

## **VARIOUS APPROACHES TO MEASUREMENT OF TECHNICAL EFFICIENCY IN NORTH-WEST FRONTIER PROVINCE OF PAKISTAN**

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An extensive research has been conducted in the past on the measurement of technical efficiency. Various approaches namely (a) non-parametric, (b) grouping method, (c) linear programming, (d) statistical frontier and (e) stochastic frontier methods are discussed. The present study proposes a comparative evaluation of these approaches using the cross-section data of 397 farms of the North-West Frontier Province of Pakistan for 1988-89. The Cobb-Douglas and Translog production functions are specified to estimate technical efficiency. The estimated farm level technical efficiency is explained by socio-economic and demographic factors. It is shown that credit, education, age and assets size contribute positively towards the improvement of efficiency.

Measurement of technical efficiency is one of the very important topics of research in both developing and developed countries. Applications vary in content because most studies in developing countries are focused on agriculture while in developed countries, the interest on technical efficiency has been confined to the industrial sector, or manufacturing sector, in general.

Farrell (1957) provides an empirical measure of technical and price efficiency. Technical inefficiency arises when actual or observed output from a given input mix is less than the maximum possible output. Allocative inefficiency arises when the input mix is different from that under profit maximization, where marginal value products are equated to input prices.

In this study, we use a cross-section data of 397 farms of North-West Frontier Province of Pakistan for the year 1988-89. The agriculture sector has made visible progress in Peshawar division of the North-West Frontier province. The objective of this study is to estimate the farm level technical efficiency using five approaches:

- i) Overall method of cost per unit of output approach,
- ii) Grouping method,
- iii) Linear programming approach,
- iv) Statistical method, and
- v) Stochastic frontier approach.

A comparative analysis of different efficiency measures is performed. Both average and farm level technical efficiency are discussed. A line of research that was initiated in the past but which, in our judgement, has remained underdeveloped and in need of additional work, is a comparative evaluation of strengths and weaknesses of five alternative efficiency measurements. Many of the potentially significant advantages of each approach are pointed out in Schmidt and Lovell (1979), and Ferrier and Lovell (1990). One of the main objectives of this study is to provide a comparative analysis of technical efficiency using various methodologies on the same cross-section data. The study uses only material inputs in the analysis. Tenancy, size of farm, education and demographic characteristics are not used in deriving the inefficiency index, but are considered as possible factors to explain the variation in inefficiency index across farms. Price efficiency is not estimated because all farmers face the same prices. Also, we did not notice much variation in prices of inputs and output at a farm level. We use a value measure of output for aggregating wheat, maize, sugarcane and other miscellaneous produced crops.

In Section I, the conceptual framework of Farrell (1957) is presented and data are described. The isoquant approach, grouping method and linear programming approaches are summarized in Section II. The statistical approach and estimates of technical efficiency are discussed in Section III. In section IV, we present the stochastic frontier approach and we derive estimates based on half-normal distributions of efficiency measure. In Section V, frequency distributions of various measures are compared. The explanatory variables are used to explain the variation in technical inefficiency in Section VI. Summary and conclusions are presented in Section VII.

## I. Conceptual Framework and Data Description

### 1. Concepts of Technical and Price Efficiency

Assume that a single output,  $Y$  is produced from two inputs,  $x_1$  and  $x_2$ . The production function for an individual farm is assumed to be linearly homogeneous,

$$Y = f(x_1, x_2) \quad (1.1)$$

The unit output isoquant defines the technical features of the farm. It represents the minimum input mix to produce one unit of output i.e., it is the best practice technique (Figure 1). If points A, B, C and D denote farms which are producing one unit of output, then farms A, B, C are technically efficient but farm D is technically inefficient. A measure of technical efficiency for farm D is given by  $OC/OD$  i.e., farm D could reduce both inputs by proportion  $CD/OD$  and still produce the same level of output. The line  $PP'$  is tangent to the isoquant  $A'A''$  and represents the minimum cost of producing one unit of output at B. Point C is technically efficient but not price efficient while point B is both technically and price efficient. A

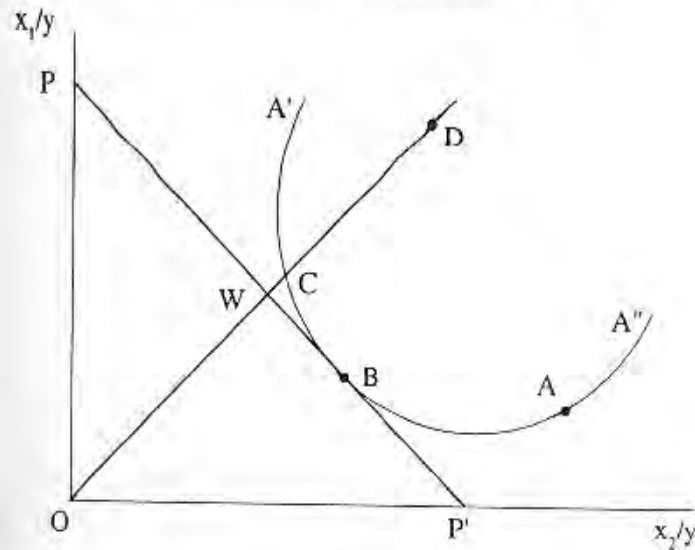


FIGURE 1

combined measure of technical and price inefficiencies is denoted as economic inefficiency. The overall economic inefficiency of farm D is measured by

$$OW/OD = OC/OD \cdot OW/OC$$

Farrell's (1957) radial measures of efficiency are input-based in that they measure differences in input use between farms for the standardized unit output on efficient unit isoquant. He also proposed output-based measure which focuses on differences in output between farms when inputs are standardized. For a non-homogeneous technology, input-based and output-based measures provide two measures of technical efficiency. In the case of linear homogeneous technology (i.e., constant returns to scale technology), both input and output-based measures provide the same measure of technical efficiency. If more output can be produced with the same input, or for a given input, the ratio of output to maximum output is smaller, then this is known as an output measure of efficiency.

## 2. Survey and Discussion of Data

The Institute of Development Studies in Peshawar (Pakistan) conducted a sample survey in the North-West Frontier Province (NWFP) of Pakistan. The sample area comprises all three districts of Peshawar division. This division is one of the most developed areas in the province and constitutes the backbone of the

provincial economy. The majority of the progressive farms, machinery and extension services is concentrated in this division.

The major crops in the province are wheat, maize, sugarcane and vegetables. The growers, however, are not specialized in cultivation of any of the major or minor crops. Each farmer grows at least one of the four crops per year. In our survey, the percentages of area under wheat, maize and sugarcane were 38, 27 and 21 per cent, respectively.<sup>1</sup>

The overall cropping intensity in the province was 121 per cent. It varied with both farm size and tenure. It was the highest at 130 per cent on tenant farms while it was lowest at 108 per cent on owner-cum-tenant farms. Out of the total farms in the NWFP survey, 51 per cent were owner-operated and accounted for 49 per cent of the area. Tenant farms were 27 per cent and accounted for 21 per cent of the area. The remaining 23 per cent of the farms and 30 per cent of the area was operated by owner-cum-tenants.

