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# Speed Control of DC Motor Using PID and FOPID Controllers Based on Differential Evolution and PSO

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**Abstract:** DC motors are widely used in industrial application for its different advantage such us high efficiency, low costs and flexibilities. For controlling the speed of DC motor, conventional controller PI and PID were the most widely used controllers. But due to empirically selected parameters $K_p, K_i, K_d$ and limitation of convention PID controller to achieve ideal control effect for higher order systems, a Fractional order Proportional-Integral-Derivative PID (FOPID) based on optimization techniques was proposed in this paper. The aim of this paper is to study the tuning of a FOPID controller using intelligent soft computing techniques such as Differential Evolution (DE) and Particle Swarm Optimization (PSO) for designing fractional order PID controller. The parameters of FOPID controller are determined by minimizing the Integral Time Absolute Error (ITAE) between the output of reference model and the plant. The performance of DE and PSO were compared with several simulation experiments. The simulation results show that the DE-based FOPID controller tuning approach provides improved performance for the setpoint tracking, error minimization, and measurement noise attenuation.

**Keywords:** DC motor, Fractional PID, Tuning parameters, Differential evolution, Particle swarm optimization, Cost function.

## 1. Introduction

The DC motors are widely used in industrial application for its different advantage such us high efficiency, low costs and flexibilities. For controlling the speed of DC motor different controllers is used, most widely used controllers are conventional controller PI and PID. But conventional PID controller has been facing lots of problem to achieve ideal control effect. For higher order systems, PID has not been working properly.

When compared with the classical three terms PID controllers, the fractional order controllers have two additional control parameters defined as integration and differentiation orders which may enable the controller to provide the more flexibility and stability.

It is quite difficult to optimize the parameters of the FOPID controller in linear and nonlinear systems. There is a need for an effective and efficient global approach to optimize these parameters automatically.

For this reason different design methods for FOPID controller have been reported in the literatures. In [1] the authors proposed a new approach for robust control by fractionalizing an integer order integrator in the classical PID control scheme and they use the Sub-optimal Approximation of fractional order transfer function to design the parameters of PID controller .In [2], the authors designed a new tuning rules for the

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tuning parameters of FOPID based on Ziegler–Nichols (Z–N) rule, A new tuning method for designing fractional order PID controllers based on radial basis function (RBF) neural networks was proposed in [3]. Sharma et al [4] proposed a Fractional Order Fuzzy Proportional Integral Derivative (FOFPID) controller for a two-link planar rigid robotic manipulator for trajectory tracking problem. For tuning of parameters of all the controllers, Cuckoo Search Algorithm (CSA) optimization technique was used. Chang et al. [5] proposed a novel adaptive GA for the multi-objective optimization design of a FOPID controller and applied it to the control of an active magnetic bearing system. They found that the fractional PID controllers have remarkably reduced the overshoot and settling time compared with the optimized conventional PID controller. Bingul [6] employed the differential evolution (DE) algorithm to tune a PID controller for unstable and integrating processes with time delay. The results showed that a faster settling time, less or no overshoot, and higher robustness were obtained with the PID tuned DE. Cao [7] demonstrated the parameter optimization of a fractional order controller based on a modified PSO. In their paper, the improved PSO could achieve faster search speed and better solution compared to the GA. Maiti et al. [8] employed PSO for designing fractional order PID controllers. They reduced significantly the percentage of overshoot, rise, and settling times using FOPID controllers compared to a PID controller. Alfi and Modares [9] found optimal system parameters for an unstable nonlinear system and optimal parameters of the PID controller using a novel Adaptive PSO (APSO). They compared the APSO with a Linearly Decreasing inertia Weight PSO (LDW-PSO) and the GA. The APSO has a faster convergence speed than the GA and LDW-PSO.

The controllers of the speed that are conceived for goal to control the speed of DC motor are numerous: Fractional PID Controller [10, 11], Fractional fuzzy PID Controller [12]; Genetic algorithm [13, 14], Particle Swarm Optimization [15], … etc.

