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REVISITING THE RELATIONSHIP BETWEEN β AND σ CONVERGENCE

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The paper questions the widely held assumptions about the relationship between absolute β-convergence and σ-convergence in the existing literature which states that β-convergence is a necessary condition for σ-convergence. The study utilizes the data of 136 countries for the period 1950 to 2008. The non-linear least squares estimations of the β-convergence regression, using both the cross-section and panel data, have confirmed no absolute convergence for the world sample. The σ-convergence has been studied using its two widely used measures, namely, coefficient of variation of income and standard deviation of log income. The results reveal that there is an evidence of σ-convergence using the former measure but not using the latter measure. These findings on β and σ convergence asserts that the relationship between β and σ convergence depends on the particular measure of σ-convergence being used, and hence, cannot be generalized. Specifically, contrary to the theoretical view of relationship between β and σ convergence, there is an evidence of σ-convergence without prevalence of β-convergence among the world countries.

I. Introduction

The convergence hypothesis is the core of a great deal of research work on economic growth which has now been debated in the literature, for more than half a century. Indebted to the Neo-Classical growth model (NGM) for its theoretical naissance, the debate on concept of convergence is characterized with variously diverse empirical findings which are based on a wide range of econometric methodologies. The initial studies on the topic were independent of the framework of NGM. However, Barro and Sala-i-Martin (1990) derived the convergence equation from the NGM and the catching-up hypothesis was named as β-convergence which, due to the assumption of cross-country constant steady states, is referred to as 'the absolute or unconditional β'. The absolute or unconditional β entails a negative relationship between the initial income per capita and the subsequent growth, assuming constant steady states across the cross-sectional units. The regression analysis pertaining to the unconditional β-convergence was criticized by Friedman (1992). He opposed the cross-sectional estimations of the convergence equation because it is based on an average for the entire time-period. The conclusion of income convergence based on the latter was referred as the 'regression fallacy', instead of being an indication

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for the poor becoming the rich. As an alternative to the β-convergence, Friedman (1992) has suggested the temporal study of cross-sectional dispersion of income which is more plausible. However, even prior to Friedman's recommendation, the trend in the cross-country income dispersion was analyzed in the convergence empirics together with the β-convergence. Moreover, the equation for σ-convergence, as this is called, was derived from that of the β-convergence, the latter is rendered as a necessity but does not prevail sufficient conditions for the former [Barro and Sala-i-Martin (1990), (1992), (2004)].

The preliminary empirical findings on catching up the hypothesis have validated the income convergence among industrialized countries except the income divergence among poor countries [Abramovitz (1986), Baumol (1986), Delong (1988), Baumol and Wolf (1988), and Dowrick and Nguyen (1989)]. These initial empirics on the convergence hypothesis were followed by a comprehensive analysis of β and σ convergence by Barro and Sala-i-Martin [(1990), (1992)] in their research work of varying breadth and depth. The absolute β-convergence was verified among samples of the OECD, USA and European countries, but the world as a whole had shown the absolute divergence.

The analysis on the σ-convergence began to be parallel in time with that of the β-convergence. Baumol (1986) and Abramovitz (1986), along with concluding the primary evidence for the β-convergence, have also corroborated strong σ-convergence among the industrialized countries. Subsequently, important contributions were made by Barro and Sala-i-Martin (1990); concluding σ-convergence among OECD countries and USA states but not across the world sample. Many of the other aforementioned studies on the β-convergence have also analyzed the σ-convergence and have concluded indifferent results. The frequency of intra-country σ-convergence evidence is higher than that of the cross-country; though the latter also prevails within the small country groups; such as, Dobson and Ramlogan (2002) reported that σ-convergence for Latin American Countries, but Miller and Upadhyay (2002) concluded σ-divergence for the world countries. Furthermore, Dawson and Sen (2007) and Romero-Avila (2009) also reported evidence of σ-convergence in their studies comprising samples of 29 world countries and 19 OECD countries.

Though there is a vast literature on both these types of convergence, the paper revisits the relationship between absolute β and σ convergence, keeping in perspective some shortcomings of the existing literature. As the existing literature on their relationship emphasized that β-convergence is a necessary condition for σ-convergence and therefore, σ-convergence cannot exist without the β-convergence. This paper revisits this issue, by estimating the absolute β-convergence and σ-convergence for a sample of 136 countries. Evidence for existence of σ-convergence in absence of β-convergence are presented. Thus, the estimated results contradict the commonly considered relationship between β and σ convergence. Furthermore, it

is argued that β-convergence is only related to a specific measure of σ-convergence, but not more generally.