The data for the year 1988-89 were collected through a survey. Information regarding household composition, farm size, inputs, outputs, prices and costs were gathered. Data on each of the inputs and outputs were collected for each crop. Inputs and outputs were recorded in quantities and value terms. Inputs include the use of seed, manure, fertiliser, animal labour, human labour and tractor.<sup>2</sup> Details of variables used and their definitions are given in Appendix I. Input costs were measured in terms of prices paid by households for each input. However, local market prices were used to approximate cost for some of the family-provided resources such as human labour, animal labour and manure.<sup>3</sup>

## II. The Deterministic Approaches

### 1. Overall Method

The efficient unit isoquant was derived under the assumption of constant returns to scale from a sample of observations in such a way, that no observation lies between the origin and efficient unit isoquant. Farrell (1957), and Farrell and Fieldhouse (1962) describe the empirical procedure which can be used for measuring technical efficiency for multiple inputs. The method consists of plotting inputs per unit of output in a space of the same dimension as number of inputs, and then forming a convex closure of the set of points, taking the appropriate part of this

<sup>1</sup> According to provincial statistics, in the whole province 43 per cent of the cultivable land was devoted to wheat, followed by maize product which constituted 24 per cent of the land.

<sup>2</sup> Seed was measured in kilograms (Kg) while manure and fertilizers were measured in bags (50 Kg). Irrigation was recorded in numbers, while the use of human labour, animal labour and tractor was recorded in hours.

<sup>3</sup> As there was not much variation in input or output prices over 397 farms, it was decided not to follow the well known duality approach. Instead direct production or value of output functions were attempted using cost of inputs (as variable inputs) due to heterogeneous nature of some of the inputs.

convex closure as the efficient technical frontier. The surface defining efficient frontier must satisfy two conditions: (a) the slope should not be positive, and (b) no observed point should lie between the frontier and origin. The technical efficiency is formed by facets which refer to a linear combination of efficient vector points on minimum cost unit isoquant.

In this method, cost incurred on each input is estimated. Then these costs are divided by value of output. The efficiency of individual farmer is measured as

$$E_i = \left[ \frac{\text{Lowest cost farmer}}{\text{Actual cost of farmer}} \right] \times 100 \quad i=1, 2, \dots, 397 \quad (2.1)$$

The efficiency was computed for two to five inputs. In each case the facets were derived using the most efficient farms. Consistently, it was discovered that farms numbered 26, 8 and 10 turned out to be the most efficient farms followed by 45 and 40. For the four input cases, we present the results of frequency distribution of efficiency index (Table A-1) and facets with weights (Table A-2) in the Appendix II.

## 2. Grouping Method

The previous procedure assumed constant returns to scale. This assumption is relaxed. Farrell and Fieldhouse (1962) grouped the value of output in ascending order. We distributed 397 farms into 10 groups; 40 farms each in 9 groups and 37 in the last. Table A-3 (Appendix II) shows the frequency distribution when all five inputs are used. Average technical efficiency ratings were given for each of the 40 farms arranged in ascending order of output. The main conclusion is that the cost per unit of output can be reduced by 33 to 44 per cent, if the resources are used according to the most efficient input combination of the farm.

## 3. Linear Programming Approach

In this approach, the maximization of one farm's output, with given inputs, is undertaken subject to the condition that the farm's potential output is above the observed output. The model moves away from the constant returns to scale assumption. The strong disposability of inputs is permitted and variable returns to scale technology is assumed.<sup>4</sup>

The objective is to maximise;

$$P_1 \bar{x}_1 + P_2 \bar{x}_2 + P_3 \bar{x}_3 + P_4 \bar{x}_4 + P_5 \bar{x}_5 \quad (2.2)$$

<sup>4</sup>As we did not have access to computer program to handle 397 constraints, we have grouped the data and have chosen the most efficient farm from each of the 50 groups at the most.

subject to

$$P_1 x_{12} + P_2 x_{22} + \dots + P_5 x_{52} \geq y_2$$

$$P_1 x_{13} + P_2 x_{23} + \dots + P_5 x_{53} \geq y_3$$

$$P_1 x_{1n} + P_2 x_{2n} + \dots + P_5 x_{5n} \geq y_n$$

$$P_1, P_2, P_3, P_4, P_5 \geq 0, \quad \bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4 \text{ and } \bar{x}_5 \text{ are known means of inputs.}$$

There will be 396 constraints and as the problem is too large, two modifications were considered. Firstly, the most efficient farm from a group of 40 farms was chosen and 10 such farms were selected.<sup>5</sup>

The solution to this problem yields the most efficient output levels from the constraints, i.e.,

$$P_1 x_{1i} + P_2 x_{2i} + \dots + P_5 x_{5i} \geq y_i \quad (2.3)$$

The left hand side of the above inequality yields ( $y_i^*$ ) the maximum output level.  $P_1, P_2, \dots, P_5$  are the optimizing prices for the objective function. The efficiency index was compiled as  $(y_i/y_i^*) \cdot 100$ .

To get the maximum number of farms, farmers were distributed into 50 groups and the most efficient farmer from each group was selected. These results are presented in Table A-4 (Appendix II). Average efficiency remains in the range of 78–87 per cent and this estimate is comparable with other estimates derived from other approaches.

### III. The Statistical Approach

In this model, the frontier production function is estimated with one-sided disturbance terms. The frontier output estimated by  $f(x)$  is always greater than or equal to actual output.

$$y = f(x) e^u, \quad u \leq 0 \quad (3.1)$$

The random disturbance term  $u$  is assumed to follow a one-sided distribution either truncated normal, gamma or exponential.

Afriat (1972) was one of the pioneers who proposed to estimate the production

<sup>5</sup>This was extended to 50 farms out of the sample of 397 farms. Instead of using the input level of first farm in the objective function, mean input levels were used.

function by using the maximum likelihood<sup>6</sup> with distributional assumption on  $u$ . An alternative method of estimation was suggested by Richmond (1974), whose approach was called *corrected ordinary least squares* by Forsund et al., (1980).

Given the mean of  $u$  by  $\mu$ , equation with Cobb-Douglas specification [equation (3.1)] can be written as:

$$\log y_i = (\alpha + \mu) + \beta_1 \log x_{1i} + \dots + \beta_n \log x_{ni} + (u_i - \mu) \quad (3.2)$$

where the new error term has zero mean. Both  $u$  and  $\mu$  can be estimated consistently from the OLS residuals and the estimate of  $\mu$  can be used to correct the constant term of model [equation (3.2)]. Forsund et al., (1980) suggest that even after correcting the constant term, some of the residuals may still have the wrong sign, so that, these observations end up above the production frontier. One way to deal with this problem is to estimate equation (3.2) by OLS and then correct the constant term by shifting it until no residual is positive and one is zero. Greene (1980) has shown that this correction provides a consistent (though biased in finite sample) estimate of  $\alpha$ .

We estimated the statistical frontier model for a Cobb-Douglas production function:

$$\begin{aligned} \log y_i = & \alpha + \beta_1 \log x_{1i} + \beta_2 \log x_{2i} + \beta_3 \log x_{3i} \\ & + \beta_4 \log x_{4i} + \beta_5 \log x_{5i} + u_i, \quad u_i \leq 0 \end{aligned} \quad (3.3)$$

where the random disturbances,  $u_i$  are assumed to follow a one-sided distribution. The output variable ( $y_i$ ) is value of output per acre and similarly inputs namely  $x_{1i}$  (manures),  $x_{2i}$  (fertilizers),  $x_{3i}$  (human labour),  $x_{4i}$  (animal labour) and  $x_{5i}$  (tractors) are all costs per acre. Prices are largely constant on cross-section data.

The first step was the use of OLS. Equation (3.3) will be corrected by appropriate shift in the intercept. The OLS residual (largest positive) turns out to be 0.198 which is added to the constant term estimated by OLS under the usual assumptions. The estimated relationship is:

$$\begin{aligned} Y^* = & 6.895 + 0.0379 x_1 + 0.0427 x_2 + 0.045 x_3 \\ & (4.556) \quad (3.076) \quad (2.939) \\ & + 0.0755 x_4 + 0.0501 x_5 + u \quad (u \leq 0) \\ & (7.285) \quad (6.177) \end{aligned} \quad (3.4)$$

$R^2 = 0.55$ ,  $SEE = 0.082$ ,  $F(5, 391) = 99.7$  per cent,  $\chi^2(20) = 29.69$ , number of observations = 397.

