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An Unsteady MHD Free Convection Flow of Casson Fluid Past an Exponentially Accelerated Infinite Vertical Plate Through Porous Media in The Presence of Thermal Radiation, Chemical Reaction and Heat Source or Sink.

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Abstract:

The aim this paper is to study An Unsteady MHD free convection flow of casson fluid past an exponentially accelerated infinite vertical plate through porous media in the presence of thermal radiation and heat source or sink. Also the first order chemical reactions are taking into an account .At the same time, the plate temperature and concentration of the plate are raised to $T_w^* and C_w^*$ The system of partial differential equation are solved by the analytically by using Laplace Transform Technique procedure. The results for various fields like velocity, temperature, and concentration are displayed and discussed. Numerical values of the skin friction coefficient, the Nusselt number and the Sherwood number for different values of physical parameters are constructed and analyzed. The convergence of the series solutions is examined. The velocity profiles decrease with increases the values of magnetic parameter, Prandtl number, heat source, thermal radiation and Casson parameter.

Keywords — **Casson fluid, MHD, free convection, thermal radiation, chemical reaction, porous medium and heat source/sink.**

1. Introduction:

The study of an unsteady MHD free convective flow of non-Newtonian Casson fluids through porousmedia has many applications in engineering and industry especially in extraction of crude oil from petroleum products. In this category of non-Newtonian fluids Casson fluid has several features. Many researchers have investigated on non-Newtonian Casson fluids like Casson N (1959) investigated a flow equation for pigment oil suspensions of the printing ink type. Dash R. K "et al" (1996), analyzed by Casson Fluid flow in a Pipe Filled With a Homogeneous Porous Medium,Mernone A. V., Mazumdar J. N "et al"(2002), studied byA Mathematical Study of Peristaltic Transport of a Casson Fluid. Boyd .J "et al"(2007), presented by Analysis of The Casson and Carreau-Yasuda Non-Newtonian Blood Models in Steady and Oscillatory Flows Using the Lattice Boltzmann Method. Mustafa, M "et al"(2011) studied unsteady boundary layer flow of a casson fluid due to an impulsively started moving flat plate. Hayat, T "et al"(2012), has investigated Soret and Dufour Effects on Magnetohydrodynamic (MHD) Flow of Casson Fluid. T.Hayat "et al"(2012), analyzed Soret and Dufour effects on magneto hydrodynamic (MHD) flow of Casson fluid.Bhattacharyya, K (2013) presented the boundary layer stagnation-point flow of casson fluid and heat transfer towards a shrinking/stretching sheet.Vajravelu, K. "et al"(2013) discussed the diffusion of chemically reactive species in casson fluid flow over an unsteady permeable stretching surface.Pramanik, S, (2014), presented the casson fluid flow and heat transfer past an exponentially porous stretching surface in the presence of thermal radiation.I.L. Animasaun,(2014) investigated the effects of thermophoresis, variable viscosity and thermalconductivity on free convective heat and mass transfer ofnon-darcian MHD dissipative Casson fluid flow with suction andnth order of chemical reaction.Hussanan A "et al"(2014) studied the unsteady boundary layer flow and heat transfer of a Casson fluid past an oscillating vertical plate with newtonian heating. Emmanuel Maurice "et al"(2015)discussed the analysis of Casson fluid flow over a vertical porous surface with chemical reaction in the presence of magnetic field.Gnaneswara Reddy M (2015) investigated the unsteady radiativeconvectiveboundary layer flow of a Casson fluid with variable thermal conductivity.Kirubhashankar, C. K., Ganesh, S. (2015),analyzed the Casson fluid flow and heat transfer over an unsteady porous stretching surface. A. Naveed "et al" (2015) presented the effectson magnetic field in squeezing flow of a Casson fluid betweenparallel plates. M.Das "et al"(2015) investigated Newtonian heating effect on steady hydro magnetic Casson fluid flow a plate with heat and mass transfer.C.S.K. Raju "et al"(2016)analyzedthe heat and mass transfer in magnetohydrodynamic Casson fluid over an exponentially permeable stretching surface,engineering science and technology. Hari R “et al”(2016) investigated the Soret and heat generation effects on MHD Casson fluid flow past an oscillating vertical plate embedded through porous medium.Hamid Khan “et al”(2016) studied the unsteady Squeezing Flow of Casson Fluid with Magneto hydrodynamic Effect and Passing through Porous Medium.S.Harinath Reddy “et al”(2016) discussed the radiation absorption and chemical reaction effect on MHD flow of heat generating casson fluid past oscillating vertical porous plate.C. Veeresha“et al”(2017) analyzed the Joul heating and thermal diffusion effect on MHD radiative and convective casson fluid flow past an oscillating semi-infinite vertical porousplate,I.L. Animasaun “et al.” (2016) studied the motion of temperature dependent plastic dynamic viscosity and thermal conductivity of steady incompressible laminar free convective magneto hydrodynamic (MHD) Casson fluid flow over an exponentially stretching surface with suction and exponentially decaying internal heat generation.N.Sandeep “ et al.” (2016) Three-dimensional Casson fluid flow towards a stagnation-point and a surface on which the heat energy falls at lower limit of thermodynamic temperature scale in the presence of cross diffusion is investigated. T. M. Ajayi ‘et al.” (2017) examined a non-Newtonian fluid flow past an upper surface of an object that is neither a perfect horizontal/vertical nor inclined/cone in which dissipation of energy is associated with temperature-dependent plastic dynamic viscosity is considered. An attempt has been made to focus on the case of two-dimensional Casson fluid flow over a horizontal melting surface embedded in a thermally stratified medium.

The main aim of this study is to investigate An Unsteady MHD free convection flow of casson fluid past an exponentially accelerated infinite vertical plate through porous media in the presence of thermal radiation, chemical reaction and heat source or sink. The transformed governing are solved analytically by using Laplace Transform technique. The behavior of non-dimensional parameters on the velocity, the temperature and the concentration profiles along with the friction factor, local Nusselt and Sherwood numbers are discussed with the help of graphs and tables.