The paper proceeds with a brief review of literature on relationship between β and σ convergence in Section II. This is followed by the results and discussions in Section III, while Section IV discusses the restricted nature of relationship between β and σ convergence. Finally, Section V concludes the paper.

II. Relationship between β and σ Convergence in the Existing Literature

The convergence empirics in the last twenty-five years have broadly discussed the two popular notions, namely β and σ convergence. The former, in its absolute form, entails a negative relationship between the initial income per capita and the subsequent growth, assuming constant steady states across the cross-sectional units, whereas, a decreasing trend in income dispersion defines the latter. Both the GDP per worker and per capita income are used to investigate, both types of convergence. The seminal work on these two concepts is contributed by Barro and Sala-i-Martin in a series of studies (discussed above). Based on the neo-classical growth model (NGM) the authors derived the following regression equation for an absolute β-convergence:

$$ln\left(\frac{\hat{y}_{it}}{\hat{y}_{it-1}}\right) = a - [1 - e^{-\beta}]\, ln\,(\hat{y}_{it-1}) + u_{it} \qquad (1)$$

where $\hat{y}_{it}$ is the income per capita variable and β-convergence implies $\beta > 0$. In Equation (1) above, the negative signs attached with β, the exponent term and the coefficient $[1 - e^{-\beta}]$ together, are indicative of negative relationship between initial income and the subsequent growth implying convergence among the countries. Given the β-convergence in Equation (1), the following σ-convergence equation is derived by Barro and Sala-i-Martin (1990):

$$\sigma_t^2 \cong e^{-2\beta}\, \sigma_{t-1}^2 + \sigma_u^2 \qquad (2)$$

where σ^2 indicates the variation in the log income.

Equation (2) explains the relationship between the two widely discussed types of income convergence namely β and σ. Based on this equation, Barro and Sala-i-Martin (1990) have rendered β-convergence, a necessary pre-requisite for the σ-convergence. In other words, σ-divergence is plausible with β-convergence but not vice versa. Lichtenburg (1994) has endorsed similar relationship between these two concepts of convergence. Utilizing equation (2), and based on the estimates of β

coefficient and R^2 of linear convergence regression, the following F-distributed test-statistic has been developed for σ-convergence hypothesis:

$$\frac{\sigma_1^2}{\sigma_T^2} = \frac{R^2}{(1+\beta)^2} \tag{3}$$

The σ_1^2 and σ_T^2 depicts variances of log-transformed income in the first and last periods, respectively. However owing to the dependency between these two variances, Carree and Klomp (1997) have indicated a bias in the test-statistic and alternatively developed the likelihood ratio and the adjusted ratio of variances test statistics in their following, respective forms:

$$T_1 = (N - 2.5)\, ln \left[1 + \frac{1(\hat{\sigma}_1^2 - \hat{\sigma}_T^2)^2}{4\,(\hat{\sigma}_1^2\,\hat{\sigma}_T^2 - \hat{\sigma}_{1T}^2)} \right] \tag{4}$$

$$T_2 = \left[\frac{\sqrt{N}\,(\hat{\sigma}_1^2 / \hat{\sigma}_T^2 - 1)}{2\sqrt{1 - \hat{\pi}^2}} \right] \tag{5}$$

Lately, Furceri (2005) has analyzed the causality between these two types of convergence in an alternative manner, while utilizing equation (1). Focusing on the β coefficient for a linear regression, Furceri (2005) derived the following:

$$sign\,(\beta) = sign\,[\sigma_{t-1}^2 - \sigma_t^2 + var\,(ln\,(GDP_{t-1}) - ln\,(GDP_t)] \tag{6}$$

In Equation (6), the declining patterns of variance over time (σ convergence) are expected to cause a positive β coefficient (β-convergence). Therefore, in the presence of σ-convergence there must always be β-convergence but the latter does not confirm the existence of the former.

All the derived relationships assert that in the presence of σ-convergence there must always be β-convergence, but the latter does not confirm the existence of the former. Several empirical studies have also endorsed this relationship between β and σ convergence [Sala-i-Martin (1996), Dobson and Ramlogan (2002)]. It is worth noting that all these studies endorsing relationship between the two concepts of convergence have utilized the standard deviation of log income as the basis for measuring σ-convergence.

III. Results and Discussions

A sample of 136 world countries is used to analyze β-convergence of the form in Equation (1). It is based on Maddison's data and covers the period 1950-2008.

The convergence equation is estimated with both the cross-section and panel data by applying the non-linear least square technique with dependent variables real per capita income and the real GDP per working age person.[1] The estimated convergence coefficients are reported in Table 1. The insignificant estimated β coefficients confirm no β-convergence for the world sample with either of the income variables; since β-convergence requires coefficient to be positive and significant.