<sup>6</sup>Schmidt (1976) pointed out that the range of the dependent variable depends on the parameters to be estimated in the maximum likelihood procedure. ML estimators do not have the usual properties of consistency and asymptotic efficiency.

In equation (3.4),  $Y^*$  denotes the maximum value of output obtained from a given level of inputs.

Technical efficiency can be defined as:

$$e_i = \log y_i - \log y_i^*, \quad i = 1, 2 \dots 397 \quad (3.5)$$

$$e_i \leq 0$$

$$TE_i = \exp(e_i) = y_i / (y_i^*) \leq 1 \quad (3.6)$$

There is considerable variation in the technical efficiency index over farms. Most of the farms were found in the range of 78-80 per cent efficiency. Table A-5 (Appendix II) gives the frequency distribution of technical efficiency. The mean efficiency index is about 82.32 per cent with a standard deviation of 7 per cent. The maximum efficiency was 100 per cent while the minimum was 60 per cent.

#### IV. Stochastic Frontier Approach.

##### 1. Stochastic Production Function Approaches

Timmer (1971) draws attention to the efficiency index defined by the deterministic approaches namely: (a) overall method and grouping method, (b) linear programming approach, and (c) statistical approach. The first two are deterministic approaches and the last defines all variations not attributable to differential input use as part of the efficiency index. As a result, Timmer (1971) casts doubt on the value of an efficiency estimate based on any one of the frontier techniques that uses one year's data. The stochastic production function based on the composed error model of Aigner et. al., (1977), Meeusen and Van den Broeck (1977), and subsequently the method of calculating farm specific technical efficiency by Jondrow et. al., (1982), overcomes Timmer's (1971) criticism. We consider a stochastic production function model with multiplicative disturbance term of the form:

$$y = f(x, \beta) e^\varepsilon \quad (4.1)$$

where  $\varepsilon$  is a stochastic error term. We specify

$$\varepsilon = u + v \quad (4.2)$$

consisting of two independent elements. The symmetric component,  $v$ , takes care of random variation in output due to factors outside the control of one farm, such as weather and diseases. It is assumed to be independently and identically distributed.



as  $N(0, \sigma^2)$ . A one-sided component  $u \leq 0$  reflects technical efficiency relative to the stochastic frontier,  $f(x) e^v$ . Thus,  $u = 0$ , for any farm's output lying on the frontier, and is strictly negative for any output lying below the frontier. Assume that  $u$  is identically and independently distributed as  $|N(0, \sigma_u^2)|$ , i.e., the distribution of  $u$  is half-normal.

The stochastic production frontier model can be used to analyse cross-section data. The frontier of the farm is given by combining equations (4.1) and (4.2)

$$y = f(x_i, \beta) e^{(u+v)} \quad (4.3)$$

in which variances of  $(u+v)$ , are  $\sigma_u^2 + \sigma_v^2$ , denoted as  $\sigma^2$

The variances of  $\varepsilon$  is

$$\sigma^2 = \sigma_u^2 + \sigma_v^2 \quad (4.4)$$

and the ratio of two standard errors<sup>7</sup> is defined by

$$\lambda = (\sigma_u / \sigma_v) \quad (4.5)$$

Jondrow et. al., (1982) have shown that measures of efficiency at the individual farm level can be obtained from the error terms  $\varepsilon = u + v$ . For each farm, it is the expected value of  $u$  conditional on  $\varepsilon$ , i.e.,

$$E(u / \varepsilon) = \frac{\sigma_u \sigma_v}{\sigma} \left[ \frac{\phi(\varepsilon\lambda/\sigma)}{1 - \phi(\varepsilon\lambda/\sigma)} \right] \frac{\varepsilon\lambda}{\sigma} \quad (4.6)$$

where  $\phi(\cdot)$  and  $\Phi(\cdot)$  are the standard normal density function and the standard normal distribution function evaluated at  $(\varepsilon\lambda/\sigma)$ . For  $\varepsilon$ ,  $\lambda$  and  $\sigma$  estimated values used to evaluate density and distribution functions. Measures of efficiency for each farm can be calculated as

$$TE = \exp. [E(u/\varepsilon)] \quad (4.7)$$

## 2. Estimation of Translog Production Function

The Cobb-Douglas production function is very restrictive and if the farm behaviour is not according to the specifications there can be biases in the estimation of technical efficiency frontier. If there are systematic biases, by ignoring quadratic and cross-product terms of a flexible functional form (such as translog), it is likely

<sup>7</sup>Battese and Corra (1977) define  $\gamma$  as the total variation in output from the frontier which is attributable to technical inefficiency i.e.,  $\gamma = (\sigma_u^2 / \sigma^2)$  so that  $0 \leq \gamma \leq 1$ .

that technical inefficiency is bound to be over-estimated. Our specification of translog production function for stochastic frontier approach amounts to

$$\log y = \beta_0 + \sum_{i=1}^S \beta_i \log x_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \log x_i \log x_j + \varepsilon \quad (4.8)$$

where  $\varepsilon = u + v$ , and  $u \leq 0$ .

The results of the estimated stochastic frontier approach are presented in Table A-6 (Appendix II). As expected, technical efficiencies for every farm are higher than the ones estimated using the Cobb-Douglas production function. In the results presented in Table A-7 (Appendix II), technical efficiency index is based on the assumption that  $u$  is half-normal. We also attempted exponential and truncated normal distributions for  $u$ , but the results did not change both at a farm level or at an aggregate level. The overall mean efficiency level was unchanged.

Our level of efficiency for farms was very high. The minimum estimated efficiency was 91 per cent while maximum was 98 per cent with mean efficiency level of 95.6 per cent. In a stochastic frontier approach, it may not be justified to test the nested hypothesis with the Cobb-Douglas restrictions within a translog framework. In general, the Cobb-Douglas is nested within the translog function but because of the half-normal distribution of the error term, it may not be appropriate to use nested hypotheses tests. If such a test was to be applied, we find that the restrictions of Cobb-Douglas are rejected. A direct estimation of translog involves too many parameters and hence the estimates are not highly reliable. We, therefore, decided to present the results of the Cobb-Douglas for comparison purposes.

### 3. Estimation of Cobb-Douglas Frontier

We specified  $f(x, \beta)$  in equation (4.1) to be Cobb-Douglas i.e.,

$$\log Y_i = \alpha + \beta_1 \log x_{1i} + \beta_2 \log x_{2i} + \beta_3 \log x_{3i} + \beta_4 \log x_{4i} + \beta_5 \log x_{5i} + (u + v)_i, \quad i=1, 2, \dots, 397 \quad (4.9)$$

The output measure  $Y_i$  is value of output per acre for all crops at a farm level, while  $x_{1i}$ ,  $x_{2i}$ ,  $x_{3i}$ ,  $x_{4i}$  and  $x_{5i}$  are cost of manures, cost of fertilizers, wages of human labour, cost of animal labour and tractor costs per acre. There was very small variation in prices and hence it was decided to use value measure for both outputs and inputs.