2. Mathematical formulation:

In this problem, we consider an unsteady MHD free convection heat and mass transfer flow of a viscous, incompressible, electrically, conducting, radiating and chemically reacting fluid past an exponentially accelerated infinite vertical plate in the presence of uniform magnetic field B_0 applied in a transverse direction to the fluid flow. Let x^*-axis is taken along the plate in vertical upward direction to the fluid flow and y^*-axis is taken normal to it in the direction of applied transverse magnetic field. Initially, when$t^* \leq 0$, both the fluid and plate are at stationary condition having constant temperature and concentration. When t^*>0, the plate is exponentially accelerated with the velocity $u^* = u_0 e^{a^* t^*}$. At the same time, the plate temperature and concentration of the plate are raised to $T_w^* and C_w^*$A uniform magnetic fieldB_0 is assumed to be applied normal to the flow. For free convection flow, it is also assumed that, the induced magnetic field is assumed to be negligible as the magnetic Reynoldsnumber of the flow is taken to be very small. The viscous dissipation is taken in the energy equation. The effects of variation in density (ρ) with temperature and species concentration areconsidered only on the body force term, in accordance with usual Boussinesq approximation. The fluid considered here is gray, absorbing/emitting radiation but a non-scatteringmedium. Since the flow of the fluid is assumed to be in the direction of x^*-axis, so the physicalquantities are functions of the space co-ordinate $y^*$and t^*only. The rheological equation of state for an isotropic and incompressible flow of a casson fluid is as follows.

$$\tau_{ij} = \begin{cases} 2\left(\mu_\alpha + \frac{P_y}{\sqrt{2\pi}}\right) e_{ij}, & \pi > \pi_c \\ 2\left(\mu_\alpha + \frac{P_y}{\sqrt{2\pi}}\right) e_{ij}, & \pi < \pi_c \end{cases}$$

here $\pi = e_{ij} e_{ij}$and $e_{ij}$are the $(i,j)^{th}$component of the deformation rate, π is the product of the component of the deformation rate with itself, $\pi_c$is a critical value of this product based on the non-Newtonian model, $\pi_\alpha$is plastic dynamic viscosity of non-Newtonian fluid, and $P_y$is the yield stress of the fluid.

Then by usual Boussinesq’s approximation, the flow is governed by the following equations:
Momentum Equation:

$$\frac{\partial u^*}{\partial t^*} = g\beta(T^* - T_\infty^*) + g\beta^*(C^* - C_\infty^*)$$

$$+\nu\left(1+\frac{1}{\alpha}\right)\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2}{\rho}u^* - \frac{\nu}{k}u^*$$

(1)

Energy Equation:

$$\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y^*} - \frac{Q}{\rho C_p}^*(T^* - T_\infty^*)$$

(2)

Diffusion Equation:

$$\frac{\partial C^*}{\partial t^*} = D_m \frac{\partial^2 T^*}{\partial y^{*2}} - K_r (C^* - C_\infty^*) \quad (3)$$

The initial and boundary conditions are:

$$t^* \le 0, u^* = 0, T^* = T_\infty^*, C^* = C_\infty^* \; for\, all\; y^*$$

$$t^* > 0,\; u^* = u_0 e^{a^* t^*}, T^* = T_w{}^*, C^* = C_w{}^* \; at\; y^* = 0$$

$$u^* = 0, T^* = T_\infty^*, C^* = C_\infty^* \; as\; y \to \infty$$

(4)

$$A = \frac{u_0^2}{v}$$

Where $\alpha, u^*, \beta, \beta^*, B_0, v, \kappa, \rho, T^*, C^*, C_p, C_s, K_T, q_r, Q, \sigma, D_m,$

$t, a and K_r$ are respectively casson parameter, the fluid in the x^*-direction, coefficient of thermal expansion, coefficient of expansion with concentration, external magnetic field, kinematic viscosity, thermal conductivity, fluid density, temperature of the fluid near the plate, Species concentration, Specific heat at constant pressure, Concentration susceptibility, radiative heat flux, heat absorption, electric conductivity, Coefficient of mass diffusivity ,time, acceleration parameter and chemical reaction parameter.

The radiative heat flux q_r, under Rosseland approximation of the form

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^*}{\partial y^*} \quad (5)$$

Where σ*is the Stefan-Boltzmann constant and $k^*$is the mean absorption coefficient.

It is assumed that the temperature differences within the flow are sufficiently small and that T^{*4} may be expressed as a linear function of the temperature. This is obtained by expanding T^{*4} in a Taylor series about T_∞^* and neglecting the higher order terms, thus we get

$$T^{*4} = 4T_\infty^{*3} T^* - 3T_\infty^{*4}. \quad (6)$$

From Equations (5) and (6), Equation (2) reduces to

$$\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{16\sigma^* T_\infty^*}{3\rho C_p k^*} \frac{\partial^2 T^*}{\partial y^{*2}}$$

$$-\frac{Q}{\rho C_p}^* (T^* - T_\infty^*) \quad (7)$$

On introducing the following non- dimensional quantities

$$u = \frac{u^*}{u_0}, y = \frac{u_0}{v} y^*, t = \frac{u_0^2}{v} t^*, K = \frac{k^* u_0^2}{v},$$

$$M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \theta = \frac{(T^* - T_\infty^*)}{(T_w{}^* - T_\infty^*)}, \phi = \frac{(C^* - C_\infty^*)}{(C_w{}^* - C_\infty^*)},$$

$$Gr = \frac{vg\beta(T_w{}^* - T_\infty^*)}{u_0^3}, Gm = \frac{vg\beta^*(C_w{}^* - C_\infty^*)}{u_0^3},$$

$$\Pr = \frac{\rho v C_p}{\kappa}, k = \frac{k^* u_0^2}{v}, Q = \frac{Q^* v}{\rho C_p u_0^2}, R = \frac{k^* k}{4\sigma^* T_\infty^*},$$

$$a = \frac{v}{u_0{}^2} a^*,\; Sc = \frac{v}{D}, \quad (8)$$

In view of (8) the Equations (1), (3) and (7), reduce to the following non-dimensionalforms

$$\frac{\partial u}{\partial t} = \left(1 + \frac{1}{\alpha}\right) \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right) u + G_r \theta + G_m \phi \quad (9)$$

$$\Pr \frac{\partial \theta}{\partial t} = \left(1 + \frac{4}{3R}\right) \frac{\partial^2 \theta}{\partial y^2} - \Pr Q \theta \quad (10)$$

$$Sc \frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial y^2} - ScKr\phi \quad (11)$$

