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Adaptive Fractional Order $PI^{\lambda}D^{\mu}$ Control for Multi-Configuration Tank Process (MCTP) Toolbox

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Abstract: This paper presents an adaptive fractional order $PI^{\lambda}D^{\mu}$ control for multi-configuration tank process (MCTP) toolbox which aims at demonstrating the problem of reference tracking and cross coupling rejection in multi-input-multi-output system. Moreover, we investigate the cases where the system is in the mode of minimum phase and non-minimum phase configuration. Besides providing theoretical control system analysis and design, we develop the multi-configuration tank process software toolbox for providing the non-linear functions of dynamic models of multi-configuration tank process which is the advantage tool for investigating the performances of the controllers. The software toolbox is developed from discrete state P-file S-function which is operated within the MATLAB environment for providing many non-linear functions of dynamic models of multi-configuration tank process such as multi-input multi-output quadruple tank full-interacting process, multi-input multi-output quadruple tank process, multi-input single-output triple tank interacting process, multi-input multi-output coupled tank interacting process. All actual process attributes are encapsulated in S-function as the input parameters, thus the multi-tank function block can be simply adjusted by specifying the physical properties of the tank system. The study explains about the mathematical model of multi-configuration tank process, nonlinear dynamic characteristic, minimum phase and non-minimum phase configuration, software toolbox features and also describes the design of the adaptive fractional order $PI^{\lambda}D^{\mu}$ controller including the performance validation. The results have been illustrated that the proposed controller design scheme can provide the sufficient effectiveness in the performance, stability and robustness. Furthermore, these tests reinforce the usefulness of MCTP toolbox as a complete simulation tool for users to perform an engineering research of multi-configuration tank control system analysis and design, moreover, it contains very useful for validating the control algorithm of multi-input-multi-output system.

Keywords: Multi-configuration tank process, Multi-variable physical system, Control system analysis & design, Adaptive fractional order $PI^{\lambda}D^{\mu}$ control, Software toolbox.

1. Introduction

The multi-tank system has been widely used in engineering research for validating various concepts of multi-input-multi-output control system design. A well-known configuration as a quadruple-tank process was originally developed by Johansson in 1999 [1]. Due to its dynamic behavior includes non-linearity, inverse response, process interaction, zero dynamics, parameter perturbation, the various theoretical control approaches have been implemented and proven in their performance. Several studies, for example a modified PI controller design with decoupling control was developed for reducing the interaction influence between 2 loops of quadruple-tank process [2]. Multi-input multi-output (MIMO) quantitative feedback (QFT) compensator and pre-filter transfer function matrices (TFMs) were proposed to illustrate the benefits of interaction treating for the quadruple-tank process [3]. Multi-controller switching control mechanism was applied with a tank level process for illustrating the switching strategy in nonlinear control problem [4]. A stable adaptive sliding mode based on tracking control was applied to quadruple-tank process for demonstrating adaptive control performance in a multi-input multi-output (MIMO) system with external disturbances [5]. Distributed multi-parametric model predictive controller design was implemented in a quadruple tank process [6] A multi-loop centralized fractional order PID controller was designed and implemented in a quadruple-tank process for validating the control performance in nonlinear minimum phase, interacting process under servo and regulatory conditions [7]. A model reference adaptive control (MRAC) using the LabVIEW program was proposed for exhibiting auto tuned controller technique for controlling interaction influence of multivariable quadruple-tank process [8]. An adaptive fuzzy compensator was applied to coupled-tank liquid level control system describing a non-linear and adaptive control action [9]. Fuzzy aggregation based multiple models explicit multi parametric model predictive control was implemented in a quadruple tank process for validating the performance of the proposed control strategy [10]. In our previous work, the PID controller design using a characteristic ratio assignment (CRA), decoupling, and two degrees of freedom (2DOF) technique were implemented in a couple-tank process for performance validation of robustness, overshoot, and settling time improvement [11 - 13]. The inverted decoupling technique was implemented in modified quadruple-tank process for illustrating performance to stabilize an unstable system in case of zero in the right of half plane system [14]. A fuzzy PID controller based on ARM7TDMI was implemented in the couple-tank process for illustrating the control performance of a non-linear fuzzy inference system comparing with conventional PID control [15]. Specifically, the several mathematical models of modified quadruple-tank process were explained clearly as well as providing the evaluation of two degrees of freedom PID control algorithm on multi-input-multi-output system [16]. Therefore, it is recognizable that numerous advantage of knowledge can be illustrated and investigated by multi-tank system.

Through the aforementioned aspects, this paper presents an adaptive fractional order $PI^{\lambda}D^{\mu}$ control for multi-configuration tank process (MCTP) toolbox which aims at demonstrating the problem of reference tracking and cross coupling rejection in multi-input-multi-output system as well as improving the limitation of the previous work, the design of PI controller using model reference adaptive control (MRAC) techniques for coupled-tanks process, [17] which has a drawback to the

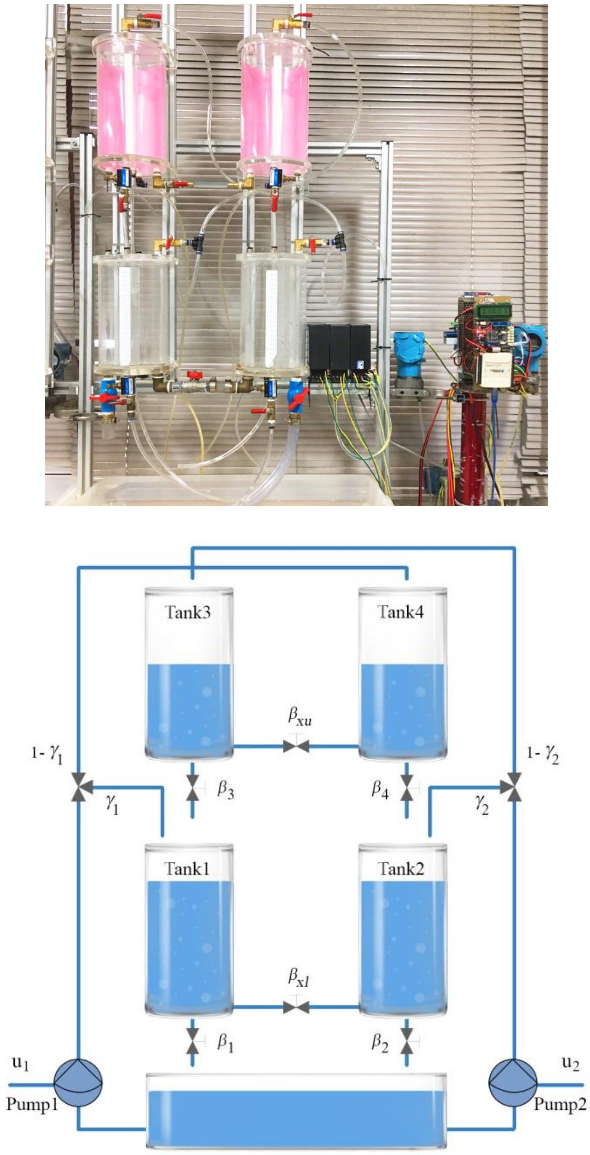

Figure. 1 The multi-configuration tank process: Schematic diagram

response adjustment by using the additional fractional operators. Besides providing theoretical control system analysis and design, we develop the multi-configuration tank process software toolbox for providing the non-linear functions of dynamic models of multi-configuration tank process which is the advantage tool for investigating the performances of the controllers.

The essential features advanced the previous studies [16] are that the software toolbox is developed from discrete state P-file S-function which offers more flexible implementation of simulation and improvement simulation times. Likewise, it is able to provide a complete non-linear function set of the dynamic models of multi-configuration tank process. In addition, all actual process attributes are encapsulated in S-function as the input parameters, thus the multi-tank function block can be simply adjusted by specifying the physical parameters of the tank system as well as it supports flexibly the variety of process characteristics such as inter-acting process, input-output cross coupling, minimum phase, non-minimum phase.

The study explains about the mathematical model of multi-configuration tank process, nonlinear dynamic characteristic, minimum phase and non-minimum phase configuration, software toolbox features and also describes the design of the adaptive fractional order $PI^{\lambda}D^{\mu}$ controller including the performance validation compared to the adaptive integer order PID control. The rest of this paper is organized as follows: The proposed multi-configuration tank process with its physical structure, mathematical modeling and nonlinear dynamic characteristic are described in the section 2; The details of software toolbox features are demonstrated in the section 3; The theoretical control system design using adaptive fractional order $PI^{\lambda}D^{\mu}$ control is explained in the section 4. The experimental results and discussions are illustrated in section 5; while the section 6 is dedicated to the conclusions.