The frontier regression model was estimated using the maximum likelihood estimation technique using three distributional assumptions on the technical efficiency disturbance term  $u$ . From the value of  $\lambda$  it can be decided whether the disturbances are due to symmetric or positive error e.g., when  $\lambda \rightarrow 0$ , it implies that  $\sigma_u^2 \rightarrow 0$  and  $\sigma_v^2 \rightarrow \infty$ . In this case, the symmetric error is dominant in determining the value of

$\varepsilon_i$ . On the other hand, when  $\lambda \rightarrow \infty$  the non-positive error becomes a dominant source of disturbance. When  $\lambda = 0$  the frontier ML is equivalent to OLS and farmers are technically efficient in the use of bundle of inputs and all errors are due to symmetric error.

The OLS and ML estimates on per acre basis are presented in Table A-8 (Appendix II). All the coefficients have positive signs as expected. The constant term is higher in ML than in OLS and  $\lambda$  is 0.7451, which is not significantly different from zero. The mean technical efficiency is calculated by using

$$E(e^{\lambda}) = 2 \exp[\sigma_u^2/2(1 - \phi(\sigma_u))] \quad (4.10)$$

The suggested approach of Jondrow et. al., (1982), [equation (4.6)] is used to derive the technical efficiency at a farm level. This estimate is defined as

$$TE_i = u_i = \log y_i - \log \hat{y}_i \quad (4.11)$$

and

$$TE_i = (y_i/\hat{y}_i) \times 100$$

Farms exhibited a range of technical efficiency from 90 per cent to 97 per cent. The largest farm efficiency was 97 per cent (see Table A-9, Appendix II).

The Cobb-Douglas was estimated with: (a) exponential and truncated normal distribution with non-zero mean for random term associated with technical efficiency but there was no significant difference in ranking of farms in their efficiency.

#### V. Comparison of Empirical Results based on Various Approaches

It is not expected that various methods of measuring technical efficiency can yield identical estimates. Deterministic methods are likely to yield lower estimate of technical efficiency since random factors are not accounted for. Despite these, the purpose of our comparison is to examine whether ordinal ranking of farms for technical efficiency remains the same across various measures. The other possible way to look at the technical efficiency at the farm level is to examine whether the farms which are most efficient, such as farm No.26, turn out to be similar in all approaches. The frontier production function is estimated with physical inputs per acre. Socio-economic, demographic and other variables pertaining to farm households are ignored. These individual characteristics are considered in the next section to explain the estimated efficiency index by farms on the basis of a stochastic frontier approach.

There are advantages and disadvantages associated with stochastic frontier approach as compared to the deterministic approaches. The biggest advantage of the stochastic frontier approach is that unlike the other three approaches, it introduces a disturbance term representing noise, measurement error and exogenous shocks beyond the control of farm unit. None of the other approaches makes any

**TABLE 1**  
Farm Specific Technical Efficiencies in Different Frontiers

Efficiency Index	L. Programming Frontier		Nonparametric Frontier		Statistical Frontier		Stochastic Frontier (Cobb-Douglas) (Half-Normal)		Stochastic Frontier (Cobb-Douglas) (Exponent) (CRS)		Stochastic Frontier (Translog) (Half-Normal) (VRS)	
	No. of Farmers	%	No. of Farmers	%	No. of Farmers	%	No. of Farmers	%	No. of Farmers	%	No. of Farmers	%
25-30	0	0.0	8	2.0	0	0.0	0	0.0	0	0.0	0	0.0
31-35	0	0.0	26	6.5	0	0.0	0	0.0	0	0.0	0	0.0
36-40	0	0.0	35	8.8	0	0.0	0	0.0	0	0.0	0	0.0
41-45	0	0.0	66	16.6	0	0.0	0	0.0	0	0.0	0	0.0
46-50	1	2.0	85	21.4	0	0.0	0	0.0	0	0.0	0	0.0
51-55	2	4.0	62	15.6	0	0.0	0	0.0	0	0.0	0	0.0
56-60	0	0.0	42	10.6	0	0.0	0	0.0	0	0.0	0	0.0
61-65	1	2.0	23	5.8	0	0.0	0	0.0	0	0.0	0	0.0
66-70	5	10.0	14	3.5	13	3.3	0	0.0	0	0.0	0	0.0
71-75	11	22.0	16	4.0	43	10.8	0	0.0	0	0.0	0	0.0
76-80	6	12.0	5	1.3	103	25.9	0	0.0	0	0.0	0	0.0
81-85	9	18.0	5	1.3	95	23.9	0	0.0	0	0.0	0	0.0
85.1-86	1	2.0	0	0.0	30	7.6	0	0.0	0	0.0	0	0.0
86.1-87	4	8.0	1	0.3	19	4.8	0	0.0	0	0.0	0	0.0
87.1-88	0	0.0	0	0.0	18	4.5	0	0.0	0	0.0	0	0.0

Continued

TABLE 1  
(Continued)

Efficiency Index	L Programming Frontier		Nonparametric Frontier		Statistical Frontier		Stochastic Frontier (Cobb-Douglas) (Half-Normal)		Stochastic Frontier (Cobb-Douglas) (Exponent) CRS		Stochastic Frontier (Translog) (Half-Normal) VRS	
	No. of Farmers	%	No. of Farmers	%	No. of Farmers	%	No. of Farmers	%	No. of Farmers	%	No. of Farmers	%
88.1-89	1	2.0	1	0.3	13	3.3	0	0.0	0	0.0	0	0.0
89.1-90	1	2.0	1	0.3	8	2.0	0	0.0	0	0.0	0	0.0
90.1-91	3	6.0	0	0.0	17	4.3	10	2.5	0	0.0	0	0.0
91.1-92	1	2.0	0	0.0	9	2.3	6	1.5	0	0.0	0	0.0
92.1-93	1	2.0	3	0.8	5	1.3	36	9.1	0	0.0	0	0.0
93.1-94	0	0.0	0	0.0	6	1.5	79	19.9	5	1.3	31	7.8
94.1-95	0	0.0	0	0.0	5	1.3	97	24.4	7	1.8	99	24.9
95.1-96	1	2.0	1	0.3	4	1.0	109	27.5	24	6.0	131	33.0
96.1-97	1	2.0	2	0.5	2	0.5	9	2.3	37	9.3	98	24.7
97.1-98	0	0.0	0	0.0	1	0.3	51	12.8	240	60.5	24	6.0
98.1-99	0	0.0	0	0.0	5	1.3	0	0.0	84	21.2	0	0.0
99.1-100	1	2.0	1	0.3	1	0.3	0	0.0	0	0.0	0	0.0
All	50.0	100.0	397.0	100.0	397.00	100.0	397.0	100.0	397.0	100.0	397.0	100.0
Mean	78.7		51.7		8.23		95.6		97.4		96.2	
S.D.	11.2		13.0		6.70		1.4		0.9		1.1	
Min	49.0		25.0		66.10		90.5		93.1		91.2	
Max	100.0		100.0		100.00		97.9		98.7		98.1	

VRS = Variable Returns to Scale. CRS = Constant Returns to Scale.

accommodation for such phenomena. The technique of estimating stochastic frontier imposes distributional assumptions on another stochastic disturbance term representing technical inefficiency. When this residual distribution is skewed to the right, the cross-section data will show high levels of inefficiency, while if residual distribution is symmetric, there is no inefficiency.

A comparative analysis of farm specific technical efficiencies is presented in Table 1. Means, standard deviations and minimum and maximum efficiency levels are different. We believe that cardinal comparisons are not possible given different assumptions of various methodologies. An attempt is made to rank the farms in descending order or provide a rank correlation matrix. On a bivariate basis, the matrix turns out to be high enough to conclude that ranking of farms across different statistical/stochastic approaches remains unchanged (Table 2). If the farms are compared on the basis of maximum efficient farm (Table 3), we find that farm 26 is the best farm across all methodologies. Both statistical frontier and stochastic frontier approaches yield the same three farms to be inefficient. Farm 385 is the worst on three methodologies in terms of efficiency index.