The benefit of FOPID controller is flexible to design, more robust [16] and the most important advantages is the better control of dynamical systems and less sensitivity to changes in parameters of a control system [17, 18]. In Fractional Order PID (FOPID) besides setting the proportional, derivative and integral constants $K_p$, $K_i$, $K_d$ we have two more parameters λ (integral order) and μ (derivative order). Hence, for designing FOPID controller, there is a need of proper tuning of five parameters $(K_p, K_i, K_d, \lambda, \mu)$ [19].

An evolutionary computation technique has become gradually popular to obtain global optimal solution in many areas. A Differential Evolution Optimization (DEO), particle swarm optimization (PSO) and Genetic Algorithm (GA) are stochastic optimization strategy from the family of evolutionary computation [20, 21].

DE has been regarded widely as a promising optimization algorithm. What's more, the optimal problems solved by genetic algorithms (GA) can be obtained better solutions with PSO in comparison with conventional methods. These are precisely the main motivations that led us to apply DE and PSO for FOPID controllers design.

This paper proposes a new method to design a speed controller of a DC motor by selection of FOPID parameters using DE. To show the efficiency of DE, the results of this method are compared with PSO method. Minimization of time domain based objective function is the main focus of design methodology.

The structure of this paper is organized as follows: Section 2 deals with mathematical modelling of DC motor. Sect. III introduces the fractional order PID controller; Section IV provides a brief overview of the DE and PSO algorithms. Section V applies the new algorithm in this paper to parameter setting of fractional order PID controller through a simulated calculation example; and Section VI draws the conclusion of the whole paper.

## 2. Modelling of DC motor

In order to experiment our proposed robust control strategy, let us apply it in numerical simulations to the general model of a DC motor (DCM) as depicted in [22]. The voltage $V_a$ is applied to command the motor angular velocity $\omega(t)$. Fig.1 shows the schematic diagram of armature controlled DC motor.

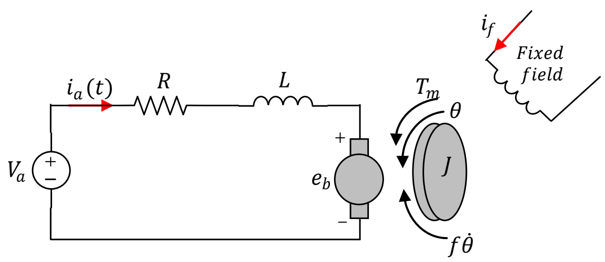


Figure.1 Closed loop response with PID controller

The DC motors are generally used in the linear range of the magnetization curve. Therefore, air gap flux $\varphi$ is proportional of field current:

$$\varphi = K_f i_f \quad (1)$$

Where $K_f$ is constant.

The torque $T_m$ developed by the motor is proportional to the armature current and air gap flux:

$$T_m = K_m i_a \quad (2)$$

Where $K_m$ is the motor torque constant.

The motor back EMF being proportional to speed is given as:

$$e_b = K_b \frac{d\theta}{dt} \quad (3)$$

Where $e_b$ is the back EMF constant.

The differential equation of armature circuit is:

$$V_a = L\frac{di_a}{dt} + Ri_a + e_b \quad (4)$$

And the dynamic equation with moment of inertia and coefficient of friction will be:

$$T_m = J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} \quad (5)$$

The resulting mathematical model for controlled DC motor is given by the following transfer function [22]:

$$G_M(s) = \frac{\theta(s)}{V_a(s)} = \frac{K_m}{s(Ls+R)(Js+f)+K_bK_m} \quad (6)$$

Where

$R$: Armature Resistance ($\Omega$).
$L$: Inductance of armature winding ($H$).
$i_a$: Armature current ($A$).
$i_f$: Field current ($A$).
$V_a$: Applied armature voltage ($V$).
$e_b$: Back emf ($V$).
$T_m$: Torque developed by motor ($Nm$).
$\theta$: Angular displacement of motor shaft ($rad$).
$\omega$: Angular speed of motor shaft ($rad/sec$).
$J$: Equivalent moment of inertia of motor and load referred to motor shaft ($kg-m2$).
$f$: Equivalent friction coefficient of motor and load referred to motor shaft ($Nm.s/rad$).