2. Mathematical Modeling

The multi-configuration tank process structure and its schematic diagram are depicted in Fig. 1. The flexible process structure composes of four 1192.5 cm^3 of cylindrical tanks, two API instrument gear pumps, four pressure transmitters, 32,000 cm^3 of water reservoir, four outlet valves and two interacting valves. The two Y-junctions are used for dividing the inlet water through the lower tanks and the upper tanks in the diagonal direction. The process structure can be flexibly adjusted by closing or opening the two Y-junctions, the four outlet valves (β_i) and the two interacting valves (β_{xu}, β_{xl}), for example the quadruple–tank non-interacting process can be created by closing the two interacting valves (β_{xu}, β_{xl}), then opening the four outlet valves (β_i). The process characteristics can be adjusted conveniently by changing the percent valves opening, for example the inter-acting of water between lower tanks can be created by increasing the percent valves opening of lower interacting valves (β_{xl}). Furthermore, the process can be adjusted to operate in non-minimum phase, zeros transition in the right half plane, by decreasing the percent valves opening of two Y-junctions (γ_1, γ_2) so that creating the input-output cross coupling action. The target is to control the level in specified tanks with two pumps. The process inputs are pumps input voltages (u_1, u_2) and the outputs are water level in the tanks (h_i). Referring to Mass balances and Bernoulli's law, non-linear plant equations describing about the characteristics of the water in each tank are obtained as

$$\begin{aligned}\frac{dh_1(t)}{dt} = &-\frac{\beta_1 a_1}{A}\sqrt{2gh_1(t)} + \frac{\beta_3 a_3}{A}\sqrt{2gh_3(t)} \\ &-\frac{\beta_{xl} a_x}{A}\operatorname{sgn}\left[h_1(t)-h_2(t)\right]\sqrt{2g\left|h_1(t)-h_2(t)\right|} \\ &+\frac{\gamma_1 k_{p1}}{A}u_1(t)\end{aligned} \tag{1}$$

$$\begin{aligned}\frac{dh_2(t)}{dt} = &-\frac{\beta_2 a_2}{A}\sqrt{2gh_2(t)} + \frac{\beta_4 a_4}{A}\sqrt{2gh_4(t)} \\ &+\frac{\beta_{xl} a_x}{A}\operatorname{sgn}\left[h_1(t)-h_2(t)\right]\sqrt{2g\left|h_1(t)-h_2(t)\right|} \\ &+\frac{\gamma_2 k_{p2}}{A}u_2(t)\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{dh_3(t)}{dt} = &-\frac{\beta_3 a_3}{A}\sqrt{2gh_3(t)} + \frac{(1-\gamma_2)k_{p2}}{A}u_2(t) \\ &-\frac{\beta_{xu} a_x}{A}\operatorname{sgn}\left[h_3(t)-h_4(t)\right]\sqrt{2g\left|h_3(t)-h_4(t)\right|}\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{dh_4(t)}{dt} = &-\frac{\beta_4 a_4}{A}\sqrt{2gh_4(t)} + \frac{(1-\gamma_1)k_{p1}}{A}u_1(t) \\ &+\frac{\beta_{xu} a_x}{A}\operatorname{sgn}\left[h_3(t)-h_4(t)\right]\sqrt{2g\left|h_3(t)-h_4(t)\right|}\end{aligned} \tag{4}$$

A : Cross section area of tank (cm^2)

a_i : Cross section area of the outlet hole (cm^2)

a_x : Cross section area of the connection hole (cm^2)

h_i : Water level (cm)

u_j : Voltage input of pump $(volt)$

β_i : Outlet valve ratio

β_{xl} : Lower connected valve ratio

β_{xu} : Upper connected valve ratio

γ_j : Inlet valve ratio

k_{pi} : Gain of pump $(cm^3/volt/\text{sec})$

g : Specific gravity $(981\, cm/s^2)$

The transfer matrix $G(s)$ of multi-configuration tank process in Eq. (5) is the linearized dynamic model of multi-configuration tank process expressed the relations between input pump voltage (U_1, U_2) and output water level (H_1, H_2) which is used for designing the controller.

$$\boldsymbol{G}(s)=\begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \tag{5}$$

$$g_{11}(s)=\frac{H_1(s)}{U_1(s)}$$
$$=\frac{\left[\dfrac{T_1T_4(1-\gamma_1)k_{p1}}{T_3A}\right]+T_{3XU}\dfrac{T_1T_2T_{xu}(1-\gamma_1)k_{p1}}{T_{xl}T_3AT_{2XL}}}{T_{4XU}\left[\dfrac{T_{xu}}{T_3}T_{3XU}-\dfrac{T_4}{T_{xu}}\right]\left[T_{1XL}-\left(\dfrac{T_1T_2/T^2{}_{xl}}{T_{2XL}}\right)\right]}+ \tag{6}$$
$$\frac{+T_{4XU}\left[\dfrac{T_{xu}}{T_3}T_{3XU}-\dfrac{T_4}{T_{xu}}\right]\dfrac{T_1\gamma_1k_1}{A}}{T_{4XU}\left[\dfrac{T_{xu}}{T_3}T_{3XU}-\dfrac{T_4}{T_{xu}}\right]\left[T_{1XL}-\left(\dfrac{T_1T_2/T^2{}_{xl}}{T_{2XL}}\right)\right]}$$

$$g_{12}(s)=\frac{H_1(s)}{U_2(s)}$$
$$=\frac{\left[T_{2XL}\dfrac{T_{xl}(1-\gamma_2)k_{p2}}{A}\right]+\left[\dfrac{T_2T_3(1-\gamma_2)k_{p2}}{AT_{xu}T_{4XU}}\right]}{\left[T_{3XU}-\dfrac{T_3T_4}{T_{xu}^2T_{4XU}}\right]\left[T_{2XL}\left(\dfrac{T_{xl}}{T_1}\right)T_{1XL}-\dfrac{T_2}{T_{xl}}\right]}+ \tag{7}$$
$$\frac{\left[T_{3XU}-\dfrac{T_3T_4}{T_{xu}^2T_{4XU}}\right]\dfrac{T_2\gamma_2k_{p2}}{A}}{\left[T_{3XU}-\dfrac{T_3T_4}{T_{xu}^2T_{4XU}}\right]\left[T_{2XL}\left(\dfrac{T_{xl}}{T_1}\right)T_{1XL}-\dfrac{T_2}{T_{xl}}\right]}$$

where

H_1 : Water level in tank1 (cm)

H_2 : Water level in tank2 (cm)

U_1 : Voltage input of pump1 $(volt)$

U_2 : Voltage input of pump2 $(volt)$

$$g_{21}(s)=\frac{H_2(s)}{U_1(s)}$$
$$=\frac{\left[T_{1XL}T_{3XU}T_{xu}T_{xl}(1-\gamma_1)k_{p1}+T_1T_4(1-\gamma_1)k_{p1}\right]/T_3A}{\left[\dfrac{T_{1XL}T_{2XL}T_{xl}}{T_2}-\dfrac{T_1}{T_{xl}}\right]\left[\dfrac{T_{4XU}T_{3XU}T_{xu}}{T_3}-\dfrac{T_4}{T_{xu}}\right]}+ \tag{8}$$
$$\frac{\left[\dfrac{T_{4XU}T_{3XU}T_{xu}}{T_3}-\dfrac{T_4}{T_{xu}}\right]\dfrac{T_1\gamma_1k_1}{A}}{\left[\dfrac{T_{1XL}T_{2XL}T_{xl}}{T_2}-\dfrac{T_1}{T_{xl}}\right]\left[\dfrac{T_{4XU}T_{3XU}T_{xu}}{T_3}-\dfrac{T_4}{T_{xu}}\right]}$$

$$g_{22}(s)=\frac{H_2(s)}{U_2(s)}$$
$$=\frac{\dfrac{T_2T_3(1-\gamma_2)k_{p2}}{AT_{xu}T_{4XU}}+\dfrac{T_1T_2(1-\gamma_2)k_{p2}}{AT_{xl}T_{1XL}}}{\left[T_{3XU}-\dfrac{T_3T_4}{T_{xu}^2T_{4XU}}\right]\left[T_{2xL}-\dfrac{T_1T_2}{T_{1xL}T_{xl}^2}\right]}+ \tag{9}$$
$$\frac{\left[T_{3XU}-\dfrac{T_3T_4}{T_{4XU}T_{xu}^2}\right]\dfrac{T_2\gamma_2k_{p2}}{A}}{\left[T_{3XU}-\dfrac{T_3T_4}{T_{4XU}T_{xu}^2}\right]\left[T_{2xL}-\dfrac{T_1T_2}{T_{1xL}T_{xl}^2}\right]}$$

where $\dfrac{1}{T_i}=\dfrac{\beta_i a_i}{A}\sqrt{\dfrac{g}{2\overline{h_i}}}$,

$$\frac{1}{T_{xu}}=\frac{\beta_{xu}a_x}{A}\sqrt{\frac{g}{2\left|\overline{h_3}-\overline{h_4}\right|}},\quad \frac{1}{T_{xl}}=\frac{\beta_{xl}a_x}{A}\sqrt{\frac{g}{2\left|\overline{h_1}-\overline{h_2}\right|}},$$

$$T_{1XL}=\left[T_1s+1+\frac{T_1}{T_{xl}}\right],\quad T_{2XL}=\left[T_2s+1+\frac{T_2}{T_{xl}}\right],$$

$$T_{3XU}=\left[T_3s+1+\frac{T_3}{T_{xu}}\right],\quad T_{4XU}=\left[T_4s+1+\frac{T_4}{T_{xu}}\right]$$