TABLE 2

The Spearman Rank Correlation Calculated for  
Different Methodologies of Frontier Functions

Methodologies	Rank Correlation Coefficients
NP with ST	0.234
NP with SCDG	0.235
NP with SCDE	0.235
NP with SCTH	0.233
NP with SCDH	0.999
ST with SCTH	0.941
SCDH with SCDE	0.999
SCDH with SCTH	0.941
SCDE with SCTH	0.942

NP = Non parametric frontier using Farrell's method.

ST = Statistical frontier.

SCDH = Stochastic (Cobb-Douglas half normal) frontier.

SCDE = Stochastic (Cobb-Douglas half normal) frontier.

SCTH = Stochastic (Translog half normal) frontier.

Note: Computed t-value for the lowest rank correlation is 4.787, which means that all rank correlations are significantly different from zero.

## VI. Determinants of Technical Efficiency

Given a technology to transform physical inputs into outputs, some farmers are able to achieve maximum efficiency upto 100 per cent while other farmers are technically inefficient. This discrepancy could be due to the latter group not having adequate technical knowledge (to produce the maximum output with a given level of input) compared to efficient farmers. Kalirajan and Shand (1989), and Shapiro and Muller (1977) have suggested that the technical efficiency of farmers is determined by socio-economic and demographic factors facing the farm household.

We postulate a relationship (expected signs in parenthesis)

$$\begin{aligned}
 TE_i = & \alpha + \overset{(+)}{\beta_1} FS_i + \overset{(+, -)}{\beta_2} AG_i + \overset{(+)}{\beta_3} ED_i + \overset{(-)}{\beta_4} OE_i + \overset{(+)}{\beta_5} AS_i \\
 & + \overset{(+)}{\beta_6} WT_i + \overset{(+)}{\beta_7} WA_i + \overset{(+)}{\beta_8} CD_i + \overset{(-)}{\beta_9} FG_i + \overset{(+)}{\beta_{10}} EX_i \\
 & + \overset{(+, -)}{\beta_{11}} SZ_i + \zeta_i, \quad i = 1 \dots 397
 \end{aligned} \tag{6.1}$$

where

TE = Technical efficiency index estimated by stochastic frontier approach using Cobb-Douglas production function.

FS = Family Size (Number).

AG = Age of the head of household (Years).

ED = Education of the head of household (Years of schooling).

OE = Off farm work (Hours per month).

AS = Value of farm assets per acre (Rupees).

WT = Value of non-farm assets per acre (Rupees).

WA = Working animals per acre (Numbers).

CD = Credit per acre (Rupees).

FG = Degree of fragmentation (Numbers).

EX = Contact and meeting of extension service per farm (Number).

SZ = Farm size (Acres).

$\beta_1 \dots \beta_{10}$  are parameters while  $\zeta$  is the random disturbance term which is normally distributed with mean zero and finite variance. The expected signs of coefficients are shown in equation (6.1). The results of the regression analysis are shown in Tables 4, 5, and 6.

A higher family-size increases efficiency because (in North West Frontier Province) at the time of peak season, there is a shortage of labour and hence family labour is a critical input. Education has a positive and significant impact on technical efficiency as expected. Credit improves farmers' liquidity and facilitates the

TABLE 3  
Efficient and Inefficient Farmers in Different Frontiers

Frontier Approaches	Efficient Farmers		Inefficient Farmers		Average Farmers	
	Farm Numbers	Per cent Efficiency	Farm Numbers	Per cent Efficiency	Farm Numbers	Per cent Efficiency
L. Programming Frontier	26	100.0	101	49.0	22	78.7
	19	100.0	75	51.0		
	272	96.3	96	53.0		
Nonparametric Frontier (Farrell's Approach)	26	100.0	76	25.3	74	51.7
	8	96.9	62	27.4		
	19	95.9	385	28.1		
Statistical Frontier	26	100.0	385	66.1	364	82.3
	36	99.4	388	66.4		
	12	99.2	366	66.5		
Stochastic Frontier (Cobb-Douglas) (Half-normal)	26	97.8	385	90.1	276	95.4
	36	97.7	388	90.5		
	12	97.6	366	90.6		
Stochastic Frontier (Cobb-Douglas) (Exponent)	26	98.7	385	93.1	295	97.4
	36	98.6	388	93.3		
	12	98.6	366	93.4		
Stochastic Frontier (Translog)(Half Normal)	26	98.1	385			
	36	98.0	388		276	96.2
	12	97.7	366			



**TABLE 4**  
 Results of Farm Specific Factors Influencing the Technical  
 Inefficiency using Statistical Frontier Approach  
 (Corrected Ordinary Least Squares in Cobb-Douglas Production Function)

Parameters	Coefficients $\times 10^{-2}$	t-ratios
Constant	13.486	7.40*
A. Demographic Characteristics		
$\beta_1$ (Family size)	-0.1476	-1.73
$\beta_2$ (Age of the head)	0.0243	1.10
$\beta_3$ (Education of the head)	-0.2797	3.19*
B. Resource factors		
$\beta_4$ (Off the farm work)	0.0613	0.96
$\beta_5$ (Farm assets)	-0.00004	-0.19
$\beta_6$ (Non farm assets)	0.00003	0.64
$\beta_7$ (Working animals)	-1.8000	-1.70
C. Institutional factors		
$\beta_8$ (Credit)	-0.00159	-2.71*
$\beta_9$ (Fragmentation)	5.5747	6.15*
$\beta_{10}$ (Extension services)	-0.2959	-1.64
$\beta_{11}$ (Farm size)	0.177	4.30*
$R^2$	0.18	
SEE	0.061	
F Statistics	(11.385)	7.57*
White $\chi^2$ (77)	64.12	
Mean of Inefficiency	0.177	
Number of observations	397	

\* Significant at 5 per cent level

**TABLE 5**  
 Results of Farm-Specific Factors Influencing the Technical  
 Inefficiency in Cobb-Douglas Production Function  
 Estimates using Stochastic frontier approach

Parameters	Coefficients $\times 10^{-2}$	t-ratios
Constant	3.599	9.24*
A. Demographic Characteristics		
$\beta_1$ (Family size)	-0.0207	-1.13
$\beta_2$ (Age of the head)	0.0067	1.42
$\beta_3$ (Education of the head)	-0.0582	-3.11*
B. Resource factors		
$\beta_4$ (Off the farm work)	0.0202	1.48
$\beta_5$ (Farm assets)	-0.000002	-0.04
$\beta_6$ (Non farm assets)	0.000001	0.14
$\beta_7$ (Working animals)	-0.2119	-1.79
C. Institutional factors		
$\beta_8$ (Credit)	-0.0004	-2.89*
$\beta_9$ (Fragmentation)	0.8992	4.64*
$\beta_{10}$ (Extension services)	-0.0577	-1.49
$\beta_{11}$ (Farm size)	0.0323	3.68*
R <sup>2</sup>	0.14	
SEE	0.0131	
F Statistics (11, 385)	5.71*	
White $\chi^2$ (77)	60.71	
Mean of Inefficiency	0.0439	
Number of observations	397	

\* Significant at 5 per cent level

**TABLE 6**  
 Results of Farm Specific Factors Influencing the Technical  
 Inefficiency in Translog Production Function  
 Estimates using Stochastic Frontier Approach