The corresponding boundary conditions reduce to

$$t \le 0,\; u = 0, \theta = 0, \phi = 0 \; for\, all\; y$$

$$t > 0, u = e^{at}, \theta = 1, \phi = 1 \; at\; y = 0 \quad (12)$$

$$u \to 0, \theta \to 0, \phi \to 0 \; as\; y \to \infty$$

3. Solution of the Problem:

The system of equations (8),(9) and (10) with subject to the boundary conditions in (11), are solved by analytically using Laplace Transform technique and the expressions for

$$\varphi(y,t)=\frac{1}{2}\left[\begin{array}{l}\exp\left(-y\sqrt{Sc\,kr}\right)erfc\left(\dfrac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Kr\,t}\right)\\+\exp\left(y\sqrt{Sc\,kr}\right)erfc\left(\dfrac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Kr\,t}\right)\end{array}\right] \quad (13)$$

$$\theta(y,t)=\frac{1}{2}\left[\begin{array}{l}\exp\left(-y\sqrt{A_1\Pr Q}\right)erfc\left(\dfrac{y\sqrt{A_1\Pr}}{2\sqrt{t}}-\sqrt{Q\,t}\right)\\+\exp\left(y\sqrt{A_1\Pr Q}\right)erfc\left(\dfrac{y\sqrt{A_1\Pr}}{2\sqrt{t}}+\sqrt{Q\,t}\right)\end{array}\right] \quad (14)$$

$$u(y,t)=\frac{e^{at}}{2}\left[\begin{array}{l}\exp\left(-y\sqrt{A_2(A_3+a)}\right)erfc\left(\dfrac{y\sqrt{A_2}}{2\sqrt{t}}-\sqrt{(A_3+a)t}\right)\\+\exp\left(y\sqrt{A_2(A_3+a)}\right)erfc\left(\dfrac{y\sqrt{A_2}}{2\sqrt{t}}+\sqrt{(A_3+a)t}\right)\end{array}\right]$$
$$+\frac{be^{ct}}{2c}\left[\begin{array}{l}\exp\left(-y\sqrt{A_2(A_3+c)}\right)erfc\left(\dfrac{y\sqrt{A_2}}{2\sqrt{t}}-\sqrt{(A_3+c)t}\right)\\+\exp\left(y\sqrt{A_2(A_3+c)}\right)erfc\left(\dfrac{y\sqrt{A_2}}{2\sqrt{t}}+\sqrt{(A_3+c)t}\right)\end{array}\right]$$
$$-\frac{1}{2}\left[\frac{b}{c}+\frac{d}{e}\right]\left[\begin{array}{l}\exp\left(-y\sqrt{A_2A_3}\right)erfc\left(\dfrac{y\sqrt{A_2}}{2\sqrt{t}}-\sqrt{A_3\,t}\right)\\+\exp\left(y\sqrt{A_2A_3}\right)erfc\left(\dfrac{y\sqrt{A_2}}{2\sqrt{t}}+\sqrt{A_3\,t}\right)\end{array}\right]$$
$$+\frac{d\,e^{et}}{2e}\left[\begin{array}{l}\exp\left(-y\sqrt{A_2(A_3+e)}\right)erfc\left(\dfrac{y\sqrt{A_2}}{2\sqrt{t}}-\sqrt{(A_3+e)t}\right)\\+\exp\left(y\sqrt{A_2(A_3+e)}\right)erfc\left(\dfrac{y\sqrt{A_2}}{2\sqrt{t}}+\sqrt{(A_3+e)t}\right)\end{array}\right]$$
$$-\frac{d\,e^{ct}}{2c}\left[\begin{array}{l}\exp\left(-y\sqrt{A_1\Pr(c+Q)}\right)erfc\left(\dfrac{y\sqrt{A_1\Pr}}{2\sqrt{t}}-\sqrt{(c+Q)t}\right)\\+\exp\left(y\sqrt{A_1\Pr(c+Q)}\right)erfc\left(\dfrac{y\sqrt{A_1\Pr}}{2\sqrt{t}}+\sqrt{(c+Q)t}\right)\end{array}\right]$$
$$+\frac{b}{2c}\left[\begin{array}{l}\exp\left(-y\sqrt{A_1\Pr Q}\right)erfc\left(\dfrac{y\sqrt{A_1\Pr}}{2\sqrt{t}}-\sqrt{Q\,t}\right)\\+\exp\left(y\sqrt{A_1\Pr Q}\right)erfc\left(\dfrac{y\sqrt{A_1\Pr}}{2\sqrt{t}}+\sqrt{Q\,t}\right)\end{array}\right]$$
$$-\frac{d\,e^{et}}{2e}\left[\begin{array}{l}\exp\left(-y\sqrt{Sc(e+Kr)}\right)erfc\left(\dfrac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{(e+Kr)t}\right)\\+\exp\left(y\sqrt{Sc(e+Kr)}\right)erfc\left(\dfrac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{(e+Kr)t}\right)\end{array}\right]$$
$$+\frac{d}{2e}\left[\begin{array}{l}\exp\left(-y\sqrt{ScKr}\right)erfc\left(\dfrac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Kr\,t}\right)\\+\exp\left(y\sqrt{ScKr}\right)erfc\left(\dfrac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Kr\,t}\right)\end{array}\right] \quad (15)$$

Where $A_1=\dfrac{3R}{3R+4}, A_2=\dfrac{\alpha}{1+\alpha}, A_3=M+\dfrac{1}{K},$

$$b=\frac{GrA_2}{A_1\Pr-A_2}, c=\frac{A_2A_3-A_1\Pr Q}{A_1\Pr-A_2},$$
$$d=\frac{GmA_2}{Sc-A_2}, c=\frac{A_2A_3-ScKr}{Sc-A_2},$$

4. Nussle number:

The rate of change of heat transfer is given by

$$Nu=-\left[\frac{\partial\theta}{\partial y}\right]_{y=0} \quad (16)$$

From equations (14) and (16), we get Nussle number as follows

$$Nu=\sqrt{\frac{A_1Pr}{\pi t}}e^{-Qt}+\sqrt{A_1PrQ}\,erf\left(\sqrt{Qt}\right)$$

5. Sherwood Number:

The rate of change of mass transfer is given by

$$Sh=-\left[\frac{\partial\phi}{\partial y}\right]_{y=0} \quad (17)$$

From equations (13) and (17), we get Nussle number as follows

$$Sh=\sqrt{\frac{Sc}{\pi t}}e^{-Kr\,t}+\sqrt{ScKr}\,erf\left(\sqrt{Kr\,t}\right)$$