TABLE 1

Absolute β Convergence
Evidence from the World Sample (1950-2008)

Results/ Variable	GDP per Capita		GDP per Working Age Person	
	Cross-section, n=136	Panel, n=1632	Cross-section	Panel
Estimated β	0.00004	-0.0013	0.0012	0.00005
	(0.02)	(-1.73)	(0.66)	(0.07)

(•) denotes the t-statistics. In the cross-section data estimation. The initial year is 1950 while the last period 'T' for the sample is 2008. The panel estimations involve 12 time-series data points by taking 5 yearly intervals' 'n' indicates the number of total observations which are identical with both variables.
The cross-section data estimations are tested for heteroscedasticity utilizing the White Test and in both cases the White heteroskedasticity-consistent standard errors based t-statistics are reported. Similarly in the panel regressions, following the tests for heteroskedasticity and autocorrelation, a suitable form of the panel corrected standard errors (PCSE) is used instead of the original ones.

We now turn to σ-convergence for identical sample of the world countries and use the variable of GDP per capita.[2] In the earlier literature, the coefficient of variation of absolute income (c) and the standard deviation of log income (s) have both been used to study σ-convergence. For example, Sala-i-Martin (1996) and, Dobson and Ramlogan (2002) used s, while, Ferreira (2000) and, Dawson and Sen (2007) have undertaken two recent studies, which used c. Based on these, the graphical representation of income dispersion is the most commonly employed method for analyzing the issue. The graphical depiction of dispersion for the world sample is with both c, and s is given in Figure 1.[3] As far as the inferential statistics for the topic is concerned, not only the regression analysis on the subject is rare but ma-

[1] Because of the non-availability of longer datasets on employed labor, the variable of GDP per worker is constructed, using the data of working age population.

[2] Owing to a similar tendency of dispersion for the two income variables and because of a few initial missing observations for working age population, the graph and regression estimations for σ-convergence are confined to the variable of per capita income only.

[3] Plotting the over time trend of any of the dispersion measure, is the most commonly used method for analyzing the σ-convergence in the existing literature.

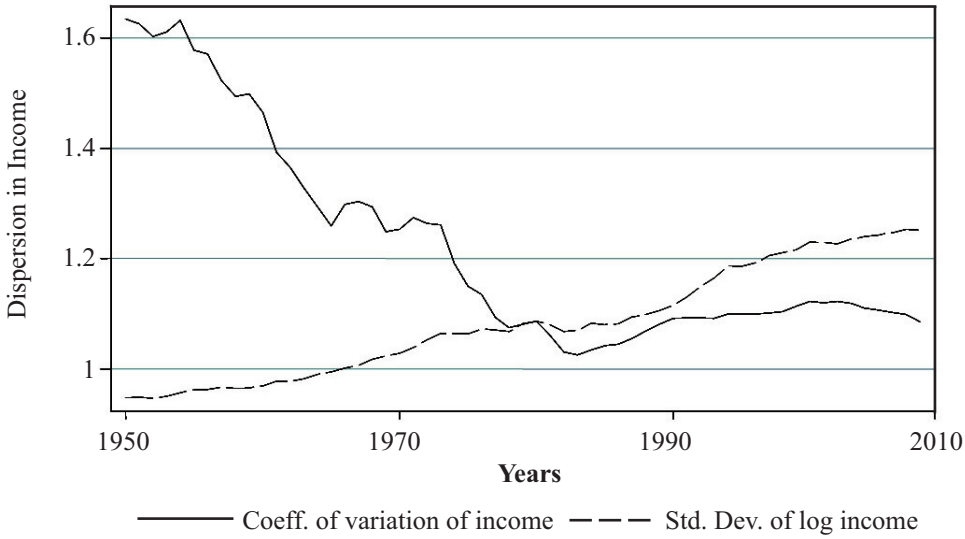

FIGURE 1

Income Dispersion in the World Countries

jority of the existing test statistics of the *σ*-convergence are based on the *β*-convergence estimations. However, the possibility of difference between trends of these two measures of *σ*-convergence and their relationship with *β*-convergence may necessitate a statistical test independent of the *β*-convergence regression. This paper utilizes regression analysis for the test of *σ*-convergence hypothesis. Thus, to evaluate the *σ*-convergence hypothesis, the logarithmic and linear trend equations of the following forms are estimated for *c* and *s*, respectively:

$$\sigma_t^c = \alpha_0 + \alpha_1 \ln t + \varepsilon_t \quad (7)$$

$$\sigma_t^s = \delta_0 + \delta_1 t + \varepsilon_t \quad (8)$$

where, σ_t^c and σ_t^s denote *c*, and *s* based income dispersion respectively, where *t* is a time trend. Using the ordinary least square method, Equations (7) and (8) are estimated - the results of which are reported in Table 2. Figure 1 suggest *c* an overall declining tendency while *s* is increasing. Consistent with this, the negative and significant trend coefficient for *c* (in Table 2) suggests significant *σ*-convergence. The positive and significant coefficient for *s* indicates *σ*-divergence. Therefore, both the traditional graphical analysis and the regression analysis suggest *σ*-convergence in the former case but suggest *σ*-divergence in the latter case.