Parameters	Coefficients $\times 10^{-2}$	t-ratios
Constant	3.0868	9.76*
A. Demographic Characteristics		
$\beta_1$ (Family size)	- 0.0088	- 0.59
$\beta_2$ (Age of the head)	0.0056	1.47
$\beta_3$ (Education of the head)	- 0.0379	- 2.49*
B. Resource factors		
$\beta_4$ (Off the farm work)	0.0098	0.89
$\beta_5$ (Farm assets)	- 0.00002	- 0.52
$\beta_6$ (Non farm assets)	0.0000003	- 0.05
$\beta_7$ (Working animals)	- 0.1571	- 1.41
C. Institutional factors		
$\beta_8$ (Credit)	- 0.0003	- 2.23*
$\beta_9$ (Fragmentation)	0.5963	3.78*
$\beta_{10}$ (Extension services)	- 0.0191	- 0.60
$\beta_{11}$ (Farm size)	0.0232	3.23*
$R^2$	0.104	
SEE	0.0107	
F Statistics (11, 385)	4.07*	
White $\chi^2$ (77)	65.51	
Mean of Inefficiency	0.0376	
Number of observations	397	

\* Significant at 5 per cent level

purchase of inputs. Credit may encourage farmers to introduce HYV to improve the yield per acre. Fragmented land reduces the efficiency index. Extension services will improve efficiency as better management and information utilization should lead to greater benefits to farmers. The coefficients of other variables are not significant although they have expected signs.<sup>8</sup>

The measurement of technical efficiency under different forms of tenure was extended to owner-cultivators and sharecroppers. The farm-specific efficiency was calculated using the stochastic frontier approach. The technical efficiency estimates range from 85-94 per cent for owner cultivators, to 87-92 per cent for share croppers. The technical inefficiency on individual farms varies from 6-14 per cent in owner cultivated farms and about 8 per cent to 12 per cent in share-cropped farms. No significant differences in technical efficiency were observed between share-croppers and owners. The Marshallian approach claims that owner cultivators are likely to be more efficient than share-croppers. Cheung (1969) had argued that "resource allocation under private rights will be the same whether the landowner cultivates the land, hires his holding on a fixed rent basis, or shares the actual yield with his tenant". Our empirical evidence does not favour any significant differences in measured efficiency for share-cropped and owner cultivated farms.

#### *Characteristics of Most Efficient and Least Efficient Farms*

It will be useful to examine the characteristics of most efficient and least efficient farms which can provide the reasons of inefficiency. There can be environmental, climatic and locational factors besides the family size, age of the farmer, education of the household, availability of off-farm work, capital assets, working animal, availability of credit, fragmentation of land, extension contacts, size of land, location of farm, type of irrigation and distance from village and Tehsil markets. These are listed in Table 7. For the most efficient farm 26, it is shown that farm assets, wealth, extension contact, and size of holding are important factors. The entire sample comes from the most fertile areas of the valley. The climate is sub-tropical and it is most intensively cultivated region of the country. Besides these factors, the managerial input which can vary systematically may be responsible for greater inefficiency observed on farms 366, 385 and 388. On inefficient farms, family size is low, fragmentation is high and extension visits are none; size of holding is large and farms are tenant cultivated, and all these farms are located far away from village and Tehsil markets. All these characteristics are listed in Table 7.

<sup>8</sup>Test for heteroscedasticity does not suggest the existence of the problem.  $R^2$  is expected to be low on a large number of observations with variability on a cross-section data.

**TABLE 7**  
 Characteristics of Efficient and Inefficient Farms

Characteristics	12	26	36	366	385	388	Mean/STD
Farm No.							
Family Size (Numbers)	11	10	10	7	6	5	7.96 (3.85)
Age (Years)	38	37	35	80	70	60	44.81 (19.13)
Education (Years of Schooling)	8	10	7	0	0	0	2.16 (3.59)
Off Farmwork (Hours)	0	0	0	65	45	60	25.33 (22.37)
Farm Asst. (Rs)	700	2500	500	7000	3000	1800	7199.41 (17497.14)
Wealth (Rs)	18000	26900	32000	20300	14900	52900	46213.87 (35170.89)
Working Animal (Number)	6	3	2	2	0	0	2.27 (2.63)
Credit (Rs)	12000	0	0	0	0	6500	1868.07 (5400.88)
Characteristics/ Farm No.	12	26	36	366	385	388	Mean (Std)
Fragmentation (Numbers)	3	3	2	19	10	13	7.70 (6.27)
Extension Contact (Numbers)	4	6	5	1	0	0	4.29 (1.72)
Size of Land (Acres)	3	3	4	19	15	16	10.56 (9.68)
Tenancy Status	Owner	Owner	Owner	Tenant	Tenant	Tenant	
Location:	Pandubala	Pirsabak	Rajar,	Khashki	Prang Mera	Musa Kelly	
Village/Tehsil	Peshawar	Nowshera	Charsada	Nowshera	Charsada	Charsada	
Type of Irrigation	canal	canal	canal	canal	canal	canal	
Distance from Village market (miles)	3	2 1/2	6	4	10	7	
Distance from Tehsil market (miles)	8	6	9	12	16	13	

## VII. Summary and Conclusions.

Various approaches to the measurement of technical efficiency have provided us with a number of estimates of technical efficiency. It remains problematic that in the absence of any direct observations on technical frontier function, which estimate of technical efficiency should be accepted? Two criteria can be used: (a) the criterion of the flexible and general functional form, and (b) the approach based on stochastic frontier. The latter approach takes account of random factors separately, compared to inefficiency caused by input misallocations or farmers not following the best practice technique.

As the cross-section data did not have price variations, it was not possible to estimate a translog cost or profit function defined over price space. The study is confined to the direct estimation of the Cobb-Douglas and the translog production frontier. Our results indicate that farms which are most efficient with a deterministic frontier remain the most efficient with stochastic approach as well. Deterministic approaches such as grouping method, overall cost per unit of output approach and linear programming approach, produce high levels of mean inefficiency. Statistical approach of corrected least squares provides almost the same level of technical inefficiency as the stochastic frontier approach and these are considerably less than those provided by the other deterministic approaches. This means that the deterministic (statistical, corrected OLS) and the stochastic frontier approaches yield comparable estimates of average and farm level technical efficiency.

Once the inputs are accounted for, the technical inefficiency at a farm level is related to various socio-economic and demographic factors. Socio-economic factors present the outcome that farms with greater availability of credit, higher education of farmers, younger farmers and larger assets of farm community contribute positively towards the improvement of efficiency. Appropriate input use in adequate proportions can improve technical efficiency, which can be induced (at a farm level) through the price system, since allocation of inputs and its use are likely to be affected by the relative prices of inputs. The analysis of the estimated technical efficiency by various social, economic and demographic factors suggests that if more resources are invested in farmers' education, improved availability of credit, if younger farmers become owners of land, and there is less fragmentation of land, then there will be an increase in technical efficiency in Pakistan. There is a great deal of collinearity between fragmentation and holding size. As expected, large-sized holdings have more fragmented land and, this in turn, yields lower efficiency as expected.

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## APPENDIX I

### Definitions of Variables Used

#### *Value of Output (Y)*

As it was not possible to aggregate physical quantity, gross value per acre in Rupees was calculated by converting both the main product and by-product of all crops into value terms.

#### *Manures ( $x_1$ )*

Manures consist of cattle dung, rotten leaves and ashes. There is no proper weighing system for manure which is usually transported on donkey back or in bullock carts. The prevailing conversion rate used is: one donkey load equivalent to 50 kgs. Figures are recorded as costs converted to per acre of land.

#### *Fertilizer ( $x_2$ )*

In estimating the amount of fertilizer used, the physical quantity of different kinds of fertilizers were recorded and 50 kg bag prices was used to estimate the total cost.

