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MONEY-INFLATION RELATIONSHIP: BAND SPECTRUM ANALYSIS APPROACH

Muhammad REHMAN*

Researchers endorse causal relationship between monetary growth and inflation. However timing as well as magnitude of response of inflation to monetary shock is open to question. Timing and the magnitude has serious monetary policy implications. Considering this the band spectrum regression applying conjugate analysis is used to examine the relationship. This estimation technique permits to analyze the relationship over different time horizons. Band spectral analysis isolates different components of variables. Low frequency components correspond to long term cycles in the variable, while high frequency components correspond to short term regular movements. Estimation results illustrate that money growth is positively and output growth is negatively related to inflation in low frequencies. Results also elucidate that only output gap explains inflation in higher frequencies.

I. Introduction

Inflation is a serious problem in many economies. Economists have done extensive research on causes and cures of inflation but have failed to establish consensus on it. One of the main causes of inflation highlighted in the literature is high money growth. Based on the quantity theory of money, a major part of literature analyzes the relationship between money and inflation. For example, Grauwe and Polan (2005) studied 160 countries for more than 30 years and found strong association between high (hyper) inflation or deflation and the money growth rate. In countries with low and stable inflation, this association was found to be weak. Pakistan has a volatile inflation history[1] and therefore, most of the literature supports the above hypothesis[2] with a strong relationship between the money growth and inflation.

In this paper, a new approach is adopted to study the problem. The spectral analysis is used to isolate different components of money growth. Low frequency components correspond to long term cycles in monetary growth, while high frequency components correspond to the short term regular movements. The relation between inflation and the monetary growth is studied at different frequencies, separately. For policy makers this analysis is very important; e.g., it informs them that the relationship holds only for low frequencies and that they need not change the policies against temporary shocks that fade away within policy target time horizon. In other words, policy makers just need to filter out high frequency money growth rate as a meaningful indicator of inflation.

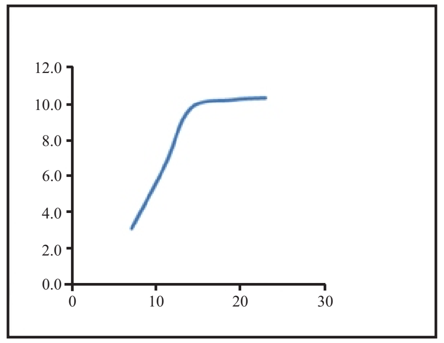
* Joint Director, State Bank of Pakistan, Karachi.

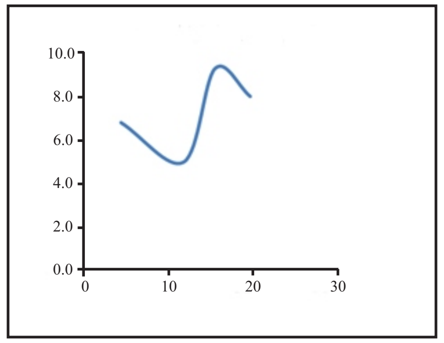
[1] In 1967 annual inflation rate was -4.85 per cent and in 1974 it reached to 30 per cent.

[2] Qayyum (2006), and Kemal (2006).

This study follows Assenmacher-Wesche and Gerlach's (2006) work who investigate the relationship between money growth and inflation over different band of frequencies for Japan and Euro area. They use the band spectral analysis technique and demonstrate that the relationship between money growth and inflation holds only for low frequencies; for high frequencies output gap causes inflation. The present study divides annual data from 1969 to 2008 into six bands of money growth [(0 50), (6 10), (11 15), (16 20), (21 25) and (26-above)] to develop a more precise relationship between the low/high frequency inflation and low/high frequency money growth. For each band of money growth, average lead of low/high frequency inflation is calculated.

Figure 1(a) shows an average lead of low frequency inflation against the band of low frequency money growth from zero to 25 in the period extending from1960 to 2008. The initial years in which money growth was in particular intervals, say (0 5) are identified; than the average of inflation of the years following the year in which the growth rate of money was in that particular range, is calculated. It is clear from Figure 1(a) that money and inflation have strong positive (89 per cent) correlation at low frequencies. Figure1(b) on the other hand shows a weak relationship between these variables at high frequencies. This is consistent with the literature[3] and motivates for further analysis.