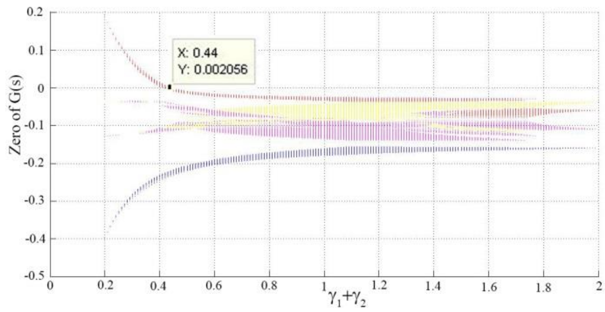

Figure. 2 Zero of $G(s)$ related with γ_1, γ_2

The mathematical model of multi-configuration tank process in Eqs. (6) - (9) were developed in the general form so that they are able to be conveniently transformed to the mathematical model of others tank system as described in [16].The zeros of multi-configuration tank transfer matrix in Eq. (5) are characterized as the zeros of det $G(s)$. As illustrating zeros position mapping in Fig. 2, the zeros of $G(s)$ can be located on the s-plane either in the left or in the right half-plane depending on the values of inlet valve ratio (γ_1, γ_2). If $0.44<\gamma_1+\gamma_2<2$, The system is minimum phase, all multivariable zeros are placed in the left half-plane. If $0<\gamma_1+\gamma_2<0.44$, the system is non-minimum phase, a multivariable zeros are placed in the right half-plane. Moreover, if

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$\gamma_1 + \gamma_2 = 0.44$, a multivariable zeros are located at the origin.

3. Multi configuration tank process (MCTP) toolbox

Multi configuration tank process toolbox is the software toolbox representing the non-linear functions of dynamic models of multi-configuration tank process. The aim of the toolbox is to provide the advantage tool of the multi-tanks process simulation which is suitable for investigating the performances of the multi-input multi-output (MIMO) control system design. In this paper, an overview of the MCTP toolbox and its functions with the used theoretical aspects are described, as well as the example of multi-tank control system is illustrated.

3.1 Toolbox features

The MCTP toolbox in Fig. 3., the essential features are that the software toolbox is developed from discrete state P-file S-function which is operated within the MATLAB environment. Likewise, it is able to provide a complete non-linear function set of the dynamic models of multi-configuration tank process such as multi-input multi-output (MIMO) quadruple tank full-interacting process, multi-input multi-output (MIMO) quadruple tank process, multi-input single-output (MISO) triple tank interacting process, multi-input multi-output (MIMO) coupled tank interacting process. In addition, all actual process attributes are encapsulated in S-function as the input parameters, thus the multi-tank function block can be simply adjusted by specifying the physical parameters of the tank system as well as it supports flexibly the variety of process characteristics such as inter-acting process, input-output cross coupling, minimum phase, non-minimum phase. The MCTP toolbox relies on the MATLAB 2015. The package includes the following 3 files as:

mct_init.m - the initial parameters

MCTP_Toolbox.mdl - Simulink library

model.p – S-function for MCTP toolbox

The function block of a multi-input multi-output (MIMO) quadruple-tank full-interacting process is illustrated in Fig. 4. It provides a dynamic behavior of the level height in four liquid tanks connected with the full-interactions, upper and lower tanks. The input variables are voltages to the pumps, while the outputs are water level in the lower tanks (tank1, tank2). A function block parameters are the process attributes which can be specified the required physical parameters of the tank system. In addition, the process characteristics can be adjusted to operate in non-minimum phase, zeros transition in the right half plane, by decreasing the percent valves opening of two Y-junctions (γ_1, γ_2) so that creating the input-output cross coupling action.

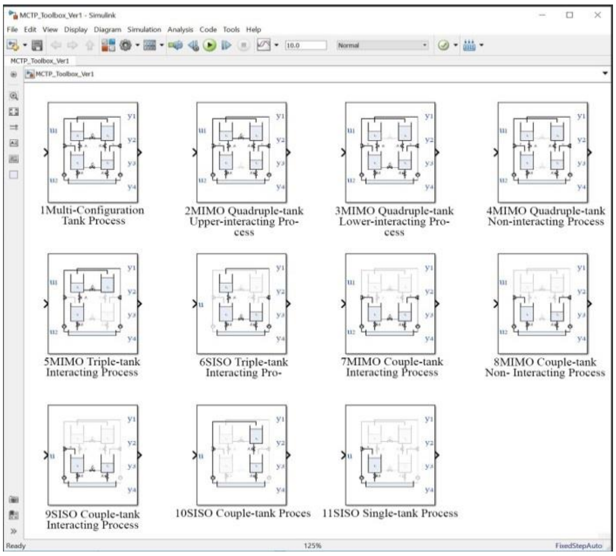

Figure. 3 MCTP Toolbox 1.0

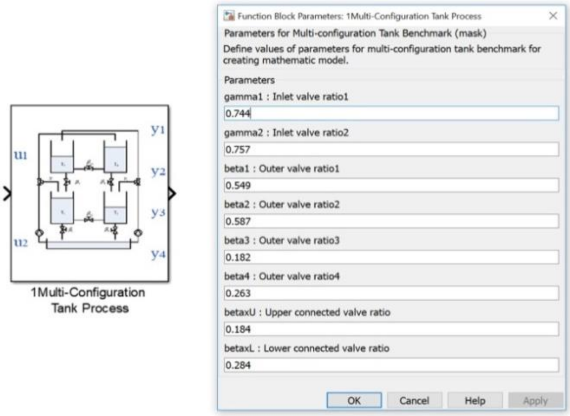

Figure. 4 Block parameters - multi-input multi-output (MIMO) quadruple tank full-interacting process

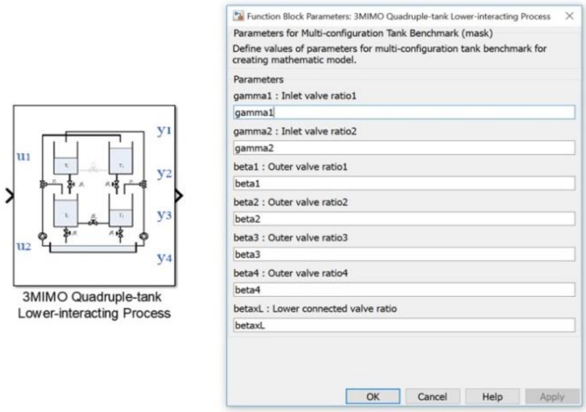

Figure. 5 Block parameters – multi-input multi-output (MIMO) quadruple tank lower-interacting process

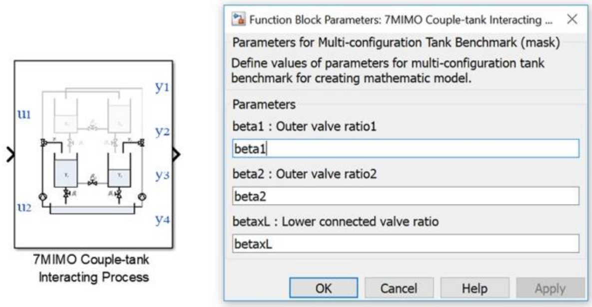

Figure. 6 Block parameters – multi-input multi-output (MIMO) couple tank interacting process

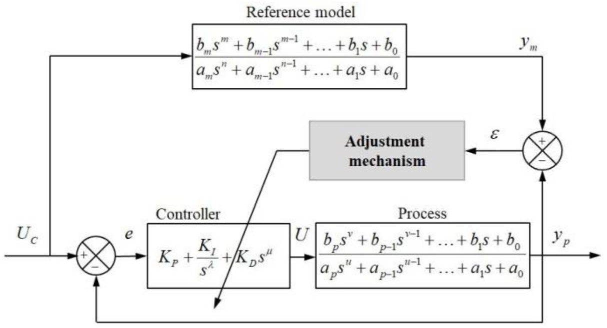

Figure. 7 Block diagram of adaptive fractional order $PI^\lambda D^\mu$ control

The function block of multi-input multi-output (MIMO) quadruple tank lower-interacting process in Fig. 5 provides a dynamic behavior of the level height in four liquid tanks connected with interactions from lower tanks. It was implemented for validating the proposed control strategy of 2DOF PID control in case of zero in the right of half plane system [14]. The process characteristics of function block are useful for evaluating the performance of stability control, especially the dynamic response can be reflected the performance for feed-forward controller effectively.