As the armature time constant for most DC Motor ($M$) is negligible we can simplify the model (6). The resulting simplified mathematical model form is:

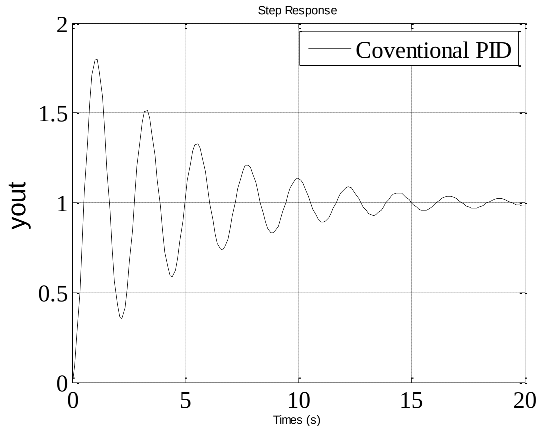


Figure.2 Closed loop response with PID controller: $K_p = 10$, $K_i = 100, K_d = 0.25$

$$G_M(s) = \frac{\theta(s)}{V_\alpha(s)} = \frac{K_m}{s[R(J_s+K_f)+K_bK_m]}$$

$$G_M(s) = \frac{[K_m/(RK_f+K_bK_m)]}{s(\tau_s+1)} = \frac{K_M}{s(\tau_s+1)} \quad (7)$$

Where:
$\tau = K_m/(RK_f + K_bK_m)$: is the time constant.
$K_M = K_m/(RK_f + K_bK_m)$: is the gain.
with $K_m = K_b$.

The specifications are: min. volt-age $1.5\,V$, max. voltage $2.5\,V$, nominal voltage 2 V, max rated current $0.08\,A$, no load speed $3830\,r/min$ and rated load speed $3315\,r/min$.

For our mini DC motor the physical constants are $R = 6\Omega$, $K_m = K_b = 0.1$, $K_f = 0.2\,Nms$ and $J = 0.01\,kgm^2/s^2$.

For the considered motor parameters the transfer function (7) becomes:

$$G_M(s) = \frac{\theta(s)}{V_\alpha(s)} = \frac{0.08}{s(0.05s+1)} \quad (8)$$

Fig. 2 shows the closed loop response with conventional PID controller. Thus the system (Eq. (8)) is marginally stable.

## 3. Fractional PID Controller Design

### 3.1 Fractional Calculus

Fractional calculus is a generalization of the differentiation and integration to non-integer-order fundamental operator ${}_aD_t^\mu$, where $a$ and $t$ are the bounds of the operation. The definition of the basic operator which includes the derivative and integration is [23]:

$$ {}_aD_t^{\mu} = \begin{cases} \frac{d^{\mu}}{dt^{\mu}} & \mu > 0 \\ 1 & \mu = 0 \\ \int_a^t (d\tau)^{-\mu} & \mu < 0 \end{cases} \quad (9) $$

Where $\mu$ is a fractional order of differentiation or integration, generally $\mu \in \mathbb{R}$. The negative sign of $\mu$ indicates integration while positive one means derivation [24].

There are many mathematical definitions of fractional derivatives [25]. One of the most important used definitions is Grunwald-Letnikov definition which is perhaps the most popular because of its suitability for the realization of discrete control algorithms.

The Grünwald-Letnikov definition of fractional-order derivatives is expressed as [26]:

$$ {}_aD_t^{\mu} f(t) = \lim_{h \to 0} \frac{1}{h^{\mu}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\mu}{j} f(t - jh) \quad (10) $$

With $\binom{\mu}{j} = \frac{\Gamma(\mu+1)}{\Gamma(j+1)(\mu-j+1)}$

while the definition of fractional-order integral is expressed as:

$$ {}_aD_t^{-\lambda} f(t) = \lim_{h \to 0} \frac{1}{h^{-\lambda}} \sum_{j=0}^{\left[\frac{t-\lambda}{h}\right]} \binom{\lambda}{j} f(t - jh) \quad (11) $$

With:
$\binom{\lambda}{j} = \frac{\Gamma(n-\lambda+1)}{j!\Gamma(\lambda)}$, $\Gamma(1) = 1$ and $\Gamma(x+1) = x\Gamma(x)$
for $\lambda \in \mathbb{N}, \Gamma(\lambda + 1) = \lambda!$

where:
$\binom{\lambda}{j} = \frac{\lambda!}{j!(\lambda-j)!}$ and $\binom{\mu}{j} = \frac{\mu!}{j!(\mu-j)!}$ are the binomial coefficients $(j > 0)$.