6. Skin friction:

The rate of change of velocity is given by

$$\tau=-\left(1+\frac{1}{\alpha}\right)\left[\frac{\partial u}{\partial y}\right]_{y=0} \quad (18)$$

From equations (15) and (18), we get Nussle number as follows

$$\tau = -e^{at}\left[\sqrt{\frac{A_2}{\pi t}}\,e^{-\left(A_3+a\right)t} + \sqrt{A_2(A_3+a)}\,erf\left(\sqrt{(A_3+a)t}\right)\right]$$

$$-\frac{b\,e^{ct}}{c}\left[\sqrt{\frac{A_2}{\pi t}}\,e^{-\left(A_3+c\right)t} + \sqrt{A_2(A_3+c)}\,erf\left(\sqrt{(A_3+c)t}\right)\right]$$

$$+\left(\frac{b}{c}+\frac{d}{e}\right)\left[\sqrt{\frac{A_2}{\pi t}}\,e^{-A_3 t} + \sqrt{A_2 A_3}\,erf\left(\sqrt{A_3 t}\right)\right]$$

$$-\frac{d\,e^{et}}{e}\left[\sqrt{\frac{A_2}{\pi t}}\,e^{-\left(A_3+e\right)t} + \sqrt{A_2(A_3+e)}\,erf\left(\sqrt{(A_3+e)t}\right)\right]$$

$$+\frac{b\,e^{ct}}{c}\left[\sqrt{\frac{A_1 \Pr}{\pi t}}\,e^{-\left(c+Q\right)t} + \sqrt{A_2 \Pr(c+Q)}\,erf\left(\sqrt{(c+Q)t}\right)\right]$$

$$-\frac{b}{2c}\left[\sqrt{\frac{A_1 \Pr}{\pi t}}\,e^{-Qt} + \sqrt{A_2 \Pr Q}\,erf\left(\sqrt{Qt}\right)\right]$$

$$+\frac{d\,e^{et}}{e}\left[\sqrt{\frac{Sc}{\pi t}}\,e^{-\left(Kr+e\right)t} + \sqrt{Sc(Kr+e)}\,erf\left(\sqrt{(Kr+e)t}\right)\right]$$

$$-\frac{d}{e}\left[\sqrt{\frac{Sc}{\pi t}}\,e^{-Kr\,t} + \sqrt{ScKr}\,erf\left(\sqrt{Kr\,t}\right)\right]$$

erfc = Complementary Error function
erf = Error function

6.Result and Discussion:

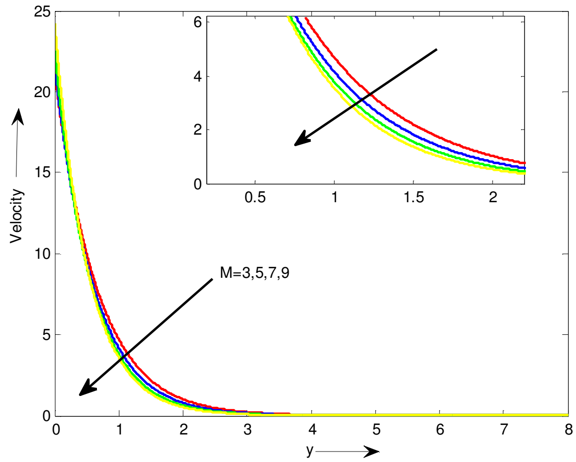
The systems of linear differential equations, with the boundary conditions are solved analytically by using Laplace Transform technique. The obtained results show the effect of the various non-dimensional governing parameters, such as Casson parameter (α), thermal radiation parameter (R), magnetic parameter (M),Schmidt number (Sc),), thermal Grashof number(Gr), mass Grashof number (Gm), chemical reaction(Kr), heat absorption parameter (Q), acceleration parameter *(a),* permeability parameter(K) and time parameter t on the flow ,velocity, temperature and concentration profiles. Also the Nusselt number and the Sherwood number are presented in the tabular form. Though out the computations we employ $M=2,\ Pr=0.71,\ K=0.5,\ Gr=3,\ Gm=3,\ t=0.6,\ Q=0.1,\ Sc=0.6,\ R=3,\ Kr=0.5, \alpha=0.3,\ a=2$

The effect of magnetic parameter *(M)* on the velocity field is shown in figure (1). It is observed that the velocity decreases with increasing the values of magnetic parameter. This is due to fact that an increase in magnetic field develops the opposite force to flow direction, which is called Lorentz force. This force has the tends to slow down the fluid motion. The influences of Prandtl number *(Pr)* on velocity field are shown figure (2). It is observed that the velocity decreases with increasing the values of Pr.The velocity profiles for different values of permeability parameter *(K)* are shown in Figures (3), from this, it is seen that the velocity increases with increasing values of permeability parameter.The effects of thermal Grashof number *(Gr)* on the velocity are shown figure (4). From these figures it is observed that the velocity decreases with increasing values of *Gr*.The effects of mass Grashof number *(Gm)* on the velocity are shown figure (5). From these figures it is observed that the velocity increases with increasing values of *Gm.*It is possible because with increasing the values of *Gr* and *Gm* has tends to increases the thermal and mass buoyancy forces. From figure (6), depicts the velocity profiles increases as increasing the values of time *(t).* From figures (7) and (8) show the effect of heat absorption parameter *(Q),* radiation parameter*(R)* on the velocity field. From these figures it is observed that the velocity decreases with increasing the values of *Q* and *R*. The influences of Schmidt number *(Sc)* and chemical reaction parameter*(Kr)* on velocity profiles are shown figures (9) and (10). It is observed that the velocity decreases with increasing the values of, *Sc* and *Kr*.The influence of casson parameter αon the velocity profiles in figure (11).depicts the velocity profiles for various values of the casson parameter decreases yield stress and suppress the velocity field. The fluid behavior as Newtonian fluid as casson parameter becomes very large. From this figure it is observed that the velocity decreases with increasing the values of casson parameter. From figure (12), depicts the velocity increases with increasing the values of acceleration parameter *(a).*If acceleration parameter vanish we get uniform velocity.

