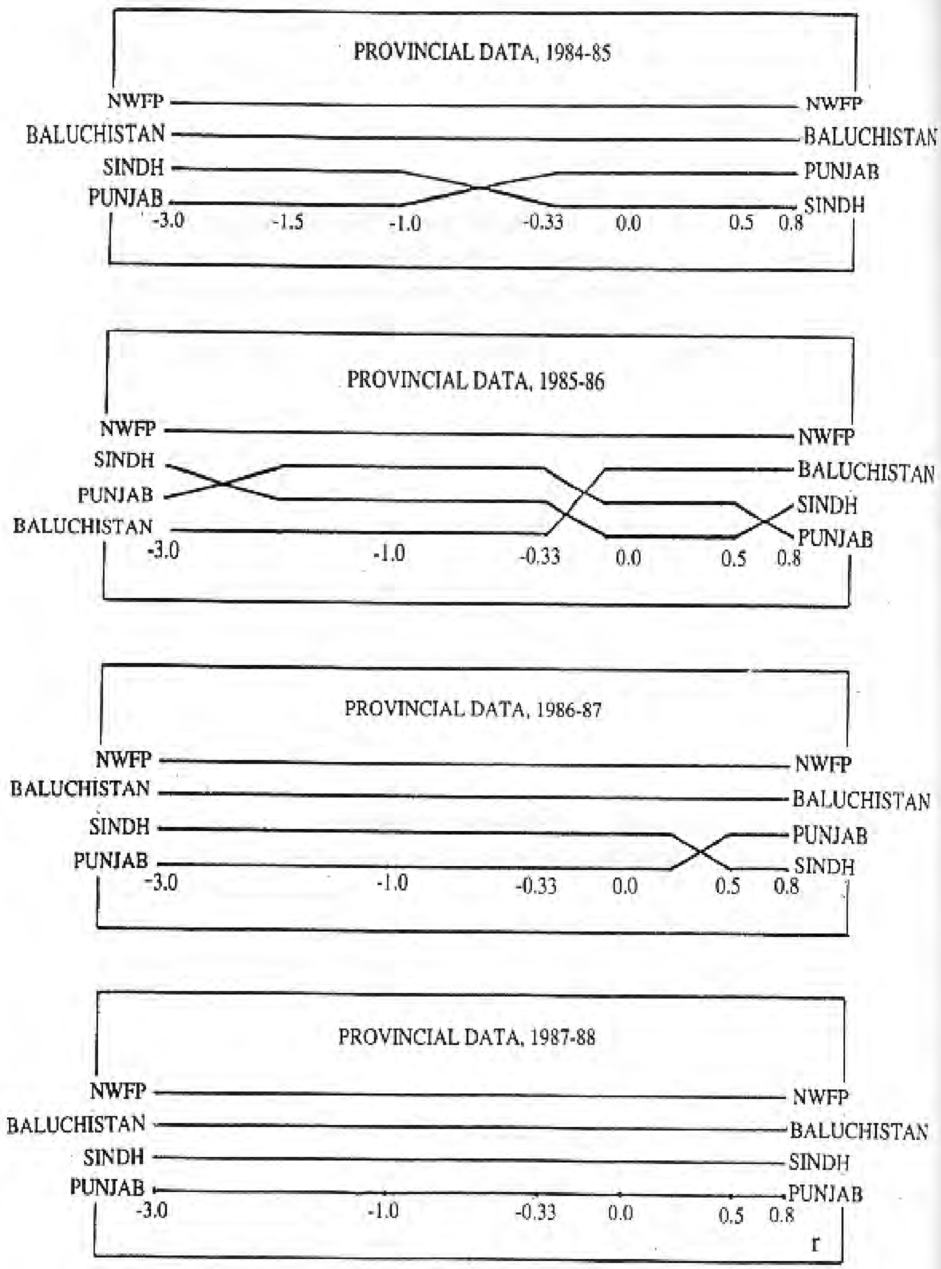

FIGURE 7

Relative Rankings of Pakistan's Provinces, by AKS Index, Annually

Rs.265.4 in 1985-86 before returning back to Rs.332.8 in 1986-87. This may well reflect a problem in the data, more than anything else, and it could certainly by itself account for the sharp relative drop in the value of the index for low values of the ethical parameter.

In 1985-86, as in 1984-85, (and to some lesser extent in 1986-87), the relative ranking of Sindh and Punjab depends importantly upon your choice of value for the ethical parameter.The income distribution in Sindh welfare-dominates the distribution in Punjab for low values of that parameter in 1984-85, and Punjab dominates Sindh for higher values. In 1985-86, Sindh welfare-dominate Punjab for both very low and very high parameter values. Over the broad intermediate range, however, the ranking is the reverse.

V. Conclusion

This study has reported some initial findings on the income distribution in Pakistan and each of its four provinces over the four years 1984-85, 1985-86, 1986-87, and 1987-88. Estimated Lorenz curves and cardinally significant Atkinson-Kolm-Sen (AKS) ethical relative indices of (real expenditure) inequality are computed from the full *HIES* data tapes for each of the four years. For each of these calculations, the statistical unit is the individual "adult equivalent" computed, following Havinga et al., (1990), from an equivalence scale due to Wasay (1977).