#### *Human Labour ( $x_3$ )*

The labour input was recorded in terms of man hours employed for different operations during the agricultural year. It included family labour; permanent, exchange and casual-labour. The prevailing wage rate was used to convert to wage costs per acre.

#### *Bullock Power ( $x_4$ )*

This was measured in pair of bullock hours. It includes both own and hired bullock power used for ploughing and other farm operations. This was converted to bullock costs per acre using price per hour of bullock labour.

#### *Tractor Use ( $x_5$ )*

This was measured in hours used for different operations and the rate paid by farmer per hour was recorded.

The information about the number of irrigations per crop was recorded in a season. Major source of irrigation was canal. In the survey villages, the water was proportionately distributed on the basis of land. Every farmer was assured of water once a week in proportion to his land. The water rate was fixed by the government and varied from crop to crop. All the areas of the survey were considered irrigated because there was no way to distinguish unirrigated farms from irrigated farms. Expenditure on insecticide and pesticide was recorded but the number of users were few.

## APPENDIX 2

TABLE A-1

Frequency Distribution of Technical Efficiency With Different Combinations of Four Inputs

Index	$X_1 X_2 X_3 X_4$	$X_1 X_2 X_4 X_5$	$X_2 X_3 X_4 X_5$	$X_3 X_4 X_5 X_1$	$X_5 X_1 X_3 X_2$
0 - 10	0	0	0	0	0
10 - 20	0	0	0	0	0
20 - 30	8 (2.01)	11 (2.77)	5 (1.26)	16 (4.03)	10 (2.52)
30 - 40	68 (17.13)	78 (19.65)	50 (12.59)	68 (17.13)	87 (21.91)
40 - 50	147 (37.04)	158 (39.80)	140 (35.27)	143 (36.02)	121 (30.48)
50 - 60	101 (25.44)	89 (22.42)	124 (31.23)	93 (23.43)	88 (22.17)
60 - 70	40 (10.07)	39 (9.82)	36 (9.08)	31 (7.81)	51 (12.85)
70 - 80	20 (5.04)	13 (3.27)	26 (6.55)	24 (6.04)	21 (5.29)
80 - 90	7 (1.76)	6 (1.51)	8 (2.01)	14 (3.53)	14 (3.53)
90 - 100	6 (1.51)	3 (0.76)	8 (2.01)	8 (2.01)	5 (1.26)

Note:  $X_1$  = Manures,  $X_2$  = Fertilizers,  $X_3$  = Human Labour,  $X_4$  = Bullock Power, and  $X_5$  = Tractor Use

Figures in parenthesis are percentages of total farms.

**TABLE A-2**  
Various Combinations of Inputs for Estimation of Facets

Farm Numbers		Facets				
		a <sub>1</sub>	a <sub>2</sub>	a <sub>3</sub>	a <sub>4</sub>	a <sub>5</sub>
8	10	18.40	3.96	0.84	5.84	0.00
10	20	2.15	11.01	0.00	6.39	6.59
10	12	9.52	0.00	5.56	3.14	1.05
10	19	26	0.00	4.43	5.26	9.59
12	19	26	0.00	5.10	7.20	4.24
10	8	19	8.21	2.99	4.80	2.73
8	17	20	11.96	0.00	0.00	11.12
17	20	44	11.90	0.00	0.00	11.23
44	45	8	7.75	0.00	0.00	18.23
269	277	286	0.00	0.00	0.00	29.75
277	286	287	12.37	0.00	16.07	2.38
287	296	269	46.17	0.00	9.74	16.47
8	19	365	17.32	0.00	16.79	0.00
2	19	26	0.00	3.23	4.57	0.00
17	19	44	0.00	7.46	4.35	0.00
17	19	40	6.77	2.72	18.12	0.00
19	40	314	5.86	0.00	17.44	10.83
40	340	10	4.24	0.00	22.05	8.90
12	19	17	9.74	0.00	14.08	7.22
314	10	17	4.21	0.00	17.10	22.50
8	26	D2	0.00	6.31	7.91	7.04
8	365	D2	3.46	0.00	0.00	0.00
49	300	D2	18.11	0.00	5.74	0.00
49	277	D2	32.40	0.00	0.00	0.00
17	20	D2	45.66	0.00	0.00	0.00
20	37	D2	38.02	0.00	0.00	0.00
		D3	35.37	0.00	0.00	0.00

continued

TABLE A-2  
(Continued)

Farm Numbers		Facets				
		$a_1$	$a_2$	$a_3$	$a_4$	$a_5$
293	D2	46.42	0.00	0.00	32.38	0.00
17	D3	31.28	0.00	0.00	46.19	0.00
10	D1	0.00	13.15	0.00	9.17	0.00
14	D3	0.00	8.87	0.00	15.20	0.00
17	D1	0.00	14.01	0.00	6.34	0.00
10	D3	0.00	10.49	0.00	19.48	0.00
8	D1	0.00	18.28	0.00	0.00	8.82
13	D3	0.00	15.34	0.00	0.00	14.87
17	D1	0.00	11.02	0.00	0.00	42.00
18	D3	0.00	19.88	0.00	0.00	16.89
8	D1	0.00	12.88	0.00	0.00	26.15
12	D2	0.00	0.00	7.92	3.62	0.00
10	D1	0.00	0.00	8.05	4.28	0.00
19	D2	0.00	0.00	7.91	0.00	6.28
149	D1	0.00	0.00	8.06	0.00	3.80
8	D1	0.00	0.00	2.56	0.00	49.55
269	D1	0.00	0.00	0.00	78.59	1.24
277	D1	0.00	0.00	0.00	22.56	37.10
281	D1	0.00	0.00	0.00	17.34	39.92
10	D1	0.00	0.00	0.00	49.29	7.69

Notes: For the first row, farm nos. 8, 10, 12 and 19 provide the efficient frontier with four inputs.

D's imply 0 input for one of the inputs.

In the four input space, with given input combinations, to produce a unit output, the following problem is solved  $\sum_{i=1}^4 a_i X_i = 1$ ,  $G = 8, 10, 12, 19$  farms, where  $X_i$  is the cost per unit of output of farm  $i$ , and  $a_i$  are unknowns determined from the solution of equation system.

**TABLE A-3**  
Grouping of Farmers by Value of Output  
(Rs./Acre)

Group	No. of Farms	Cost per Unit of Output			Efficiency Rating			Average
		Range of Value (Rs./Acre)	Maximum	Minimum	Difference Per cent	Maximum Per cent	Minimum Per cent	
1	40	2226-2617	0.638	0.282	44.130	100	44.130	72.060 (14.320)
2	40	2632-2816	0.835	0.282	33.730	100	33.730	67.580 (12.260)
3	40	2820-2928	0.743	0.271	36.410	100	36.410	62.720 (12.980)
4	40	2938-3035	0.723	0.266	36.870	100	36.780	58.618 (14.880)
5	40	3038-3132	0.725	0.244	33.650	100	33.640	56.470 (17.350)
6	40	3134-3277	0.699	0.242	34.660	100	34.650	55.511 (14.740)
7	40	3278-3412	0.763	0.235	30.800	100	30.807	52.110 (14.530)
8	40	3418-3502	0.709	0.307	43.310	100	43.311	62.110 (11.110)
9	40	3504-3646	0.858	0.362	42.240	100	42.230	70.800 (13.440)
10	37	3650-3997	0.929	0.385	41.470	100	41.467	66.980 (12.960)

Figures in parentheses are the standard deviations.