The function block of multi-input multi-output (MIMO) couple tank interacting process in Fig. 6 provides a dynamic behavior of the level height in two tanks connected with interactions from lower tanks. The input variables are voltages to the pumps, while the outputs are water level in the lower tanks (tank1, tank2). It was implemented for validating the proposed control methodology of characteristic ratio assignment for two-input-two-output (TITO) system [13]. The process characteristics of function block are useful for evaluating the performance of adjusting the shape of dynamic response both of the percent overshoot and settling time.

4. Adaptive fractional order $PI^\lambda D^\mu$ control

In this paper, the dynamic adjustment of the adaptive fractional order $PI^\lambda D^\mu$ controller has been validated to achieve the control performance and robustness for a multi-input multi-output (MIMO) quadruple tank full-interacting process. The structure of control system is depicted in Fig. 7 which composes of a feedback loop of tank process and order $PI^\lambda D^\mu$ controller as well as a feedback loop of the reference adaptive mechanism. The *PID* parameters are changed by the adjustment mechanism based on the value of feedback error, which is the deviation between the output of the system and the output of the reference model. Meanwhile an integrator of order λ and a differentiator of order μ are determined the appropriate values using Integral Square Error (ISE) tuning [18, 19].

4.1 Fractional order PID

The differentiation and integration operators in the fractional calculus are given as [20, 21]

$$ {}_aD_t^q = \begin{cases} \dfrac{d^q}{dt^q} & q > 0 \\ 1 & q = 0 \\ \displaystyle\int_a^t (dt)^{-q} & q < 0 \end{cases} \tag{10} $$

where q is the fractional-order of the operation. a and t are the limits of the operation.

A definition of fractional derivatives given by Grunwald–Letnikov is demonstrated as follows:

$$ \begin{aligned} {}_aD_t^q f(t) &= \frac{d^q f(t)}{d(t-a)^q} \\ &= \lim_{h \to 0} \frac{1}{h^q} \sum_{i=0}^{[(t-a)/h]} (-1)^i \binom{q}{i} f(t - ih) \end{aligned} \tag{11} $$

where D is the mathematical operator. q is the fractional-order of the operation. t and a are limits of the operation. h is the small step size. $\binom{q}{i}$ is the function of $\frac{(q)(q-1)(q-2)\ldots(q-i+1)}{\Gamma(i-1)}$.

The implementation of fractional calculus in *PID* controller design has been widely proposed as well as applied to many process applications due to the aims of increasing a capability of control

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performance using the five parameters $(K_P, K_I, K_D, \lambda, \mu)$. Referring to a block-diagram in a Fig. 8, the transfer function of the fractional-order $PI^{\lambda}D^{\mu}$ controller composed with fractional operators (λ, μ) can be defined as

$$G_{FOPID}(s) = K_p + K_I \frac{1}{s^{\lambda}} + K_D s^{\mu},\ (\lambda, \mu > 0) \quad (12)$$

where G_{FOPID} is the transfer function of the fractional-order. K_P is the proportional gain. K_I is the integral gain. K_D is the derivative gain. λ and μ are fractional operators.

The fractional operators are optimized by minimizing the function of integral square error (ISE) tuning method.

$$J_{ISE} = \int_0^t e^2(t)\,dt \quad (13)$$

where $e(t) = r(t) - y(t)$, $r(t)$ is the reference input, and $y(t)$ is the system response.

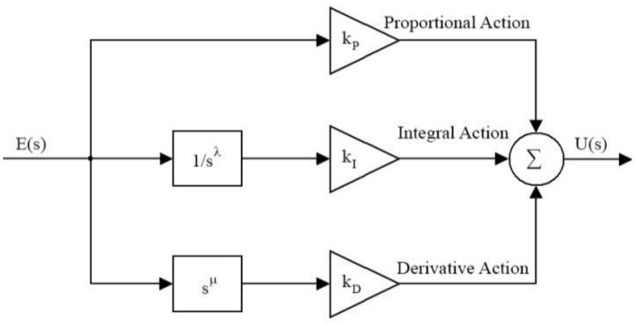

Figure. 8 Block-diagram of the fractional-order $PI^{\lambda}D^{\mu}$ controller

4.2 Model reference adaptive control

The model reference adaptive control (MRAC) is an adaptive servo system using the adjustment mechanism to automatically adjust the controller parameters so that the control signals can manipulate the behavior of process response (y_p) to closely follow the response of the reference model (y_m). From the adjustment mechanism of MRAC, the reference model is determined by the required performance of the control system such as a settling time and a percent of overshoot while the initial parameters of $PI^{\lambda}D^{\mu}$ controller are changed on the basis of the error signal (ε), which is the difference between the output of the process (y_p) and the output of the reference model (y_m). The adjustment mechanism of MRAC system which is developed by MIT adaptive control rule performs the algorithms as following details [22].

The error input signal

$$\varepsilon = y_p - y_m \quad (14)$$

The function of square error

$$J(\theta) = \frac{1}{2}\varepsilon^2(\theta) \quad (15)$$

The changing rate of θ

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma \varepsilon \frac{\partial \varepsilon}{\partial \theta} \quad (16)$$

where θ is the vector of controller parameter. $\frac{\partial \varepsilon}{\partial \theta}$ is sensitivity derivatives of the error with respect to θ. γ is an adaptation gain. ε is the error input signal. y_p is the process response. y_m is the response of the reference model.

The aim of the MIT adaptive control rule is minimizing the function of square error $J(\theta)$ which reflects the capability of controller to track the reference model output signal.

The transfer function of the reference model

$$\frac{Y_m(s)}{U_c(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{a_m s^n + a_{m-1}s^{n-1} + \ldots + a_1 s + a_0} \quad (17)$$

where Y_m is response of the reference model. U_c is the control signal. b_m ~ b_0 are numerator coefficients of transfer function. a_m ~ a_0 are denominator coefficients of transfer function.

The transfer function of the process

$$\frac{N_p(s)}{D_p(s)} = \frac{b_p s^v + b_{p-1}s^{v-1} + \ldots + b_1 s + b_0}{a_p s^u + a_{p-1}s^{u-1} + \ldots + a_1 s + a_0} \quad (18)$$

where $N_p(s)$ is numerator polynomial of the process transfer function. $D_p(s)$ is denominator polynomial of the process transfer function. b_p ~ b_0 are numerator coefficients of the process transfer function. a_p ~ a_0 are denominator coefficients of the process transfer function.

The transfer function of fractional $PI^{\lambda}D^{\mu}$ controller

$$G_c(s) = K_P + \frac{K_I}{s^{\lambda}} + K_D s^{\mu} \quad (19)$$

Apply MIT gradient rules for determining the value of $PI^{\lambda}D^{\mu}$ controller parameters K_p, K_i, K_d . The gains of $PI^{\lambda}D^{\mu}$ controller are computed as

$$K_p = -\gamma_p s \frac{\varepsilon}{s} \frac{N_p(s)}{B(s)} y_p \tag{20}$$

$$K_i = -\gamma_i \frac{\varepsilon}{s} \frac{N_p(s)}{B(s)} \left[U_c - y_p \right] \tag{21}$$

$$K_d = -\gamma_d s^2 \frac{\varepsilon}{s} \frac{N_p(s)}{B(s)} y_p \tag{22}$$

where γ_p is an adaptation gain of the proportional gain (K_P). γ_i is an adaptation gain of the integral gain (K_I). γ_d is an adaptation gain of the derivative gain (K_D). s denotes the differential operator. $1/s$ denotes the integral operator. ε is the error input signal. $N_p(s)$ is numerator polynomial of the process transfer function. $B(s)$ is the denominator polynomial of the reference model. y_p is the process response. U_c is the control signal.

5. Simulation results

This section explains the simulation results obtained from the multi-configuration tank process controlled by the adaptive fractional order $PI^{\lambda}D^{\mu}$ control strategy described in section 4. The tests have been performed for validating the effectiveness of the proposed control methodology in the performance, stability and robustness in case of minimum phase and non-minimum phase process configuration. The target is to control the level in specified tanks with two pumps. The process inputs are pumps input voltages (u_1, u_2) and the outputs are water level in the tanks (h_i). The results are compared with the conventional model reference adaptive control (MRAC). Besides, the using of multi-configuration tank process toolbox is explained and presents through simulations.