Note: x-axis has money growth while y-axis reflect inflation rate

Figure 1(a)
Average Low Frequency Inflations

Figure 1(b)
Average High Frequency Inflations

The remaining paper is arranged as follows. Section II presents a brief review of literature on the money inflation relationship in Pakistan. In Section III, the econometric model and the methodology based on Assenmacher-Wesche and Gerlach (2007) is presented. Econometric issues relating to data are also discussed in this section. Section IV, presents the econometric results of the analysis. Conclusion and relevant policy implications are discussed in Section V.

[3] Assenmacher-Wesche and Gerlach [(2006), (2007)], Grauwe and Polan (2005).

II. Review of the Literature

Hossain (1990) tested the Monetarist vs. New Keynesian visions for high inflation in Pakistan, Sri Lanka, Bangladesh, India, and Nepal. The important finding of this study was that bond-financed government expenditures had no significant impact on the acceleration of inflation, independently. Their results support Monetarist view that bond financing has no real impact on aggregate demand and prices, until the money is held constant. Their study consistently holds up the Monetarist view that growth of real money balances is the sole determinant of long run inflation. The study tested the growth of money balances as an explanatory variable in two different scenarios and found that only in low frequency band, it correspondence one to one with inflation. Ahmad and Ram (1991) also found money growth as an important determinant of inflation in their research. Khan and Schimmelpfennig (2006) determined the drivers of inflation in Pakistan using monthly data They mainly used wheat support price, exchange rate, interest rate, money supply, credit to private sector as a test variable in their model. Their results show that monetary factors play a dominant role in the headline Consumer Price Index (CPI) inflation. The impact of these variables on prices is felt with a one year lag. They showed that growth rate of broad money and private sector credit can be used as leading indicators of inflation in Pakistan. In this study only money as a leading indicator of inflation in low frequency band is tested.

Qayyum (2006) attempted to examine whether the Quantity Theory of Money is applicable in Pakistan. He found that changes in money supply affect the real GDP growth which in turn affects inflation in Pakistan. The significant conclusion from the study is that excess money supply growth is a leading indicator of inflation. Similarly, Kemal's (2006) study found that Quantity Theory of money holds for Pakistan in the long run. However, the impact of money growth is felt after 9 months. A significant result that followed from this study was that the system takes a long time to converge to equilibrium, whenever it is faced with shocks in any of the variables, prices, money supply or GDP. The results of this study show that the Quantity Theory of Money holds for Pakistan in band of low frequency inflation. In low frequency inflation both the assumptions of Quantity Theory of Money – proportionality and neutrality of money hold but in high frequency band (a band of frequencies that needs to complete its cycle in three or more than three years) only neutrality of money holds. Husain and Abbas (2006) stated comparable results according to which, there exist unidirectional causality from money to prices in the long run and no causality in the short run.

III. Econometric Model and Methodology

1. The Empirical Model

Numerous researchers in Pakistan, have investigated the relationship between money and inflation and concluded that inflation is a monetary phenomenon. If inflation is indeed "a monetary phenomenon" then it can best be described by quantity theory of money. But, according to Bordo and Filardo (2005), a study conducted for number of countries to examine inflation experiences, concluded that money growth can only explain high inflation. However, in periods of low inflation money does not contain useful information for predicting the behaviour of inflation. Gerlach (2004) investigated this phenomenon for different countries and concluded that low frequency component of inflation is explained by the quantity theory of money[4] and high inflation component is explained by output gap.[5] This study follows Gerlach (2004) approach, the inflation could be decomposed into low and high frequencies. i.e.

$$\pi_t = \pi_t^l + \pi_t^h, \qquad (1)$$

It is ascertained that, the high-frequency movements (short run moments) of inflation are related to positive movements in the output gap, g_t

$$\pi_t^h = \beta_g g_{t-1} + \vartheta_t^h; \qquad \beta_g > 0 \qquad (2)$$

Where as the low-frequency variation (long run moments) of inflation can be understood in terms of quantity theory of money (QTM) i.e.