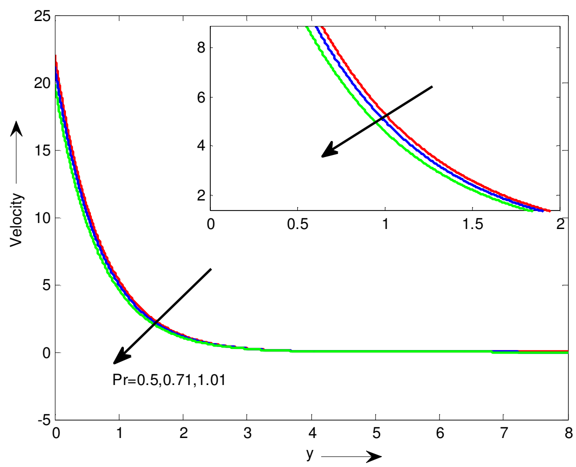
The influences of various flow parameters on the fluid temperature are given in figures (13)-(16). The Prandtl number *(Pr)* is defined as the ratio of kinematic viscosity to thermal diffusivity. The effect of Prandtl number on the temperature profile is shown in figure. (13).It is observed from this figure that increasing the values of Pr decreases the fluid temperature. From figure (14), depicts the temperature profiles increases as increasing the values of time *(t).* From figure (15) displays the effect of heat source parameter on temperature profiles of the flow. It is noticed that an increase in the heat absorption parameter (Q) reduces the temperature profiles of the flow. It is expected that an increase in heat source parameter will release the heat energy to the flow. This causes the temperature profiles to enhance. Due to the domination of the external heat compared with the heat source supplied to the flow. We noticed that reverse results to the expected result. The effect of thermal radiation parameter (R) on the fluid temperature in figure (16).It is observed that increasing the thermal radiation parameter decreases the temperature of the fluid.

The concentration profiles for different vales of Schmidt number (Sc), Chemical reaction (Kr) and time (t) are presented in figures (17) and (19). Schmidt number is defined as the ratio of kinematic viscosity to the thermal diffusivity.

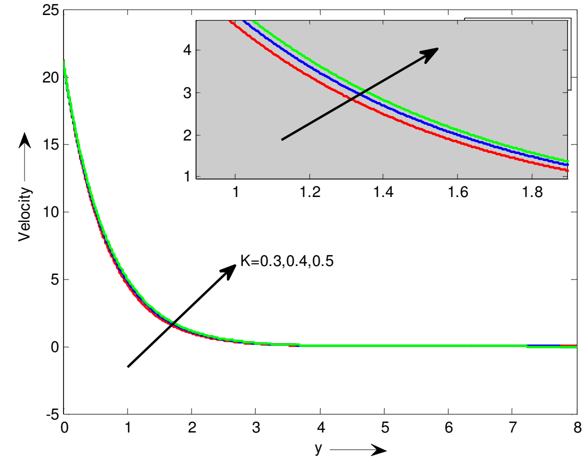
From figure (17), displays the influence of Schmidt number Sc increases, the concentration profiles decreases. The effect of chemical reaction parameter Kr on the concentration profiles presented in figure. (18). It is clear that increasing the chemical reaction parameter decreases the concentration profile on the fluid. From figure (19), depicts the concentration profiles increases as increasing the values of time *(t)*.



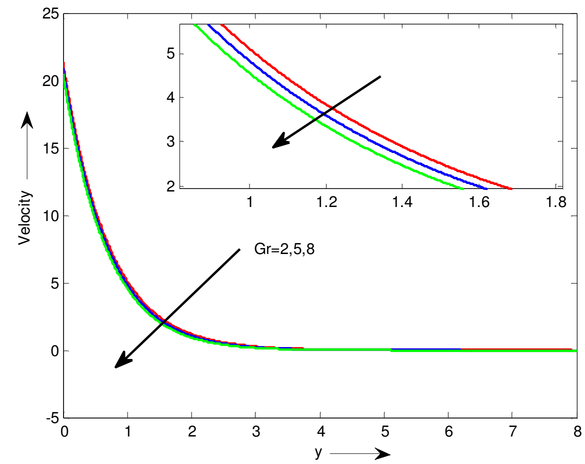
1. The influence of M on Velocity field



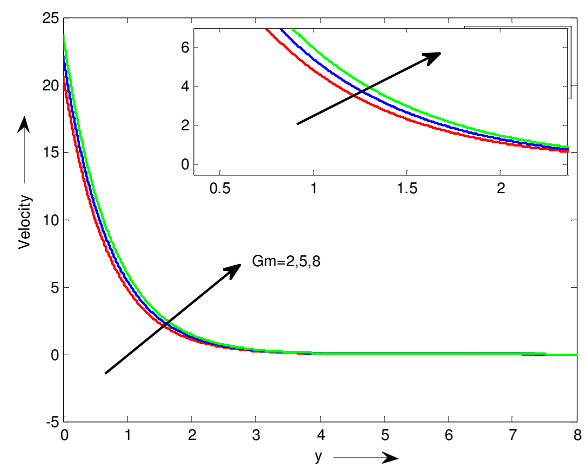
2. The influence of Pr on Velocity field



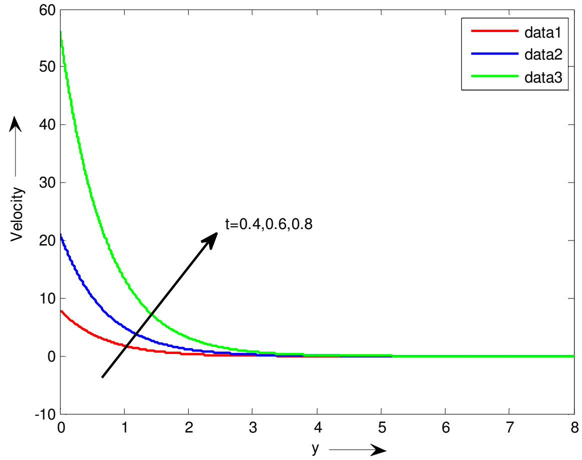
3. The influence of K on Velocity field



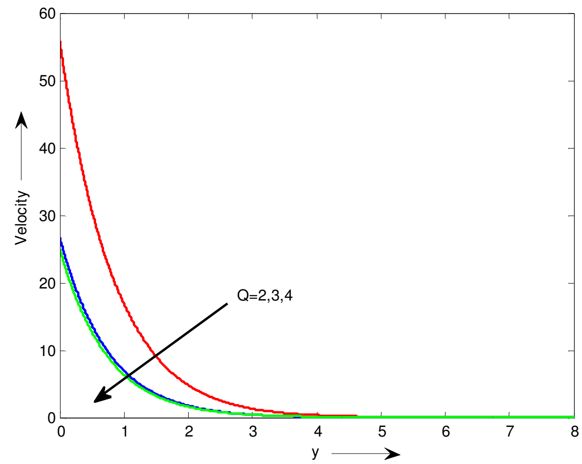
4. The influence of Gr on Velocity field