5.1 Process Modeling

Referring to the multi-configuration tank process parameters in Table 1, minimum phase configuration, the mathematical model of multi-tank process $(g_{11}, g_{12}, g_{21}, g_{22})$ developed from Eqs. (6) – (9) are demonstrated as.

$$\boldsymbol{G_{process}(s)} = \begin{bmatrix} \boldsymbol{g}_{11}(\boldsymbol{s}) & \boldsymbol{g}_{12}(\boldsymbol{s}) \\ \boldsymbol{g}_{21}(\boldsymbol{s}) & \boldsymbol{g}_{22}(\boldsymbol{s}) \end{bmatrix} \tag{23}$$

$$g_{11}(s) = \frac{\begin{array}{l} 2.29e5s^4 + 6.18e4s^3 + 6.04e3s^2 \\ +2.48e2s + 3.46 \end{array}}{\begin{array}{l} 6.66e6s^5 + 2.33e6s^4 + 3.02e5s^3 \\ +1.77e4s^2 + 4.54e2s + 3.84 \end{array}} \tag{24}$$

$$g_{12}(s) = \frac{1.45e4s^3 + 3.22e3s^2 + 2.23e2s + 4.66}{\begin{array}{l} 8.31e6s^5 + 3.22e6s^4 + 4.67e5s^3 \\ +3.14e4s^2 + 9.62e2s + 10.8 \end{array}} \tag{25}$$

$$g_{21}(s) = \frac{3.38e2s^2 + 47.8s + 1.3}{\begin{array}{l} 1.76e5s^4 + 5.45e4s^3 + 5.71e3s^2 \\ +2.31e2s + 3.1 \end{array}} \tag{26}$$

$$g_{22}(s) = \frac{\begin{array}{l} 1.53e6s^5 + 5.81e5s^4 + 8.67e4s^3 \\ +6.33e3s^2 + 2.25e2s + 3.07 \end{array}}{\begin{array}{l} 4.4e7s^6 + 2.06e7s^5 + 3.84e6s^4 \\ +3.65e5s^3 + 1.84e4s^2 + 4.66e2s \\ +4.58 \end{array}} \tag{27}$$

5.2 Decoupling compensators design

The multi-tank system has controllability limitation from interference between inputs and outputs and interaction on the level between upper tanks and lower tanks. Thus the decoupling compensators have been applied for minimizing cross coupling between two loops of the full interaction system [23, 24].

$$D_1(s) = \frac{g_{21}(s)}{g_{22}(s)},\ D_2(s) = \frac{g_{12}(s)}{g_{11}(s)} \tag{28}$$

Referring to the mathematical model of multi-tank process $(g_{11}, g_{12}, g_{21}, g_{22})$ in Eqs. (24) – (27), the decoupling compensators are determined as

$$\boldsymbol{G_{decoupling}(s)} = \begin{bmatrix} \boldsymbol{1} & \boldsymbol{-D_{12}(s)} \\ \boldsymbol{-D_{21}(s)} & \boldsymbol{1} \end{bmatrix} \tag{29}$$

$$D_{12}(s) = \frac{\begin{array}{l} 2.77e8s^6 + 1.12e8s^5 + 1.78e7s^4 \\ +1.38e6s^3 + 5.53e4s^2 + 1.06e3s + 7.52 \end{array}}{\begin{array}{l} 5.47e9s^7 + 2.68e9s^6 + 5.4e8s^5 + 5.73e7s^4 \\ +3.47e6s^3 + 1.19e5s^2 + 2.14e3s + 15.7 \end{array}} \tag{30}$$

$$D_{21}(s)=\frac{\begin{array}{l}3.18e7s^6+1.43e7s^5+2.54e6s^4\\+2.23e5s^3+1.02e4s^2+2.31e2s+2.04\end{array}}{\begin{array}{l}5.78e8s^7+3.07e8s^6+6.71e7s^5+7.82e6s^4\\+5.22e5s^3+1.99e4s^2+3.98e2s+3.25\end{array}} \quad (31)$$

The compensated process models (g_{11c}, g_{22c}) of multi-tank process becomes

$$\boldsymbol{G_{comp\_process}(s)}=\begin{bmatrix} \boldsymbol{g_{11c}}(s) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{g_{22c}}(s) \end{bmatrix} \quad (32)$$

$$g_{11c}(s)=\frac{\begin{array}{l}1.33e15s^{12}+1.71e15s^{11}+4.58e14s^{10}\\+1.06e14s^9+1.59e13s^8+1.65e12s^7\\+1.2e11s^6+6.27e9s^5+2.31e8s^4\\+5.91e6s^3+1.0e5s^2+1.01e3s+4.66\end{array}}{\begin{array}{l}3.86e16s^{13}+3.71e16s^{12}+1.6e16s^{11}\\+4.14e15s^{10}+7.08e14s^9+8.48e13s^8\\+7.29e12s^7+4.56e11s^6+2.07e10s^5\\+6.71e8s^4+1.51e7s^3+2.23e5s^2\\+1.93e3s+7.39\end{array}} \quad (33)$$

$$g_{22c}(s)=\frac{\begin{array}{l}7.79e14s^{12}+6.86e14s^{11}+2.69e14s^{10}\\+6.19e13s^9+9.33e12s^8+9.67e11s^7\\+7.07e10s^6+3.68e9s^5+1.36e8s^4\\+3.47e6s^3+5.87e4s^2+5.93e2s+2.73\end{array}}{\begin{array}{l}2.24e16s^{13}+2.17e16s^{12}+9.46e15s^{11}\\+2.46e15s^{10}+4.25e14s^9+5.14e13s^8\\+4.48e12s^7+2.84e11s^6+1.31e10s^5\\+4.36e8s^4+1.01e7s^3+1.56e5s^2\\+1.42e3s+5.83\end{array}} \quad (34)$$

5.3 Model reference adaptive control (MRAC) fractional-order $PI^{\lambda}D^{\mu}$ controller design

Owing to the response specifications of the control system performance, which comply by the percent overshoot (P.O.) is less than 7%, the settling time (t_s) is under 3,000 sec, the reference models are designed by poles placement technique [25] as follows

$$\text{Model ref1}=\frac{Y_{m1}(s)}{U_1(s)}=\frac{0.0011}{s^3+s^2+0.101s+0.0011} \quad (35)$$

$$\text{Model ref2}=\frac{Y_{m2}(s)}{U_2(s)}=\frac{0.0013}{s^3+s^2+0.106s+0.0013} \quad (36)$$

Referring to the MIT adaptive control rule described in section 4.2, the adaptation gains (γ) are specified as Eqs. (37) and (38) to minimize the function of square error $J(\theta)$, so that the controllers' gains are obtained as well as enable the capability of input signal tracking effectively.

MRAC adaptation gains1:

$$\gamma_{p1}=-0.10, \gamma_{i1}=-0.0001, \gamma_{d1}=-0.000012 \quad (37)$$

MRAC adaptation gains2:

$$\gamma_{p2}=-0.10, \gamma_{i2}=-0.0001, \gamma_{d2}=-0.000015 \quad (38)$$

The design of the fractional-order $PI^{\lambda}D^{\mu}$ controller is able to perform by the optimization integral square error (ISE) function proposed in [26]. Specifications are as follows. Gain margin is set to 10 dB, and phase margin to 65 degrees.

In Fig.9, It can be seen, that the proper values of fractional parameter (λ, μ) are in the range of $\lambda=[0.90;\ 1.20]$ and $\mu=[0.30;\ 0.80]$ respectively. In this simulation, we selected the most suitable fractional parameters which the values of ISE at 3,234 cm^2 (tank1), 2,320 cm^2 (tank2) as the following controller transfer functions.

$$G_{FOPID\_1}(s)=K_{p1}+K_{i1}\frac{1}{s^{1.18}}+K_{d1}s^{0.65} \quad (39)$$

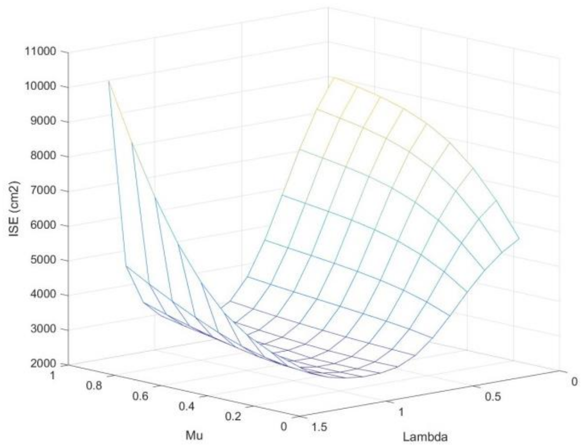

Figure. 9 Fractional order (λ, μ) for ISE minimization

$$G_{FOPID\_2}(s) = K_{p2} + K_{i2}\frac{1}{s^{1.19}} + K_{d2}s^{0.67} \quad (40)$$

5.4 The performance validation for minimum phase

The structure of the proposed control system, model reference adaptive control (MRAC) fractional-order $PI^{\lambda}D^{\mu}$, is illustrated in Fig. 10. The system consists of multi-configuration tank process toolbox, MRAC mechanism, fractional-order $PI^{\lambda}D^{\mu}$ controller and the decoupling compensators. The target is to control the level in tank1 and tank2 to track the step input signal using pumps input voltage signals (u_1, u_2).