The theory states that increase in money growth leads to equal changes in inflation rate through the Fischer Effect. It assumes that velocity and output in the long run (low frequency) remain the same. Therefore;

$$\pi_t^l = \beta_m g_{mt-1}^l + \beta_y g_{t-1}^l + \vartheta_t^l \qquad (3)$$

Equation (3) establishes a relationship between inflation and its determinants in long run. The variables g_{mt} and g_{yt} show growth of money and income at time *t* whereas superscripts *l* and *h* are used for low and high frequencies respectively. QTM predicts that $\beta_m = 1$, $\beta_y < 0$ (proportionality) and g_{mt} and g_{yt} are uncorrelated (orthogonality).

By adding equation (2) and equation (3), we get;

$$\pi_t^l + \pi_t^h = \beta_m g_{mt-1}^{\ \ l} + \beta_y\, g_{yt-1}^{\ \ l} + \beta_g g_{t-1} + \vartheta_t^l + \vartheta_t^h \qquad (4)$$

From equation (1) our complete model could be defined as:

$$\pi_t^l = \pi_t^l + \pi_t^h = \beta_m g_{mt-1}^{\ \ l} + \beta_y\, g_{yt-1}^{\ \ l} + \beta_g g_{t-1} + \vartheta_t \qquad (5)$$

Equation (5) is estimated by Band Spectrum Regression. This technique is appropriate in the analysis because the inflation determinants on different band of frequencies are analyzed.

[4]Since independent data series for velocity was not available therefore velocity in error term was included.

[5] Shahid H. Kardar, Governor State Bank of Pakistan, also emphasis out-put gap as a determinant of inflation [Bank News, January (2011)].

[6] Definition of coherence from Wikipedia.

2. *Methodology*

The Band spectrum regression is briefly discussed in this section. Spectral method, based on the Fourier Representation, examines time series in the frequency domain and converts series from time domain to frequency domain. Finite Fourier transform -$F_x(\omega)$, of a covariance-stationary time series X_t ,can be calculated for each frequency ω, as under:

$$F_x(\omega) = \sum_{t=0}^{N-1} X_{te}^{-it\omega} \qquad (6)$$

From the Fourier representation, the spectral density function can be calculated which is used to examine the univariate characteristics of the time series in the frequency domain. The spectral density function $f_x(\omega)$ can be estimated by averaging m ordinates of the periodogram $|f_x(\omega)|^2$. From spectral density coherence can be calculated which shows the strength of association between the two-series where the possible dependence between the two series is not limited to simultaneous values but may include leading, lagged and smoothed relationships.[6] Square coherence can be defined as:

$$C_{xy} = \frac{\left| f_{xy}(\omega)^2 \right|}{f_x f_y} \qquad (7)$$

Here, f_x and f_y represents spectrum densities of covariance stationary series X_t and Y_t, respectively and $f_{xy}(\omega)$ represent cross-spectrum-average of m ordinates of $|F_x(\omega)\ F_y(\omega)|^2$. Just like the correlation coefficient the value of squared coherence varies between zero and one. This measure has few limitations. It is not well defined for more than two variables. Also, it does not capture dynamic structure of time series. To overcome these problems Engle (1974) developed Band Spectrum Regression. In this analysis all the covariance stationary time series (dependent and independent) are first converted to frequency domain by Fourier transform. Specific frequencies by harmonic analysis is then removed. Harmonic analysis is a type of regression analysis in which the following equation is estimated:

$$X_t = \mu + A\cos(\omega t) + B\sin(\omega) + \varepsilon_t \qquad (8)$$

On the basis of F statistics it can be concluded whether the coefficients are collectively significant or not for each of the frequency ω. After this the band of frequencies can be made. If $\tilde{X}$,$\tilde{Y}$ represent the transform vector of series of dependent and independent variables of X and Y respectively, then the *β* of the regression:

$$Y = \beta X + \varepsilon$$

can also be consistently estimated in frequency domain as:

[6] Definition of coherence from Wikipedia.

$$\hat{\beta}=\left[\sum_{t=0}^{T-1} f_{xx}(\omega_t)\right]^{-1}\sum_{t=0}^{T-1} f_{xy}(\omega_t) \qquad (9)$$

where, T represents the sample size. This regression allows to test hypothesis on different bands of frequencies. The filtered data is reconverted to time series by inverse Fourier transformation for further analysis.