TABLE 2

σ-Convergence
Evidence from the World GDP per Capita (1950-2008)

	c		R^2	
Results/Variable	$\hat{\alpha}_1$	R^2	$\hat{\hat{\delta}}_1$	R^2
Estimated coefficients	-0.19**	0.86**	0.006**	0.96
	(-8.75)		(19.54)	

(•) denotes the t-statistics; The number of observations equals to 59 in both regressions.
** indicates significance at 1% level; Significance for R^2 is based on F-statistics. Estimations are tested for possible regression problems and the above t-statistics are based on the Newey-West (HAC) consistent standard errors. Reported results for *c* and *s* are based on Equations (7) and (8) respectively because of the higher values of R^2 in these particular formulations.

The findings on β-convergence are consistent with earlier literature on the topic. However, a more surprising finding from both, Figure 1 and the regression estimations of σ-convergence, is a completely opposite trend with different measures of dispersion. This is consistent with Dalgaard and Vastrup (2001) and Kang (2004), who have also found the contradictory results of σ-convergence with two different measures, utilizing samples of the world countries and regions within Japan.

An important question arisesing, a here the results relate to the rationales of opposite trends for income dispersion with *c* and *s*, an important question arises in g. Dalgaard and Vastrup (2001) explain it by deriving the slopes of these measures, as a function of the weighted growth rates of individual countries. The slope of *c*, is shown as convex function of $(y_i / \bar{y})$, while the slope of *s* is derived as a concave function of $(y_i / \tilde{y})$ where $\bar{y}$ and $\tilde{y}$ are arithmetic and geometric means of income.

However, the existing literature has not considered any revision to relationship between β and σ convergence in light of varying measurements of the latter. In this context, contribution of the present study is to be questioned about the extensively cited theoretical and empirical relationship between β and σ convergence; and explain its limitations when *c* is used as a measure of σ-convergence.

IV. Restricted Nature of Relationship between β and σ Convergence

A comparison of the σ-convergence results of Table 2 with those of β-convergence reported in Table 1 shows significant σ-convergence in the absence of β-convergence. This invalidates all relationships between the two types of convergence discussed in Section I, which maintained that σ-convergence is implausible without β-convergence. However, the σ-divergence evidence, when using *s*, is compatible with both results of Table 1 and the causal relationship explained in Section I.

The above argument regarding no relationship between β and c based σ-convergence can be explained further using the derived weights for the slopes of c and s as found by Dalgaard and Vastrup (2001). With its weight being a concave function of $(y_i/\tilde{y})$, s gives more importance to the growth rate of lower income countries than the richer ones. This exactly corresponds with the underlying idea of β-convergence, as shown in Equation (1), which is based on the logarithmic function and gives more weight to low values of initial income. However, the opposite holds with convex shaped weight for c with growth of rich countries having a higher relative weight. Hence, s based convergence is consistent with this but not with c based convergence. In a discussion on the measurement of income inequality, Godoy, et al. (2004) stated that s is more responsive to income variations among poor than the rich. Similarly, Atkinson (1970) was of the view that s attaches more weight to redistributive transfers at lower side, while the respective weight for is constant. Hence c, though a measure of income dispersion, may not be the measure of σ-convergence which is related to β-convergence. In other words, the relationship between β and σ convergence cannot be generalized for all measures of income dispersion which has been used in the literature to represent σ-convergence.

V. Conclusions

The paper contradicts the postulation that β-convergence is a necessary condition for the existence of σ-convergence and furnishes an alternative explanation for their relationship. The study maintains that this relationship is valid when the standard deviation of log income is the measure of σ-convergence. However, this is not the case when σ-convergence is measured by c, when it is not related to the β-convergence. All work studies pertaining to the derivation of relationship between the two types of convergence have used s as a measure of the σ-convergence. The empirical work which have used c, have done so on an ad hoc basis, whilst, the two measures of σ-convergence are similar. It has also been argued that this is not a valid approach.

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