Overall there seems to be rather clear evidence of improvement in the income distribution in Pakistan from the beginning of the period in 1984-85 to the end of the period in 1987-88. This is borne out both by analyses of Lorenz curves and an analysis of the AKS indices over a broad range of parameter values. There is less clear-cut (but nonetheless compelling) evidence that, again, over a broad range of ethical values the investigator might bring to bear, this improvement has been a fairly steady and sustained one. That same trend is most clearly identifiable within at least one of Pakistan's provinces, the NWFP. The trend in inequality over time of Pakistan's other three provinces depends more heavily on the investigator's choice of underlying social welfare criterion.

Looking across the provinces, a clear and robust relative ranking in terms of inequality emerges in the data by 1986-87 and continues through 1987-88. There we find the income distribution in NWFP to be the best, followed in order by the distributions in Baluchistan, Sindh, and finally Punjab.

The relative rankings in 1984-85 and 1985-86 depend more heavily on the investigator's choice of ethical values, and may, in 1985-86, be influenced by a sharp decline in mean real expenditure that occurs in the data for Baluchistan. In 1984-85, the findings reported here for NWFP and Baluchistan lend support to and extend the generality of the same relative ranking of these two provinces reported by Havinga et al., (1990) using the Gini coefficient as the index of inequality. They (robustly) *reverse* those authors' findings when the measure of inequality is the Theil index, One final caution is called for. The ordering of provinces in the later years, 1986-87 and 1987-88, exactly parallels the ordering of provinces by population. The NWFP is Pakistan's least populous province, followed in order by Baluchistan, Sindh and Punjab. These population differences are sometimes extreme. Roughly speaking, Punjab is three times larger than Sindh. Sindh, in turn, is something less than twice as populous as Baluchistan, and Baluchistan is roughly four times as populous as NWFP. These large differences in population – combined with the encouraging, but inconclusive, results sof examining the welfare properties of AKS indices over variable populations – suggest that caution may be appropriate in trying to draw too much significance from, the relative ranking of provinces by inequality index. While we've noted that the AKS index seems to behave "reasonably" across variable populations, we need to be able to say more than we can at present before these results can be taken as anything more than suggestive.

Finally, this paper has devoted a good deal of attention to technical measurement issues, but little attention to the economic and social forces that might account for the patterns uncovered. Hopefully, the evidence presented here can serve as a starting point for future research.

Vassar College
Poughkeepsie, New York, USA

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APPENDIX

Lorenz curves for expenditure per adult equivalent for Pakistan and its provinces were estimated from the complete *HIES* data tapes for 1984-85, 1985-86, 1986-87, and 1987-88. Following Kakawani and Podder (1973), ordinary least squares was used to estimate the parameters of the equation,

$$\log(\eta) = \alpha \log(\pi) + \beta(1 - \pi).$$

Here η, ranging from zero to unity, is the cumulative percentage of total expenditure by all adult equivalents and π is the percentage of adult equivalents accounting for η. Estimates for α and β, along with t-statistics and R^2 for the regressions, are reported below.

ESTIMATED COEFFICIENTS
(t-statistics)

	Beta	Alpha	R-Squared
Pakistan			
1984-85	–0.94572	1.04296	
	(–25.3)	(62.0)	0.99848
1985-86	– 0.9043	1.06832	
	(–24.8)	(63.9)	0.99858
1986-87	–0.97825	1.00438	
	(–27.04)	(61.5)	0.99849
1987-88	–0.91094	1.02189	
	(–25.9)	(64.3)	0.99853
Punjab Province			
1984-85	–0.91797	1.05733	
	(–25.5)	(65.8)	0.99857
1985-86	– 0.95769	1.02615	
	(–27.2)	(64.4)	0.99866
1986-87	–0.99459	1.00501	
	(–27.4)	(62.2)	0.99848
1987-88	– 0.96819	1.01308	
	(–26.1)	(61.4)	0.99842

contd.
ESTIMATED COEFFICIENTS
(t-Statistics)

	Beta	Alpha	R-Squared
Sindh Province			
1984-85	-1.05643	0.98443	
	(-26.0)	(52.5)	0.99823
1985-86	- 1.06282	0.97644	
	(-31.4)	(61.7)	0.99872
1986-87	-1.00175	0.99081	
	(-26.3)	(58.0)	0.99838
1987-88	- 0.93117	0.9953	
	(-25.8)	(60.3)	0.99847
Baluchistan Province			
1984-85	-0.82801	1.0601	
	(-21.1)	(62.0)	0.9983
1985-86	- 0.65731	1.22222	
	-23.2)	(92.4)	0.99923
1986-87	- 0.83363	1.00789	
	(-29.5)	(79.8)	0.99903
1987-88	- 0.74265	1.03712	
	(-28.7)	(88.2)	0.99916
NWFP Province			
1984-85	-1.08052	0.96231	
	(-42.6)	(78.7)	0.99943
1985-86	- 0.80565	1.03104	
	(-29.9)	(81.3)	0.99915
1986-87	-0.87534	0.96304	
	(-29.6)	(67.7)	0.99872
1987-88	- 0.6015	1.07552	
	(-20.1)	(83.3)	0.99878