3. *Inflation, Money, and Output Growth*

Evaluating/estimating the out-put gap is a controversial and difficult task for economic researchers. It involves judging the potential of economy to produce and supply for the aggregate demand of the economy. Due to these difficulties researchers usually use Hodrick-Prescott (HP)[7] filter for construction of out-put gap. The HP-filtered out-put gap series for analysis is also used.

Figure 2 shows the power spectrum for the stationary variables[8], inflation, money growth and output growth for frequencies less than 0.5 Hz[9]. Different frequencies are calculated by (*t/T*), where T represents the size of the sample and t moves from one to (*T/2*). To calculate the power spectrum the following regression is estimated first:

$$X_t(\omega)=\mu+A\cos(\omega t)+B\sin(\omega_t)+\varepsilon_t \qquad (10)$$

Periodogram is then calculated by $|X_t(\omega)|^2$The most apparent crest for inflation and money growth is approximately 16 year cycle [1/(freq)], whereas, for output growth it is 24 year cycle. The relative power (R2 of the regression) of 16 year component is 25 per cent and 26 per cent for money growth and inflation, respectively; for output growth rate it is 23 per cent. Both inflation and money growth share the same frequencies.

Figure 2 demonstrates that inflation has positive relationship with money growth and negative relationship with output growth within low frequency band.[10] It is apparent from the Figure that money growth leads inflation in low frequencies.

IV. Results and Interpretations

The model is estimated using band spectrum regression which was initially developed by Engle (1974). In present analysis, the data is first converted from time domain into frequency domain and then specific components are eliminated. By conjugate analysis significant frequencies were selected and then developed to high and low frequencies bands. This technique is appropriate for present analysis to analyze the inflation determinants on different band of frequencies. In this study the whole data is divided into two groups: high frequency and low frequency bands.

[7] The program of Hp filter is available in Appendix-A. It is a two- sided linear filter used as a mathematical tool for separating cyclic component of series. Detail is available on http://en.wikipedia.org/wiki/Hodrick-Prescott_filter.

[8] Spectrum analysis needs stationarity. We applied different test to check unit root in inflation, money growth, and out-put gap series. These variables appear to be stationary.

[9] In Figure 2, solid line shows output growth, broken line shows growth of money and dots are for inflation. In x-axis there are frequencies and along Y axis there are mean sum of square of regressions for each frequency.

[10] Following Assenmacher-Wesche and Gerlach (2006) low frequency is defined as having fluctuations with a periodicity of 3 (1/3=0.33 frequency) or more than 3 years.