$\lambda, \mu$ : Integral and derivative Order respectively.
$\Gamma(.)$ : Gamma function
$h$ : Step time.

Another popular definition is that of Riemann-Liouville definition of fractional-order derivatives given by:

$$ {}_aD_t^{\mu} f(t) = \frac{1}{\Gamma(n-\mu)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\mu-n+1}} d\tau \quad (12) $$

Where $n - 1 < \mu < n$
while the definition of fractional-order integral is expressed as:

$$ {}_aD_t^{\lambda} f(t) = \frac{1}{\Gamma(\lambda)} \int_a^t (t-\tau)^{\lambda-1} f(\tau) d\tau \quad (13) $$

The Laplace transform of the fractional derivative of $f(t)$ is given by:

$$ L\{D^{\mu} f(t)\} = s^{\mu} F(s) - [D^{\mu-1} f(t)]_{t=0} \quad (14) $$

The Laplace transform of the fractional integral of $f(t)$ is given as follows:

$$ L\{D^{-\lambda} f(t)\} = s^{-\lambda} F(s) \quad (15) $$

Where
$F(s)$ is the Laplace transform of $f(t)$.

### 3.2 Fractional PID controller

The Fractional Order PID (FOPID) Controller is the expansion of the generic control loop feedback mechanism (PID controller) widely used in industrial control systems. The FOPID Controller attempts to correct the error between a measured process variable and a desired set point by calculating and then outputting a corrective action that can adjust the process accordingly.

The transfer function of the FOPID controller is described as follows:

$$ G_c(s) = K_p + K_i S^{-\lambda} + K_d S^{\mu} \quad (16) $$

The FOPID equation has five unknown parameters, where $K_p$ is the proportional gain, $K_i$ is the integral gain, $K_d$ is the derivative gain, $\lambda$ is the fractional-order integral and $\mu$ is the fractional-order derivative and $\lambda, \mu$ are positive real numbers.

The block diagram of control system employing Soft computing FOPID control action is shown in Fig.3.

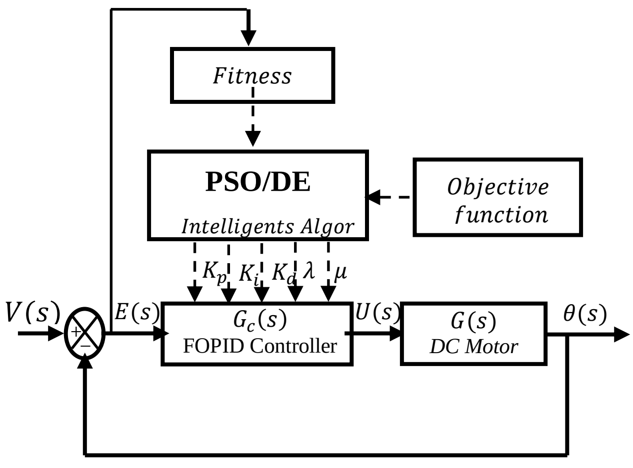


Figure.3 A block diagram of Intelligent FOPID controller

where

$V(s)$ : Input Signal
$E(s)$ : Error Signal
$G_c(s)$: Controller Transfer Function
$G(s)$ : System or plant (DC Motor)
$\theta(s)$ : Output Signal
$U(s)$ : Control Signal

### 3.3 Cost function

To evaluate the controller performance, there are always several criterions of control quality like integral of absolute error (IAE), integral of time absolute error (ITAE), integral of squared error (ISE) and integral of time squared error (ITSE) [27].