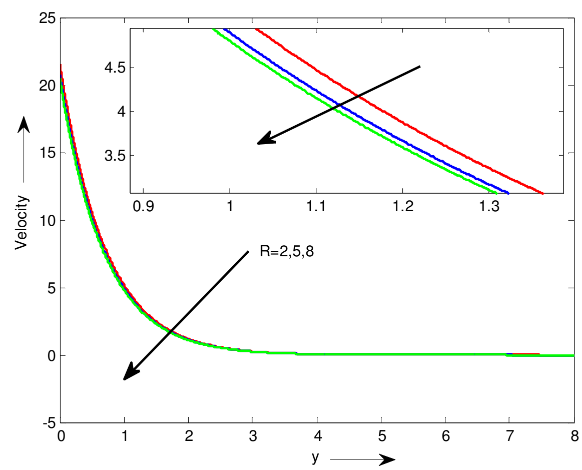
5. The influence of Gm on Velocity field



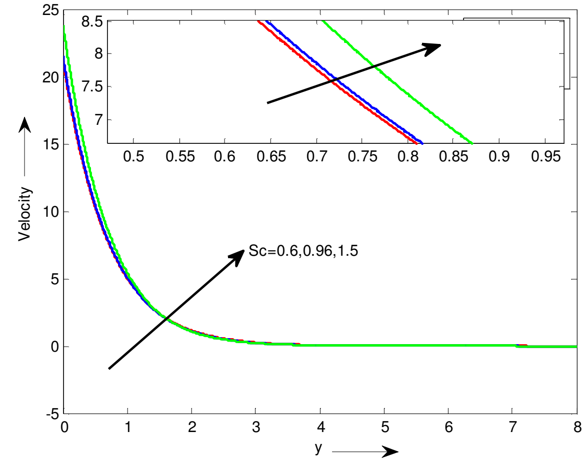
6. The influence of t on Velocity field



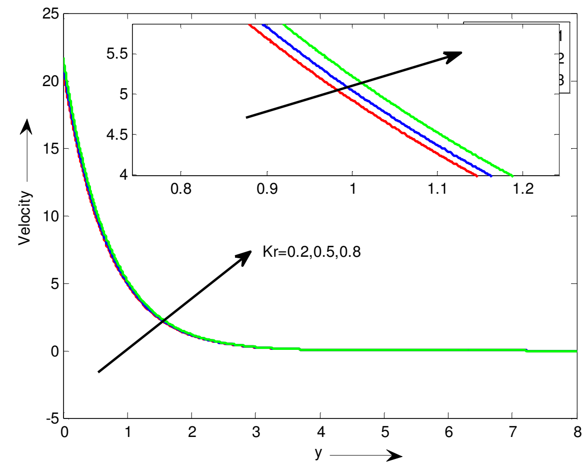
7. The influence of Q on Velocity field



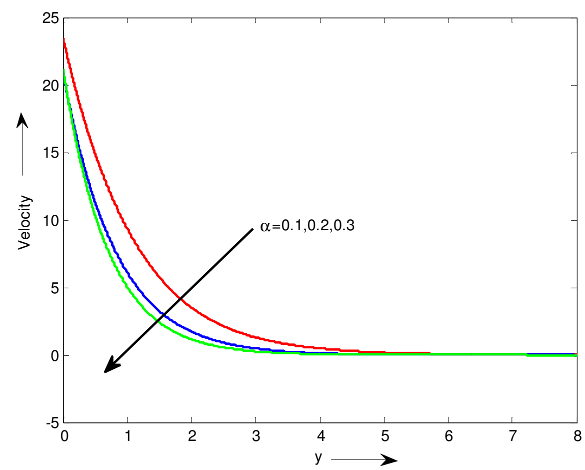
8. The influence of R on Velocity field



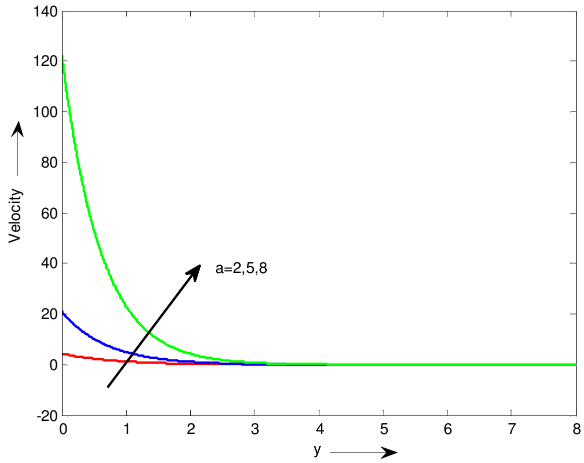
9. The influence of Sc on Velocity field



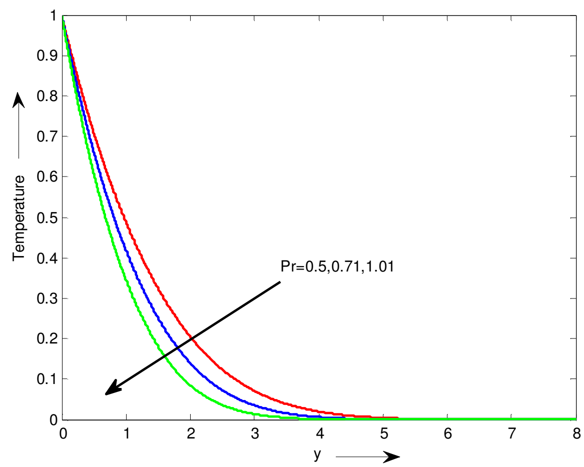
10. The influence of Kr on Velocity field