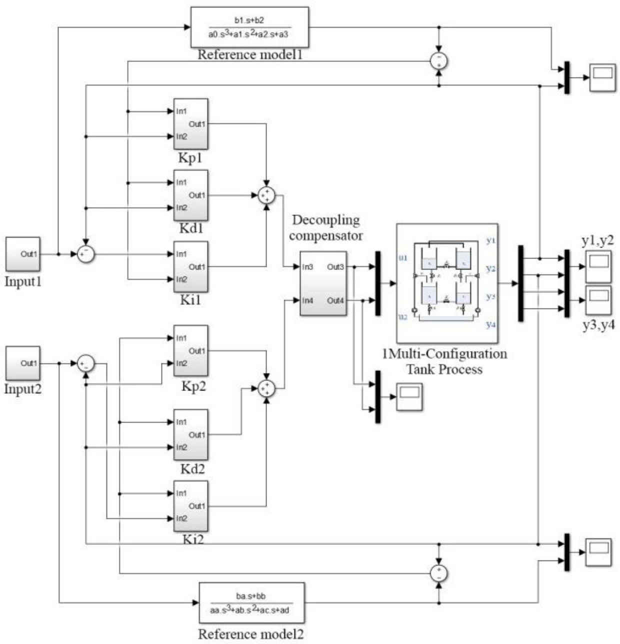

Figure. 10 Simulation of adaptive fractional order $PI^{\lambda}D^{\mu}$ control for multi-configuration tank process toolbox

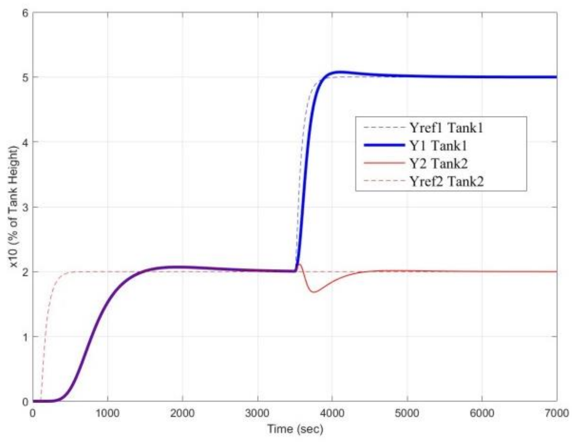

Figure. 11 Step response of multi-configuration tank process simulation: minimum phase

The simulated responses of levels of tank1 and tank2 are shown in Fig. 11. The results show that the reference signals are tracked effectively. The transitions take place approximately at the 100th, and 3500th second. The dash lines represent the outputs of the reference models (y_{m1}, y_{m2}) which are characterized by the transfer functions in Eq. (35), Eq. (36).

As mentioned in section 4, the proposed adaptive fractional order $PI^{\lambda}D^{\mu}$ controller used level outputs (y_1, y_2) and the difference between the reference model outputs (y_{m1}, y_{m2}) and level outputs (y_1, y_2) as the inputs for calculating the controller gain $\left(K_P, K_I, K_D\right)$ adaptively while the fractional-order of the operations (λ, μ) are able to support the control system for increasing the control performance.

The effectiveness of the proposed theoretical method applied to the minimum phase of multi-configuration tank process toolbox, it can be seen that at the first step tracking, the output responses have 3.45 of percent overshoot, settling time at 2,797 seconds for both of y_1 (Blue line) and y_2 (Red line), likewise the values of steady state error are zero. As well as in the second step tracking the output responses have 1.52 of percent overshoot, settling time at 889 second for y_1 (Blue line) and the values of steady state error are zero.

The test of control system without the decoupling compensator is presented in Fig. 12. The results reveal that the designed decoupling compensator has effectiveness of inter-acting rejection. While the water of tank1 raises to the higher set point, many volumes of water are fed to the tank2 causing the huge increments of the water level in the tank2. Therefore, It is proved that the decoupling compensator has been used to perform the inter-acting rejection efficiently.

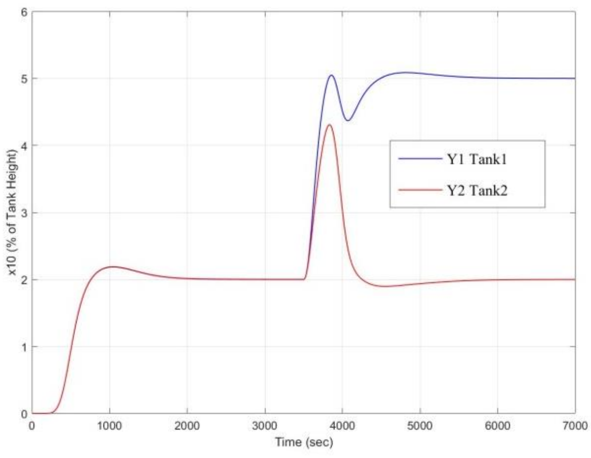

Figure. 12 Step response in case of non-decoupling

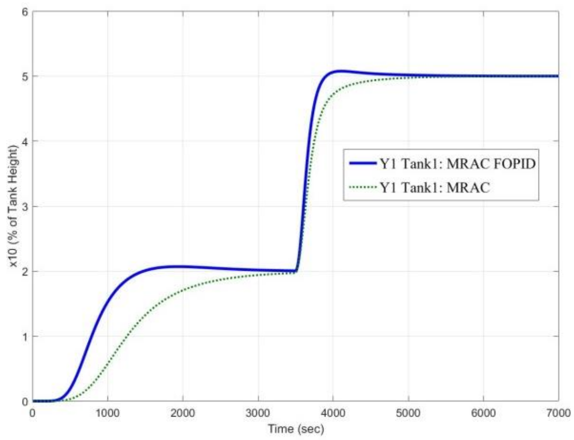

Figure. 13 Step response compared between adaptive fractional order $PI^{\lambda}D^{\mu}$ (MRAC FOPID) control and model reference adaptive control (MRAC)

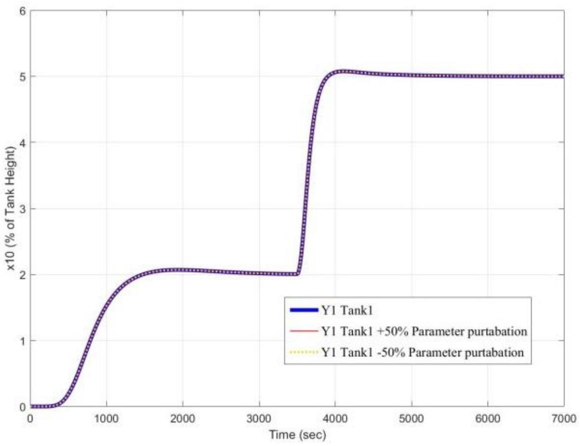

Figure. 14 Step response in case of robustness validation

The advantage of proposed technique over the conventional model reference adaptive control (MRAC) [17] clearly demonstrated in Fig.13. The responses due to a reference signal 20% of the tank height at 100th second and 50% of the tank height at 3,500th second indicate an overshoot of less than 7% and a settling time around 3,000 seconds (2% tolerance). The results clearly showed that the conventional model reference adaptive control has a limitation to adjust the response, while the adaptive fractional order $PI^{\lambda}D^{\mu}$ control can decrease the settling time over 453 seconds using the appropriate adjusting of the fractional-order of the operations (λ, μ).

The robustness of the proposed adaptive fractional-order $PI^{\lambda}D^{\mu}$ controller is demonstrated in Fig. 14. The test was done by adjusting the process parameter, the gain of pumps (k_{p1}, k_{p2}) to ±50% for validating the robustness of controller in the system that have the process parameter perturbations.

This finding validates the usefulness of the proposed technique over a non-adaptive fractional-order $PI^{\lambda}D^{\mu}$ controller in [27] that adaptive mechanism of the adaptive fractional-order $PI^{\lambda}D^{\mu}$ controller is capable of adjusting the controller gain $\left(K_P, K_I, K_D\right)$ to ensure the system to meet the same desired specifications at almost every step set-point, even though the process parameters was changed over 45% compared to the robustness evaluation of the previous work [27] which has the variation of the response in settling time and overshoot.