A disadvantage of the ISE and IAE criteria (weight all errors equally and independent of time) is that they may result in a response with a long settling time and relatively small overshoot [27]. To overcome this drawback, an integral of time weighted absolute error (ITAE) is used in this paper as fitness function.

Therefore, the controller can be evaluated using the following performance index:

$$J(K_p, K_i, K_d, \lambda, \mu) = \int_0^\infty t|e(t)|dt \qquad (17)$$

$J$ is called as ITAE. It explains indirectly the level that the controlled object is close to the reference model. Where $t$ is the time and $e(t)$ is the difference between set point and controlled variable.

## 4. PSO and DE optimization methods

In this paper, the FOPID controller is optimized to achieve the optimal behaviour of the plant. The optimizer is used to search for the optimal solution of the FOPID control gain.

### 4.1 Particle swarm optimization algorithm

PSO is a modern heuristic search method inspired by the social behavior of bird and fish schooling. PSO optimization consists of designing the optimization goal, i.e. the fitness function and then encoding the parameters to be searched.

PSO exploits a swarm of particles probing promising regions of the D-dimension search space with adaptable velocity. It runs until the stop condition is satisfied. The best particle's position gives the optimized parameters for the controller.
The flowchart of a typical PSO algorithm is shown in Fig.4.

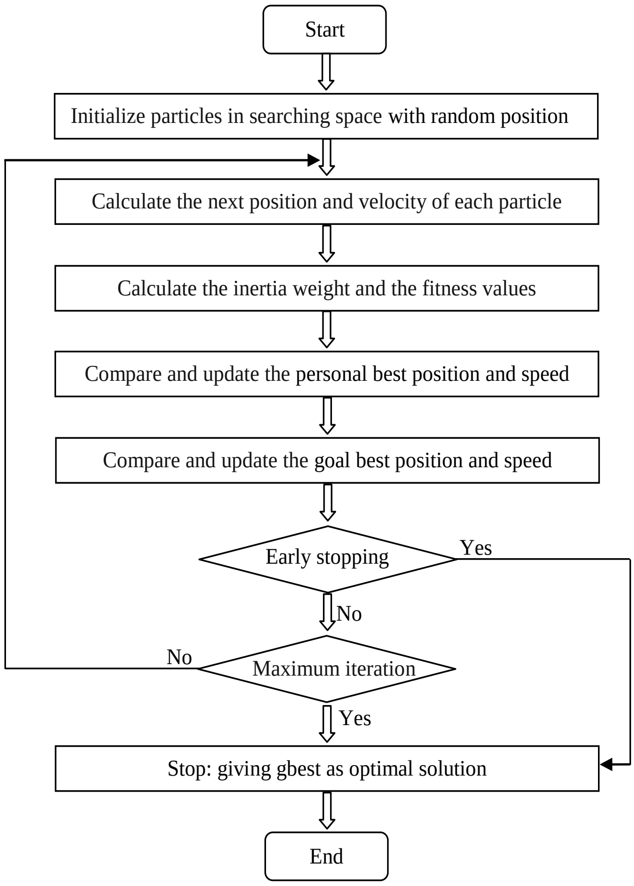


Figure.4 Flowchart of PSO algorithm procedure

The update formula of velocity and position is stated by Eqs. (18) and (19):

$$v_i^{k+1} = w_i v_i^k + C_1 a\left(P_i - x_i^k\right) + C_2 b\left(P_g - x_i^k\right) \quad (18)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1} \qquad (19)$$

Where:
$v_i^k, x_i^k$ : Velocity and positioning vectors of particle $i$ at iteration $k$ respectively.
$v_i^{k+1}, x_i^{k+1}$ : Modified velocity and position of particle $i$ at the next iteration $k+1$ respectively.
$a, b$ : Random number between 0 and 1
$C_1, C_2$ : Positive constants
$P_i, P_g$ : Best positions found by particle $i$ and $g$ respectively
$w_i$ : Weight function for velocity of particle $i$.

In order to design optimum controller, the fitness function are defined in Eq. (17).