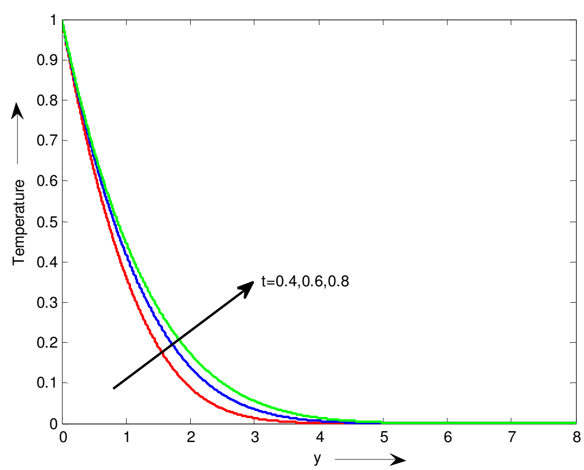
11. The influence of α on Velocity field



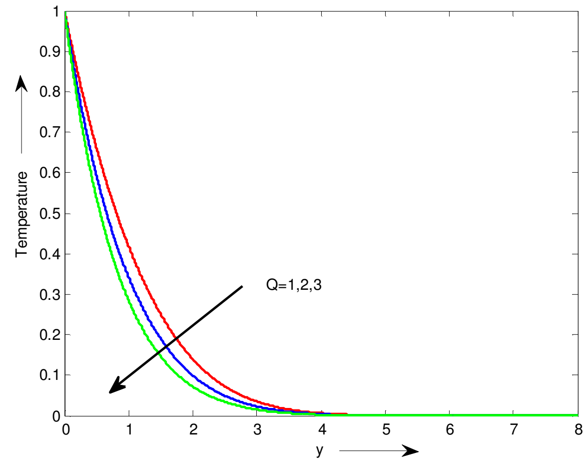
12. The influence of a on Velocity field



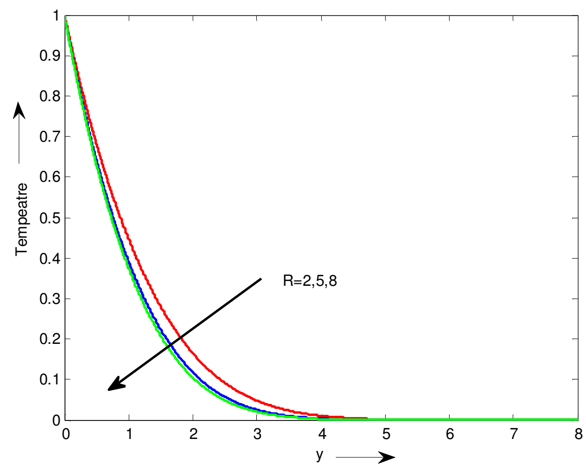
13. The influence of Pr on Temperature field



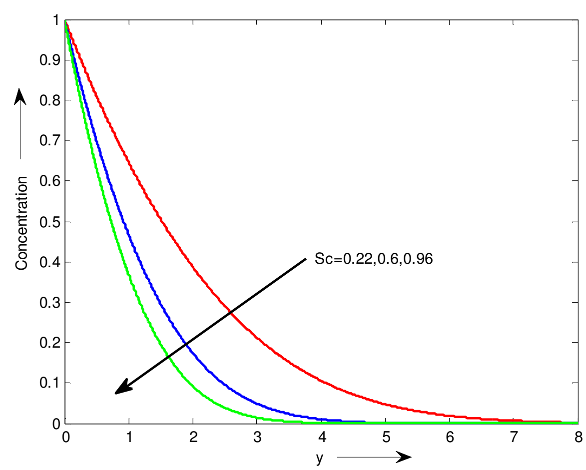
14. The influence of t on Temperature field



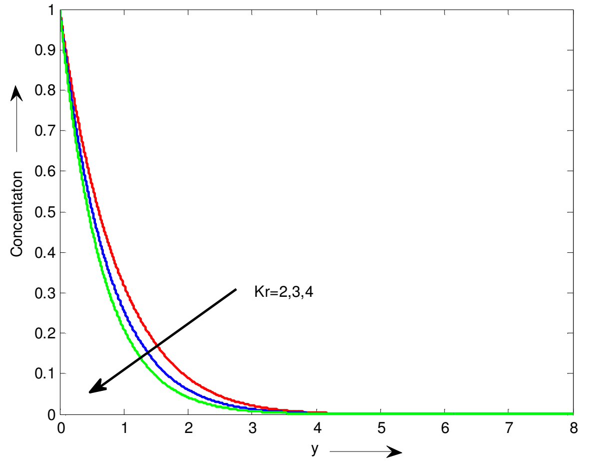
15. The influence of Q on Temperature field



16. The influence of R on Temperature field



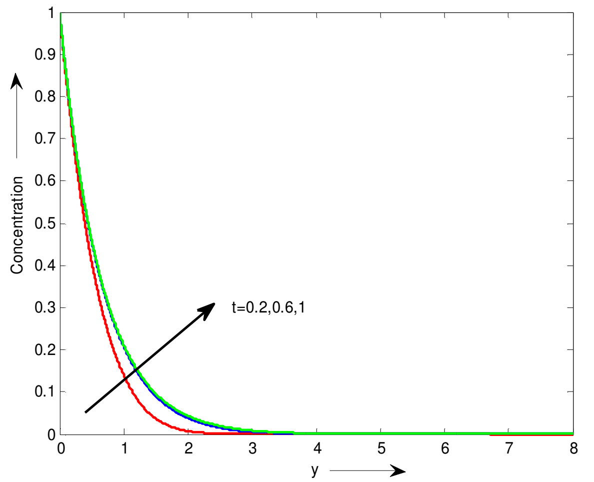
17. The influence of Sc on Concentration field



18. The influence of Kr on Concentration field

Table.1: Nusselet Number

Pr	*R*	*Q*	*t*	*Nu*
0.71	**2**	**1**	**0.8**	
0.5	2	1	0.8	**0.5591**
0.71	2	1	0.8	**0.6663**
7	2	1	0.8	**2.0936**
0.71	**2**	1	0.8	**0.6663**
0.71	**3**	1	0.8	**0.7157**
0.71	**4**	1	0.8	**0.7449**
0.71	2	**1**	0.8	**0.6663**
0.71	2	**2**	0.8	**0.9216**
0.71	2	**3**	0.8	**1.1282**
0.71	2	1	**0.4**	**0.5666**
0.71	2	1	**0.6**	**0.6308**
0.71	2	1	**0.8**	**0.6663**