5.5 The performance analysis for non-minimum phase

Due to the process parameters in case of non-minimum phase, the zeroes of the multi-tank process are located in the right half plane, which means that upper tanks get more water, while the target is to control lower tanks, making the control problem become more challenges [28, 29]. Referring to the proposed control strategies explain in the section 4, the adaptation gains (γ) are specified as

MRAC adaptation gains1:

$$\gamma_{p1} = -0.00001, \gamma_{i1} = -0.001, \gamma_{d1} = -0.0018 \quad (41)$$

MRAC adaptation gains2:

$$\gamma_{p2} = -0.00001, \gamma_{i2} = -0.001, \gamma_{d2} = -0.0012 \quad (42)$$

The fractional-order operations, which provide the values of ISE at 2,268 cm^2 (tank1), 1171 cm^2 (tank2) are demonstrated in the controller transfer functions as following.

$$G_{FOPID\_1}(s) = K_{p1} + K_{i1}\frac{1}{s^{1.02}} + K_{d1}s^{0.75} \quad (43)$$

$$G_{FOPID\_2}(s) = K_{p2} + K_{i2}\frac{1}{s^{1.04}} + K_{d2}s^{0.75} \quad (44)$$

The decoupling compensators in Eq. (30), Eq. (31) with the structure of invert-decoupling [30] were used for rejecting the inter-action disturbances.

The simulated responses of levels of tank1 and tank2 are shown in Fig. 15. The transitions take place approximately at the 100th, and 3500th second. The dash lines represent the outputs of the reference models (y_{m1}, y_{m2}). The effectiveness of the proposed model reference adaptive control (MRAC) Fractional-order $PI^{\lambda}D^{\mu}$ applied to the non-minimum phase of multi-configuration tank process toolbox, it can be seen that at the first step tracking, the output responses have 5.75, 6.9 of percent overshoot, settling time at 866, 1064 seconds for y_1 (Blue line) and y_2 (Red line) respectively, likewise the values of steady state error are zero. As well as in the second step tracking the output responses have 5.0 of percent overshoot, settling time at 869 seconds for y_1 (Blue line) and the values of steady state error are zero. The test of control system without the invert-decoupling compensator is presented in Fig. 16. The results reveal that the proposed invert-decoupling structure has effectiveness of inter-acting rejection.

The advantage of proposed technique over the conventional model reference adaptive control (MRAC) [17] clearly demonstrated in Fig.17. The results clearly showed that adaptive fractional order $PI^{\lambda}D^{\mu}$ control can decrease the value of overshoot over 21.36 percent using the appropriate adjusting of the fractional-order of the operations (λ, μ) compared to the conventional model reference adaptive control which has a massive percent of overshoot.

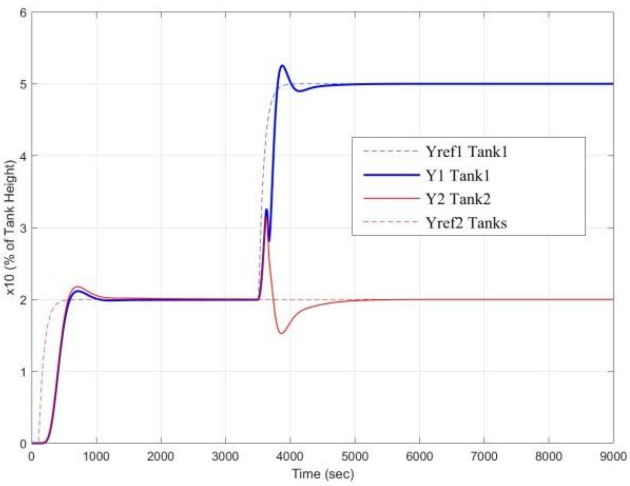

Figure. 15 Step response of multi-configuration tank process simulation: non-minimum phase

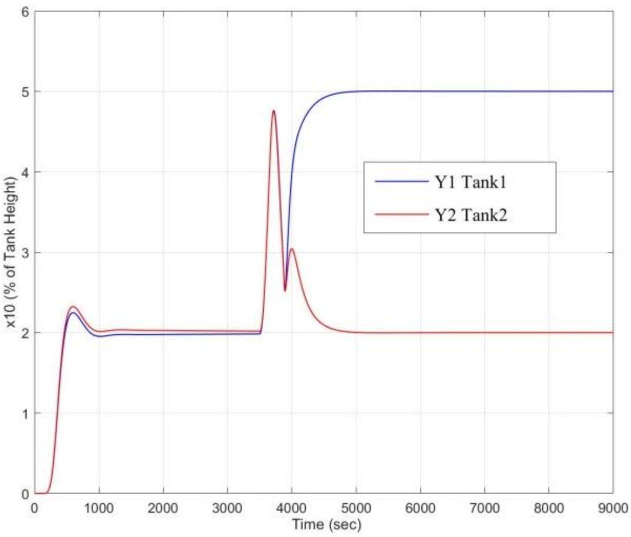

Figure. 16 Step response in case of non-decoupling

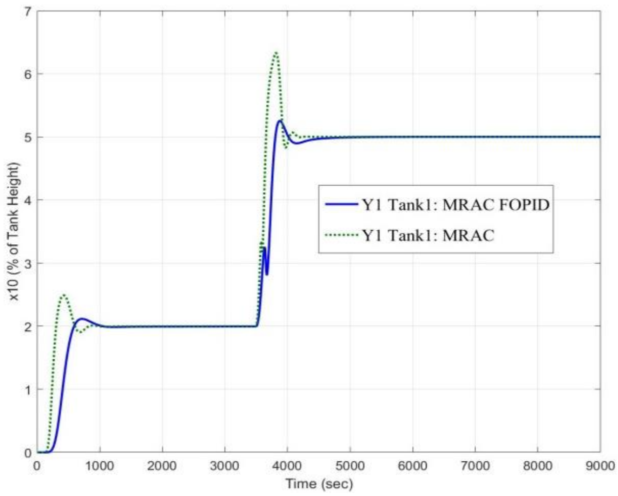

Figure. 17 Step response compared between adaptive fractional order $PI^{\lambda}D^{\mu}$ (MRAC FOPID) control and model reference adaptive control (MRAC)

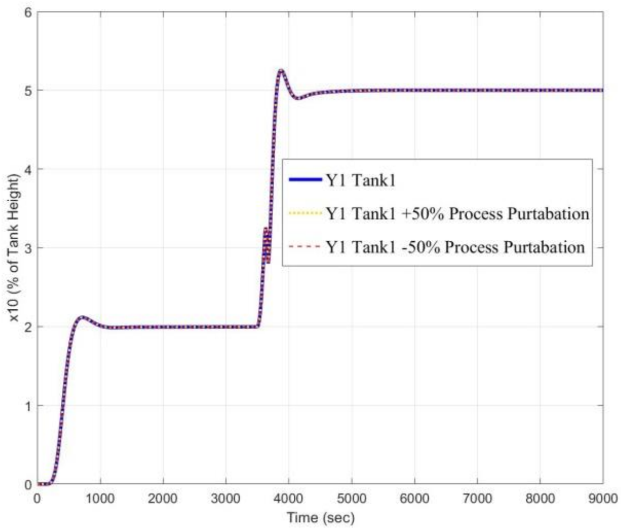

Figure. 18 Step response in case of robustness validation

The robustness of the proposed adaptive fractional-order $PI^{\lambda}D^{\mu}$ controller is demonstrated in Fig. 18. The test was done by adjusting the process parameter, the gain of pumps (k_{p1}, k_{p2}) to ±50% for validating the robustness of controller in non-minimum phase of multi-configuration tank process that have the process parameter perturbations. This finding validates the usefulness of the proposed technique over a non-adaptive fractional-order $PI^{\lambda}D^{\mu}$ controller in [27] that adaptive mechanism of the adaptive fractional-order $PI^{\lambda}D^{\mu}$ controller is capable of adjusting the controller gain $\left(K_P, K_I, K_D\right)$ to ensure the system to meet the same desired specifications at almost every step set-point, even though the process parameters was changed over 45% compared to the robustness evaluation of the previous work [27] which has the variation of the response in settling time and overshoot.