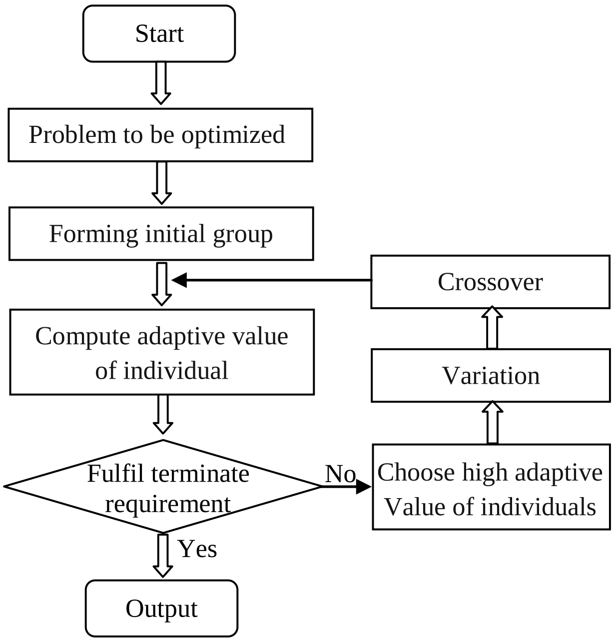


Figure.5 Flowchart of DE algorithm

### 4.2 Differential evolution

DE was introduced by Storn and Price in 1996. It is a stochastic, population based optimization algorithm like Genetic Algorithm. But one big difference is that DE is developed to optimize real parameters that are non-differentiable, non-continuous, non-linear, noisy, flat, multi-dimensional or have many local minima. As a result, the idea of mutation and crossover are substantially different in both the techniques.

DE has better convergence to global optimum, more accurate and reduced number of simulations in comparison to other optimization techniques.
Minimizing the cost function generates the controller parameters. The error criterion is considered as the cost function, and the values of $K_p$, $K_i$, and $K_d$ are continuously adjusted, until the error of the closed-loop system is minimum. Eq. (17) shows the normally considered error criterion in control system to evaluate the performance of controller.

The flowchart of DE algorithm is shown in Fig.5.

## 5. Tuning of the FOPID controller using PSO and DE optimizations

The speed control loop of DC motor (model number PN13KA12C) has been modeled in SIMULINK PSO and DE algorithms has been programmed and implemented in Matlab.

In this paper a time domain quantities such as maximum overshoot, rise time, setting time, damping ratio and undamped natural frequency of the desired dominant closed-loop poles, is used for evaluating the FOPID controller. A set of good control parameters P, I, D, $\lambda$ and $\mu$ can yield a good step response that will result in performance criteria minimization in the time domain. To control the plant model the following intelligent tuning methods PSO and DE parameters are used to verify the performance of the FOPID controller Parameters.

Table 1 summarizes the values of parameters affecting the optimization. Table 2 displays the optimization parameters for each optimization method used in this paper.

## 6. Simulation results and discussion

All optimization procedures are successful, producing gains inside the specified bounds and providing valid solutions for each case. Conventional methods of controller tuning lead to a rise time, overshoot, large settling time and steady state error of the controlled system. Hence intelligent soft computing techniques are introduces into the control loop. PSO and DE based FOPID tuning methods have proved their excellence in giving better results by improving the performance indices and the steady state characteristics.

Performance characteristics of process model (dynamic response characteristic of the closed loop) was indicated and compared with the intelligent tuning methods as shown in Figs.6 and 7.

It can be observed from the Fig.6 that, the DE algorithm method gives much better time domain performance comparatively to PSO algorithm especially for maximum overshoot, rise time, and settling time and also comparatively to [22] study.