**TABLE A-4**  
 Technical Efficiency Index of 50 Farmers in Programming Model

Farm	$X_2$	$X_3$	$Y^*$	Y	(Y/Y*100)
3	2232.8	2596.9	4829.7	3216.4	66.6
6	1042.0	2425.5	3467.4	3054.4	88.1
8	1042.0	2346.5	3388.5	3238.4	95.6
10	1190.8	2295.2	3486.0	3108.0	89.2
12	1339.7	2433.2	3772.8	3280.0	86.9
17	1339.7	2709.2	4048.9	3025.0	74.7
19	1190.8	1850.6	3041.4	3042.0	100.0
20	1042.0	2275.9	3317.8	3003.5	90.5
22	1339.7	2703.5	4043.1	3165.3	78.3
25	1324.8	3019.6	4344.4	3563.0	82.0
26	1027.9	2295.2	3323.0	3323.0	100.0
36	1339.7	2662.7	4002.4	3367.6	84.1
44	1190.8	2390.8	3581.6	3256.0	90.9
45	1042.0	4067.1	5109.0	3657.6	71.6
46	1190.8	2340.1	3530.9	3073.0	87.0
47	1339.7	2600.1	3939.8	3296.0	83.7
49	1339.7	2538.1	3877.8	3090.0	79.7
75	2977.0	4263.5	7240.5	3749.0	51.8
82	1488.5	3091.2	4579.7	3719.0	82.8
93	1488.5	3053.7	4542.2	3608.8	79.5
96	3721.3	3566.3	7287.6	3860.0	53.0
100	1190.8	3107.3	4298.1	3429.2	79.8
101	3721.3	4299.3	8020.6	3933.2	49.0
117	1488.5	3081.6	4570.1	3633.6	79.5
119	1488.5	3009.5	4498.0	3504.0	77.9

Continued

TABLE A-4  
(Continued)

Farm	X <sub>2</sub>	X <sub>3</sub>	Y*	Y	(Y/Y*100)
124	2232.8	2734.9	4967.7	3535.3	71.2
138	2232.8	3325.6	5558.3	3616.0	65.1
149	2188.1	2031.3	4219.4	2942.8	70.0
159	1570.4	2940.4	4510.7	3186.0	70.6
162	1763.9	2742.3	4605.2	3034.9	67.4
173	1496.0	2256.0	3751.9	2777.2	74.0
175	1295.0	2870.7	4165.7	2976.0	71.4
178	1324.8	2523.1	3847.8	3489.0	90.7
180	1369.4	2596.2	3965.7	3451.0	87.0
183	1302.4	2083.6	3386.1	2904.0	85.8
191	1161.0	2458.9	3619.9	2921.0	80.7
194	1845.7	2988.5	4834.3	3402.4	70.4
265	2002.0	2837.6	4839.7	3465.0	71.6
272	863.3	1796.8	2660.1	2561.0	96.3
279	922.9	2176.4	3099.3	2856.0	92.2
282	1231.1	2231.0	3444.1	2870.2	83.3
291	1228.0	2239.8	3467.8	2484.0	71.6
294	1131.3	2166.8	3298.0	2433.0	73.8
295	1488.5	2218.1	3706.6	2587.0	69.8
307	1101.5	2208.5	3310.0	2672.0	80.7
317	1071.7	2019.1	3090.8	2518.0	81.5
329	1116.4	2115.4	3231.8	2716.0	84.0
339	922.9	2241.2	3164.1	2732.0	86.3
342	1354.5	2648.3	4002.8	2822.4	70.5
343	1510.8	2622.6	4133.4	2816.0	68.1

Mean: 78.7, STD :11.2, Maximum: 100.0, Minimum: 49.0.

TABLE A-5

Frequency Distribution of Technical Efficiency Rating

Efficiency Range	Frequency	Per cent
Below 65	0.00	0.00
66-68	9.00	2.27
69-71	7.00	1.76
72-74	34.00	8.56
75-77	48.00	12.09
78-80	68.00	17.13
81-83	59.00	14.86
84-86	65.00	16.37
87-89	46.00	11.59
90-92	35.00	8.82
93-95	15.00	3.78
96-98	7.00	1.76
99-100	4.00	1.01
Total	397.00	100.00

Mean : 82.32, STD : 7.00, Maximum : 100, Minimum : 66



TABLE A-6

Maximum-Likelihood Estimates of Translog Production Frontier Function

Parameters	Coefficients	t-ratio
$\beta_0$	10.58000	7.026*
$\beta_1$	0.32150	1.420
$\beta_2$	-1.68300	-3.490*
$\beta_3$	0.39150	0.920
$\beta_4$	-0.15470	-0.620
$\beta_5$	-0.00840	-0.030
$\gamma_{11}$	0.02630	0.880
$\gamma_{12}$	0.00083	0.020
$\gamma_{13}$	-0.02167	-0.680
$\gamma_{14}$	-0.03602	-1.390
$\gamma_{15}$	-0.01770	-0.900
$\gamma_{22}$	0.23160	2.530*
$\gamma_{23}$	0.03004	0.520
$\gamma_{24}$	0.01792	0.400
$\gamma_{25}$	0.02549	0.730
$\gamma_{33}$	-0.13434	-1.500
$\gamma_{34}$	0.05526	1.510
$\gamma_{35}$	0.03748	1.110
$\gamma_{44}$	0.01678	0.410
$\gamma_{45}$	-0.03466	-1.220
$\gamma_{55}$	-0.01330	-0.410
$\lambda = \frac{\sigma_u}{\sigma_v}$	0.65830	0.740
$\sigma = \sqrt{\sigma_u^2 + \sigma_v^2}$	0.08560	4.430*
$\sigma_u^2$	0.00222	
$\sigma_v^2$	0.00512	
Log - likelihood	454.81000	

\* = significant at 5 per cent.

**TABLE A-7**

Farm Specific Technical Efficiencies in  
Stochastic Translog Production Frontier

Efficiency Index	No. of Farms	Per cent
100 - 99.01	0	0.00
99 - 98.01	3	0.76
98 - 97.01	104	26.20
97 - 96.01	148	37.28
96 - 95.01	99	24.94
95 - 94.01	29	7.30
94 - 93.01	9	2.27
93 - 92.01	4	1.01
92 - 91.01	1	0.25
91 - 90.01	0	0.00
	397	100.00

Mean: 96.2; S.D.: 1.1; Minimum: 91.2; Maximum: 98.1.

**TABLE A-8**

Estimates of Cobb-Douglas Production Function  
( OLS and Frontier Approach )

Parameters	OLS estimates		Composed error estimates	
	Coefficients	t-Value	Coefficients	t-values
$\alpha$	6.6970	72.950*	6.7470	64.90*
$\beta_1$	0.0379	4.557*	0.0379	4.40*
$\beta_2$	0.0427	3.076*	0.0427	2.81*
$\beta_3$	0.0450	2.939*	0.0450	3.10*
$\beta_4$	0.0754	7.286*	0.0754	6.91*
$\beta_5$	0.0500	6.177*	0.0500	5.47*
$R^2$	0.5500			
$\lambda = \frac{\sigma_u}{\sigma_v}$			0.74515	1.04
$\sigma = \sqrt{\sigma_u^2 + \sigma_v^2}$			0.92144	5.44*

\* = significant at 5 per cent

TABLE A-9

Frequency Distribution of Technical Efficiency

Efficiency Index	Frequency	Per cent
100 - 98.01	0	0.00
98 - 97.01	60	15.11
97 - 96.01	119	29.97
96 - 95.01	97	24.43
95 - 94.01	69	17.38
94 - 93.01	36	9.07
93 - 92.01	6	1.51
92 - 91.01	6	1.51
91 - 90.01	4	1.01
	397	100.00

Mean : 95.6; S.D. : 1.4; Maximum : 97.9; Minimum : 90.5