Table 1. Process parameters

Process parameters	Values
A : Cross section area of tank	$70\ (cm^2)$
a_i : Cross section area of outlet hole	$0.5028\ (cm^2)$
a_x : Cross section area of the connection hole between tank1, tank2	$0.5028\ (cm^2)$
$h_i(t)$: Water level	$20\ (cm)$
u_j : Voltage input of pump	$7\ (volt)$
β_i : Outlet valve ratio	0.549, 0.587, 0.182, 0.263
β_{xu} : Connected upper valve ratio	0.284
β_{xl} : Connected lower valve ratio	0.184
γ_j : Inlet valve ratio (min phase)	0.744, 0.757
γ_j : Inlet valve ratio (non-min phase)	0.214, 0.217
k_{pi} : Gain of pump	3.237, 3.2 $(cm^3/volt/\mathrm{sec})$
g : Specific gravity	$981\ (cm/s^2)$

6. Conclusion

The adaptive fractional order $PI^{\lambda}D^{\mu}$ control for multi-configuration tank process (MCTP) toolbox has been presented in this study with the purpose of demonstrating the problem of reference tracking and cross coupling rejection in multi-input-multi-output system. Furthermore the proposed multi-configuration tank process (MCTP) software toolbox is developed from the discrete state P-file S-function which is operated within the MATLAB environment for providing many non-linear functions of dynamic models of multi-configuration tank process such as multi-input multi-output (MIMO) quadruple tank full-interacting process, multi-input multi-output (MIMO) quadruple tank process, multi-input single-output (MISO) triple tank interacting process, multi-input multi-output (MIMO) coupled tank interacting processes. The paper describes the mathematical model of multi-configuration tank process, nonlinear dynamic characteristic, minimum phase and non-minimum phase configuration, software toolbox features and also explains about the design of the adaptive fractional order $PI^{\lambda}D^{\mu}$ controller including the performance validation. Significantly, the results have been illustrated that the proposed controller design scheme can provide the sufficient effectiveness in the performance, stability and robustness. In addition, these tests reinforce the usefulness of MCTP toolbox as a complete simulation tool for users to perform an engineering research of multi-configuration tank control system analysis and design, moreover, it contains very useful for validating the control algorithm of multi-input-multi-output system. To further our research intends to implement the nonlinear modeling and dynamic optimization in the actual multi-configuration tank process for extending the research capacity of the multi-variables control system analysis and design.

References

[1] K. H. Johansson, A. Horch, O. Wijk, and A. Hansson, “Teaching Multivariable Control Using The Quadruple - Tank Process”, In: *Proc. of IEEE Conference on Decision and Control*, pp. 807-812, 1999.

[2] K. J. Astrom, K. H. Johansson, and Q. G. Wang, "Design of Decoupled PI Controllers for Two-By-Two Systems", *IEE Proceedings - Control Theory and Applications*, Vol. 149, pp. 74-81, 2002.

[3] S. M. M. Alavi, A. Khaki-Sedigh, B. Labibi, and M. J. Hayes, "Improved Multivariable Quantitative Feedback Design For Tracking Error Specifications", *IET Control Theory & Applications*, Vol. 1, pp. 1046-1053, 2007.

[4] M. Pasamontes, J. D. Alvarez, J. L. Guzman, and M. Berenguel, "Learning Switching Control: A Tank Level-Control Exercise", *IEEE Transactions on Education*, Vol. 55, pp. 226-232, 2012.

[5] S. Larguech, S. Aloui, A. Chaari, A. E. Hajjaji, and Y. Koubaa, "Improved Sliding Mode Control of a Class of Nonlinear Systems: Application to Quadruple Tanks System", In: *Proc. of European Control Conference*, pp. 3203-3208, 2013.

[6] V. Kirubakaran, T.K. Radhakrishnan, and N. Sivakumaran, “Distributed Multiparametric Model Predictive Control Design for a Quadruple Tank Process”, *Measurement*, Vol. 47, pp. 841-854, 2014.

[7] U. S. Banu and S. K. Lakshmanaprabu, "Adaptive Multi-Loop Fractional Order PID Controller Tuning Using Bat Colony Optimization for Quadruple-Tank Process", In: *Proc. of International Conference on Robotics, Automation, Control and Embedded Systems*, Vol.1-8, 2015.

[8] X. Christie and P. S. Jose, "Design of Model Reference Adaptive Control for a Quadruple Tank System Using LabVIEW", In: *Proc. of International Conference on Innovations in Information, Embedded and Communication Systems*, Vol. 1-6, 2015.

[9] A. Başçi and A. Derdiyok, “Implementation of an Adaptive Fuzzy Compensator for Coupled Tank Liquid Level Control System”, *Measurement*, Vol. 91, pp. 12-18, 2016.

[10] V. Kirubakaran, T.K. Radhakrishnan, and N. Sivakumaran, “Fuzzy Aggregation Based Multiple Models Explicit Multi Parametric MPC Design for a Quadruple Tank Process”, *IFAC-Papers OnLine*, Vol. 49, pp. 555-560, 2016.

[11] A. Numsomran, J. Chaoraingern, T. Suesut, T. Trisuwannawat, and V. Tipsuwanporn, "PID Controller Design Using Characteristic Ratio Assignment Method for Coupled-tank Process", In: *Proc. of International Conference on Computational Intelligence for Modelling, Control and Automation and International Conference on Intelligent Agents, Web Technologies and Internet Commerce*, pp. 590-594, 2005.

[12] A. Numsomran, T. Suksri, and M. Thumma, "Design of 2-DOF PI Controller with Decoupling for Coupled-tank Process", In: *Proc. of International Conference on Control, Automation and Systems*, pp. 339-344, 2007.

[13] A. Numsomran, T. Wongkhum, T. Suksri, P. Nilas, and J. Chaoraingern, "Design of Decoupled Controller for TITO System Using Characteristic Ratio Assignment", In: *Proc. of International Conference on Control, Automation and Systems*, pp. 957-962, 2007.

[14] A. Numsomran, V. Tipsuwanporn, T. Trisuwannawat, and K. Tirasesth, "Design of PID Controller for the Modified Quadruple-tank Process Using Inverted Decoupling Technique", In: *Proc. of International Conference on Control, Automation and Systems*, pp. 1364-1368, 2011.

[15] A. Numsomran, R. Jaisue, J. Chaoraingern, V. Tipsuwanporn, and P. Pholkeaw, "A Design of Fuzzy PID Controller Based on ARM7TDMI for Coupled-Tanks Process", In: *Proc. of International Conference on Control, Automation and Systems*, pp. 610-613, 2012.

[16] A. Numsomran, J. Chaoraingern, and T. Trisuwannawat, “The Modified Quadruple-Tank Process: A Flexible Mathematical Model with an Adjustable Simulink Block”, *Applied Mechanics and Materials*, Vol. 541, pp. 813-821, 2014.

[17] S. Kangwanrat, V. Tipsuwannaporn, and A. Numsomran, "Design of PI Controller Using MRAC Techniques for Coupled-tanks Process", In: *Proc. of International Conference on Control, Automation and Systems*, pp. 485-490, 2010.

[18] B. M. Vinagre, I. Petras, I. Podlubny, and Y. Q. Chen, “Using Fractional Order Adjustment Rules and Fractional Order Reference Models in Model Reference Adaptive Control”, *Nonlinear Dynamics*, Vol. 29, pp. 269–279, 2002.

[19] S. Ladaci and A. Charef, “On Fractional Adaptive Control”, *Nonlinear Dynamics*, Vol. 43, pp. 365–378, 2006.

[20] Y. Chen, I. Petr´asˇ, and D. Xue, “Fractional Order Control - A Tutorial”, In: *Proc. of American Control Conference*, St. Louis, MO, USA, 2009.

[21] A. Oustaloup, B. Mathieu, and P. Lanusse, “The CRONE Control of Resonant Plants: Application to a Flexible Transmission”, *European Journal of Control*, Vol. 1 No.2, pp. 113-121, 1995.

[22] K. J. Astrom, and B. Wittenmark, “Adaptive Control”, *Addison-Wesley Publishing Company,* 1989.

[23] P. Nordfeldt and T. Hägglund, “Decoupler and PID Controller Design of TITO Systems”, *Journal of Process Control*, Vol. 16, No. 9, pp. 923-936, 2006.

[24] B. T. Jevtović and M. R. Mataušek, “PID Controller Design of TITO System Based On Ideal Decoupler”, *Journal of Process Control*, Vol. 20, No. 7, pp. 869-876, 2010.

[25] K. Ogata, “Modern Control Engineering 5th Edition”, *Pearson Education, Inc*., 2010.

[26] A. Tepljakov, E. Petlenkov, and J. Belikov, “FOMCON: a MATLAB Toolbox for Fractional-order System Identification and Control”, *International Journal of Microelectronics and Computer Science*, Vol.2, pp. 51-62, 2011.

[27] P. Kungwalrut, A. Numsomran, P. Chaiyasith, J. Chaoraingern, and V. Tipsuwanporn,“A $PI^{\lambda}D^{\mu}$ Controller Design for Peltier-Thermoelectric Cooling System”, In: *Proc. of International Conference on Control, Automation and Systems*, pp. 766-770, 2017.

[28] L. Sun, D. Li, Z. Gao, Z. Yang, and S. Zhao, "Combined Feedforward and Model-assisted Active Disturbance Rejection Control for Non-Minimum Phase System", *ISA Transactions*, Vol. 64, pp. 24-33, 2016.

[29] S. Nazrulla and H. K. Khalil, "Robust Stabilization of Non-Minimum Phase Nonlinear Systems Using Extended High-Gain Observers", *IEEE Transactions on Automatic Control*, Vol. 56, pp. 802-813, 2011.

[30] J. Garrido, F. Vázquez, and F. Morilla, “An Extended Approach of Inverted Decoupling”,

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Journal of Process Control, Vol. 21, No. 1, pp. 55-68, 2011.