Table 1. Optimization parameters

| **Optimization parameters** | **Value** |
|---|---|
| Number of Population (NP) | 50 |
| D-dimensional parameter | 5 |
| Generation number | 100 |

Table 2. Optimization methods parameters

| **Optimization Process** | | |
|---|---|---|
| Optimization method | Optimization Parameter | Value |
| Differential Evolution | $CR$ | 0.8 |
| | $F$ | 0.9 |
| Particle Swarm Optimization | $C_1$ | 1 |
| | $C_2$ | 3 |

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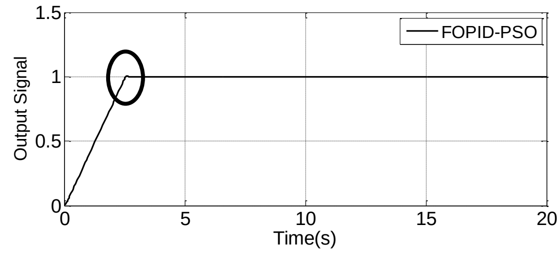
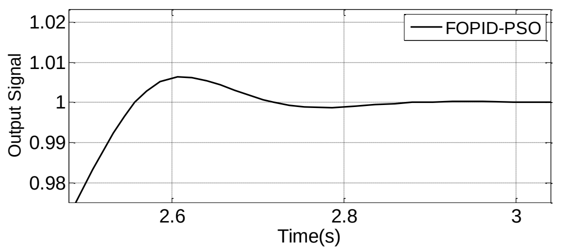



(a)




(b)

Figure.6 Unit step response of closed-loop system: (a) FOPID-PSO and (b) Zoom

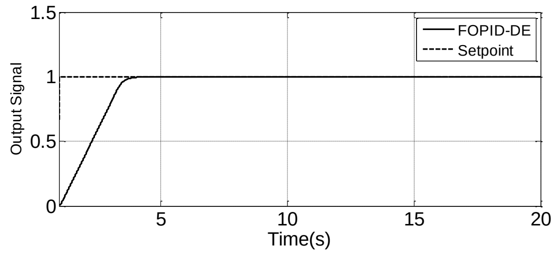



Figure.7 Unit step response of closed-loop system: FOPID-DE

As can be seen in Fig.7, the dynamic properties (overshoot and settling time) of the controlled system response obtained from the FOPID-DE are much better than those of obtained from FOPID-PSO controller.

We remark also, in all figures, a sluggish initial responses which is due to ITAE index (ITAE index reduces the settling time and absolute error but it has sluggish initial response).

Fig.8 shows the responses of the FOPID-PSO and PID-PSO controllers with the ITAE cost function.

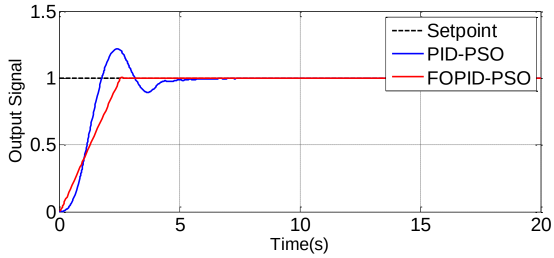



Figure.8 Comparison of PID-PSO and FOPID-PSO

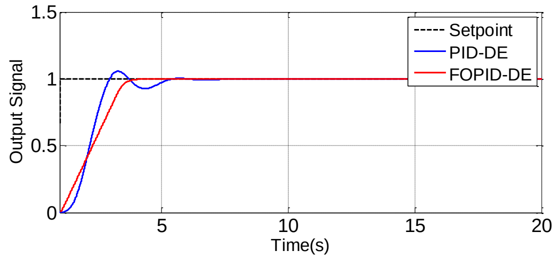



Figure.9 Comparison of PID-DE and FOPID-DE

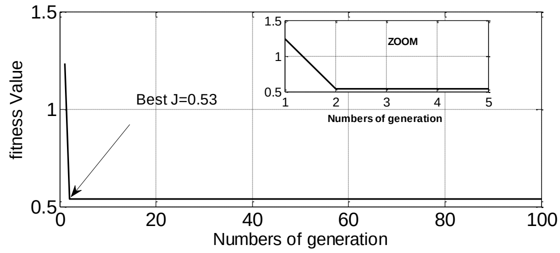



Figure.10 Convergence of behaviours of FOPID-DE

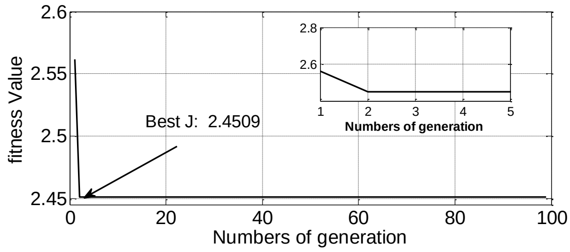



Figure.11 Convergence of behaviours of the: FOPID-PSO

As can be seen in Fig.8, the dynamic properties (overshoot and settling time) of the controlled system response obtained from the FOPID-PSO controller are much better than those of the PID-PSO controller.