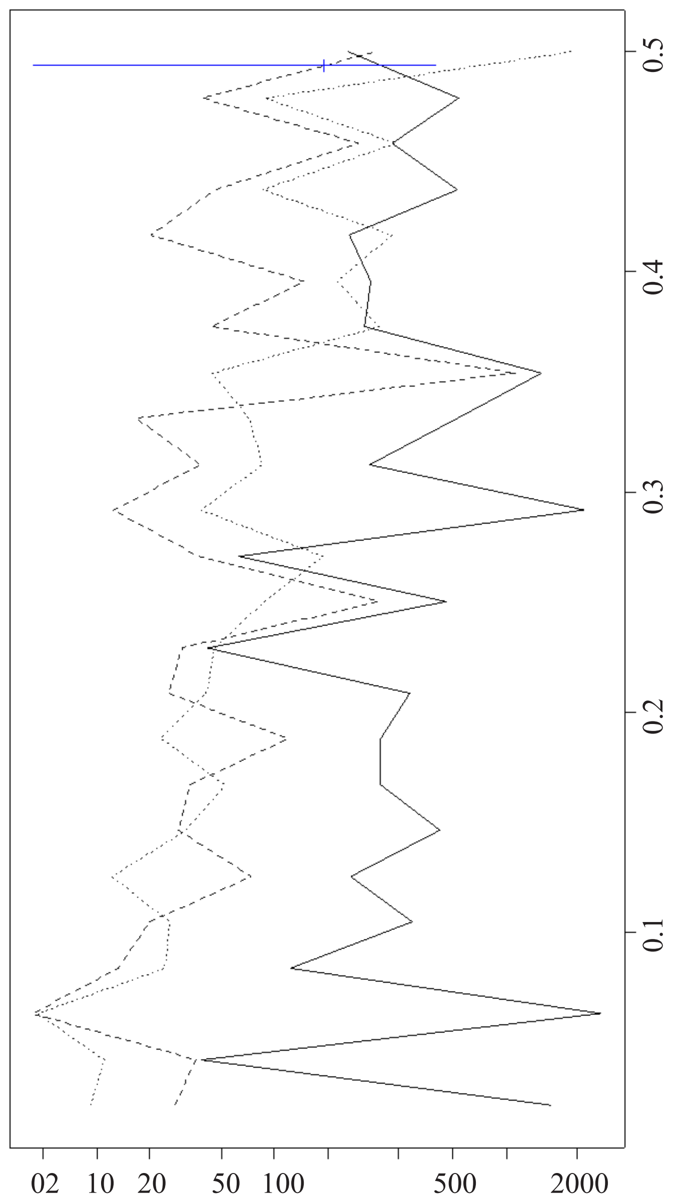


Figure 2
Series: x Raw Periodogram

After defining the bands, the inverse Fourier transform is applied to recover filtered time series data. Following spectral regression was estimated and results are presented in Table 1:

$$\pi_t^i = \sum_{j=1}^{2} \beta_{mj}^i \, g_{mt-j}^i + \sum_{j=1}^{2} \beta_{yj}^i \, g_{yt-j}^i + \sum_{j=1}^{2} \beta_{gj}^i \, g_{yt-j}^i + \sum_{j=1}^{2} (\beta_{\pi j}^i \, \pi_{yt-j}^i), i = H, L$$

Table 1

Results of Spectral Regression

	Low frequency	High frequency	System estimation
Money growth	0.74**	0.39*	0.68**
Output growth	-1.32**	-1.01	-1.38**
Output gap	0.20**	0.23**	0.27**
$\bar{R}^2$	0.45	0.23	0.52

Author's estimation. *, ** Significant at the 10% and 5% levels of significance, respectively.

Results reported in Table 1, clearly indicate varying behaviour of the key variables along different bands of frequencies. Here, money and output growth explain inflation at low frequencies. while at high frequencies only output gap has a significant effect. In low frequencies money has positive while output has negative coorrelation with inflation which is consistent with theory. At high frequency, money growth and output growth are statistically insignificant at one per cent significance level.

For robustness analysis[11] the inflation equation was also estimated through ARDL approach. The results are presented in Table 1 (column 3), under the caption of System Estimation. They match with low frequency band and confirm the results.

V. Conclusion and Policy Implication

Results from the band spectral regressions analysis indicate strong positive relationship between the money growth and inflation in the low frequency band. At this frequency band, one per cent change in money growth brings almost proportional change in inflation. Furthermore, the regression results illustrate that output growth is inversely correlated with inflation at low frequency, particularly when it is defined as frequencies of three or more years. Here, one percent change in output

[11] For robustness analysis Assenmacher-Wesche and Gerlach (2006) approached is followed.

growth depresses inflation proportionately. In the high frequency band the quantity-theoretic variables appear to be of little significance for inflation. In low frequency bands both money and output growth explain inflation while at high frequency band output gap explains inflation.

The important long run policy implication is that, if money is controllable, policy makers should target money growth to cure inflation in Pakistan.

State Bank of Pakistan,
Karachi.

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APPENDIX-A

Program for the calculation of HP filter

```
Y = [],
W = [100],
[m, n] = size(y), if m < n.
y = y', m = n,

end

s = repmat ([w -4*w ((6*w+1)/2}], m, 1);
s(1,2) = -2*w; s(m-1,2) = -2*w;
s(1,3) = (1+w)/2; s(m, 3) = (1+w)/2;
s(2,3) = (5*w+1)/2; s(m-1, 3) = (5*w+1)/2;
M = spdiags(s,-2:0, m, m);
M = M+M';
L = M\y;
L.
```