19. The influence of t on Concentration field

Table.1: Sherwood Number

Sc	*kr*	*t*	*Sh*
0.22	**0.2**	**0.8**	
0.22	0.2	0.8	**0.2916**
0.6	0.2	0.8	**0.4815**
0.96	0.2	0.8	**0.5897**
0.22	**0.2**	0.8	**0.2916**
0.22	**0.5**	0.8	**0.3672**
0.22	**0.8**	0.8	**0.4361**
0.22	0.2	**0.4**	**0.2197**
0.22	0.2	**0.6**	**0.2606**
0.22	0.2	**0.8**	**0.2916**

Table-3; Skin-fiction

M	*Pr*	*K*	*Gr*	*Gm*	*t*	*Q*	*R*	*Sc*	*Kr*	*α*	*a*	*τ*
2	**o.71**	**o.2**	**10**	**10**	**0.4**	**0.1**	**3**	**0.6**	**0.2**	**0.03**	**2**	
3	o.71	o.2	10	10	0.4	0.1	3	0.6	**0.2**	**0.03**	**2**	**5.3319**
4	o.71	o.2	10	10	0.4	0.1	3	0.6	**0.2**	**0.03**	**2**	**13.4229**
2	**0.5**	o.2	10	10	0.4	0.1	3	0.6	**0.2**	**0.03**	**2**	**16.3831**
2	**0.71**	o.2	10	10	0.4	0.1	3	0.6	**0.2**	**0.03**	**2**	**-4.9357**
2	o.71	**0.3**	10	10	0.4	0.1	3	0.6	**0.2**	**0.03**	**2**	**-32.6311**
2	o.71	**0.4**	10	10	0.4	0.1	3	0.6	**0.2**	**0.03**	**2**	**-67.4562**
2	o.71	o.2	**8**	10	0.4	0.1	3	0.6	**0.2**	**0.03**	**2**	**-5.6473**
2	o.71	o.2	**9**	10	0.4	0.1	3	0.6	**0.2**	**0.03**	**2**	**-5.2915**
2	o.71	o.2	10	**8**	0.4	0.1	3	0.6	**0.2**	**0.03**	**2**	**3.9491**
2	o.71	o.2	10	**9**	0.4	0.1	3	0.6	**0.2**	**0.03**	**2**	**-0.4933**

2	o.71	o.2	10	10	**0.4**	0.1	3	0.6	**0.2**	**0.03**	**2**	**25.8540**
2	o.71	o.2	10	10	**0.6**	0.1	3	0.6	**0.2**	**0.03**	**2**	**54.6187**
2	o.71	o.2	10	10	0.4	**0.2**	3	0.6	**0.2**	**0.03**	**2**	**-23.2528**
2	o.71	o.2	10	10	0.4	**0.3**	3	0.6	**0.2**	**0.03**	**2**	**-72.3369**
2	o.71	o.2	10	10	0.4	0.1	**4**	0.6	**0.2**	**0.03**	**2**	**-11.3845**
2	o.71	o.2	10	10	0.4	0.1	**5**	0.6	**0.2**	**0.03**	**2**	**-16.0013**
2	o.71	o.2	10	10	0.4	0.1	3	**0.6**	**0.2**	**0.03**	**2**	**-4.9357**
2	o.71	o.2	10	10	0.4	0.1	3	**0.9**	**0.2**	**0.03**	**2**	**8.3298**
2	o.71	o.2	10	10	0.4	0.1	3	0.6	**0.5**	**0.03**	**2**	**-7.0911**
2	o.71	o.2	10	10	0.4	0.1	3	0.6	**0.6**	**0.03**	**2**	**-6.6249**
2	o.71	o.2	10	10	0.4	0.1	3	0.6	**0.2**	**0.04**	**2**	**3.1852**
2	o.71	o.2	10	10	0.4	0.1	3	0.6	**0.2**	**0.05**	**2**	**7.8143**
2	o.71	o.2	10	10	0.4	0.1	3	0.6	**0.2**	**0.03**	**3**	**16.4406**
2	o.71	o.2	10	10	0.4	0.1	3	0.6	**0.2**	**0.03**	**4**	**49.8708**

The bold values in table indicate the variation of the parameters and variables present in the corresponding column

Effect of casson parameter, acceleration parameter, Schmidt number, magnetic number, thermal Grashof number, mass Grashof number ,permeability parameter, Radiation parameter, chemical reaction parameter, heat source/absorption parameter ,Prandtl number and Eckert number on Skin friction, Nusselt number and Sherwood number is presented in tables. From table-1, the effect of the rate change of temperature increases when increasing the values of Prandtl number, thermal radiation, heat source/absorption and time. From table-2,the influence of the rate of change of concentration increases when increasing the values of Schmidt number, chemical reaction and time. From table-3,it is observed that, the Skin friction increases with an increase in magnetic parameter, Grashof number, Schmidt number, casson parameter and acceleration parameter. Where as it decreases in the presence of Prandtl number, modified Grashof number, time and Eckert number,permeability parameter, radiation parameter, chemical reaction parameter and heat source/absorption parameter.

Conclusion:

In the present study the effect of An Unsteady MHD free convection flow of casson fluid past an exponentially accelerated infinite vertical plate through porous media in the presence of thermal radiation, chemical reaction and heat source or sink. The dimensions less governing equation are solved analytically by using Laplace Transform technique.The results for the velocity, the temperature and the concentration are plotted graphically and the Nussle number, Sherwood number and Skinfriction are shown in tables.

The following conclusions are made:

- ❖ The velocity increases with increasing the vales of *K, Gm, t, Sc, Kr and a.* while the velocity reduces when *M, Pr, Gr, Q, R, and*α increases.
- ❖ The temperature reduces when *Pr, Q, and R*.While it is rise only when t rises.
- ❖ The Concentration decreases with increasing the vales of *Sc and Kr.* While it is rise only when t rises.
- ❖ The Nusselt number increase when *Pr, R, Q and t* are increases.
- ❖ The Sherwoodnumber increases when *Sc, Kr and t* are increases.
- ❖ The Skin friction increases when *M, Gr, Sc,* α *and a* are increases. While it decreases when *Pr, K, Gm, t, Q, R and Kr* are increases.

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