Fig.9 shows the responses of the FOPID-DE and PID-DE controllers with the ITAE cost function. As can be seen from Fig.9, the FOPID-DE controller is more robust and has better trajectory tracking than the PID-DE. In order to compare the search performance of the different intelligent optimization methods, PSO and DE algorithms are applied to the FOPID controller optimization with ITAE cost function.

Figs.10 and 11 show the fitness values of different algorithms and as can be seen, the fitness value of the FOPID-DE is decreased to 0.53 after 2 generations. On the second hand, the fitness value of the FOPID-PSO is decreased to 2.45 after 2 generations. It is clear from Fig.10 that DE converges fast initially and requires fewer generations to reach the optimal point. As can be seen, through about 2 generations, the DE algorithm provides better convergence. Furthermore, the results obtained here show that the DE algorithm

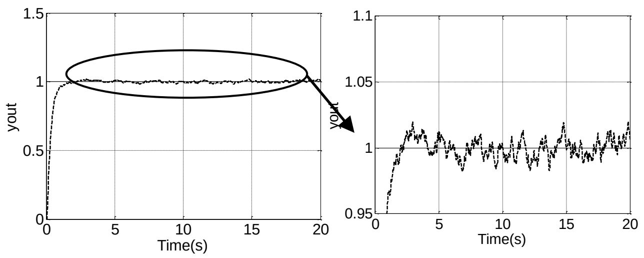



Figure.12 FOPID controller with random output noise of 5 % of the reference signal amplitude

Table 3. Parameters of controllers

| | $Kp$ | $Ki$ | $Kd$ | $\lambda$ | $\mu$ | $Fitness$ |
|---|---|---|---|---|---|---|
| $FOPID - DE$ | 300 | 719 | 120 | 1.06 | 1.01 | 1.106 |
| $FOPID - PSO$ | 180.4 | 582 | 35.7 | 0.78 | 1.20 | 2.4509 |

can search optimal FOPID controller parameters more quickly and efficiently than the PSO algorithm. Fig.12 shows the time response characteristics for a step change of the system (2) with random output noise of a magnitude equal to 5% of the reference signal amplitude.

We remark the FOPID-DE controller give them a certain diminution of the noise effect (Absolute error =2%). For DC Motor, the Evolution optimization algorithms (PSO, DE) aims to find optimal value of FOPID controller to minimize the objective function as given in Eq. (12). For ITAE cost function, the parameters of the FOPID controller tuned with two different algorithms and a comparaison in terms of the cost function are summarized in Tables 3. From Tables 3, the parameters of the FOPID-DE controller for the cost function ITAE are approximately close to that of the FOPID-PSO controller.

## 7. Conclusion

In this work, a new design of intelligent optimization-based model independent controller tuning for DC Motor plant has been attempted. All of the parameters related to the fractional order PID controller were determined using PSO and DE. The robust design of the FOPID controller is difficult to compare to the PID controller, since the FOPID controller includes more parameters. The parameters of FOPID controller were determined by minimizing the ITAE between the output of reference model and the plant. The robustness of the FOPID-DE controller was tested in the case of presence noise at the reference signal amplitude.

Considering all of the results from the simulation experiments, the FOPID-DE controller can achieve good performance, noises rejection and robustness, superior to those obtained with the FOPID-PSO controller. The FOPID-DE controller has good tracking performance in comparison with the FOPID-PSO controller. In addition, the FOPID-DE controller enhanced the flexibility and stability of the PID controller. Furthermore, the implementation of the controller tuning with DE is much easier than with the traditional methods because there is no need for derivative knowledge or complex mathematical equations. In future studies, Bacterial Foraging (BF-FOPID) will be developed using these optimized FOPID controllers.

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