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Design Optimization of Planar Mechanisms

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Abstract:

This paper presents an optimization technique to dynamically balance the planar five-bar mechanisms in which the shaking force and shaking moment are minimized using the genetic algorithm (GA). A dynamically equivalent system of point-masses that represents each rigid link of a mechanism is developed to represent link's inertial properties. The shaking force and shaking moment are then expressed in terms of the point-mass parameters which are taken as the design variables. These design variables are brought into the optimization scheme to reduce the shaking force and shaking moment. This formulates the objective function which optimizes the mass distribution of each link. The balancing problem is formulated as a multi-objective optimization problem and multiple optimal solutions are created as a Pareto front by using the genetic algorithm. The masses and inertias of the optimized links are computed from the optimized design variables. The effectiveness of the proposed methodology is shown by applying it to a problem of five-bar planar mechanism available in the literature.

Keywords — **Dynamic balancing, Shaking force and shaking moment, Equimomental system, Optimization, Genetic algorithm.**

1. INTRODUCTION

An unbalanced mechanism running at high speed transmits forces and moments to the ground known as shaking forces and shaking moments. These forces and moments are vector sum of the inertia forces and moments of all the moving links. They adversely affect the dynamic performance of the mechanism. Several techniques are presented in the literature for reducing these shaking forces and shaking moments due to inertia. The complete force balancing can be achieved by making the mass center of moving links of a mechanism stationary [1]. This is achieved either by mass redistribution or by adding counterweights to the moving links. This methodology was extended for the mechanisms having prismatic joints under certain conditions [2, 3]. Force balancing and trajectory tracking is achieved in a five-bar real-time controllable mechanism using adjusting kinematics parameter approach [4].

The complete force balancing increases other dynamic performance characteristics such as shaking moment, driving torque and bearing forces in joints [5]. Therefore, along with the full force balancing, several methods proposed in the literature to balance the shaking moment [6, 7]. The complete force and moment balancing is achieved by adding duplicate mechanism, inertia or disk counterweights [8-10]. However, this method is not recommended due to complexity and practical reasons.

Several trade-off methods were developed to minimize different dynamic quantities simultaneously [11, 12]. As the shaking force and shaking moment depend on link masses, their locations of mass centers and moment of inertias, these trade-off methods find the optimal distribution of the link masses [13].

The conventional optimization methods like gradient based search method is used to optimally balance the planar mechanisms [14,15] and to analyse the sensitivity of shaking force and shaking moment to the design variables [16]. Optimum force balancing is achieved for a five-bar mechanism using natural orthogonal complement dynamic modeling [17]. The shaking moment is minimized in five-bar manipulator through constrained nonlinear optimisation problem in which shaking force elimination is presented as the balancing constraints [18]. The conventional optimization methods require an initial guess point to start searching the optimum solution and likely to produce local optimum solution close to the start point.

The evolutionary optimization techniques like particle swarm optimization (PSO) and genetic algorithm (GA) can be applied to minimize multi-objective functions subject to some design constraints [19, 20]. The mixed mass redistribution method using genetic algorithm is applied for reducing shaking force and shaking moment in mechanisms [21].

In this paper, the formulation of optimization problem is simplified by modelling the rigid links of mechanism as dynamically equivalent system of point-masses, known as *equimomental system* [22, 23]. The balancing problem is formulated as a multi-objective optimization problem and solved using genetic algorithm. This algorithm doesn't require a start point and searches the solution in the entire design space. Therefore, it produces the global optimum solution for the optimization problem. Also, for a multi-objective optimization problem, it produces several solutions which are all pareto optimum. Any solution among these can be chosen as per the specific requirement.

The structure of this paper is as follows. Section 2 presents the equations of motion for rigid body and equimomental point-masses. Problem of minimizing shaking force and shaking moment for a five-bar mechanism is formulated in Section 3. A numerical example is solved using the proposed method and its results are presented in Section 4. Finally, conclusions are given in Section 5.

2. EQUIMOMENTAL SYSTEM OF POINT-MASSES

In this section, the concept of equimomental system of point-masses is discussed and the dynamic equation of motion for a rigid body is rewritten in terms of the point-masses.

2.1. Equations of motion of rigid body

The links of a mechanism can be modelled as rigid bodies for simplifying the kinematic and dynamic analyses. Consider an *i*th rigid link having motion in *XY* plane for which a local frame, $X_i Y_i$, is fixed at O_i on the body. The Newton-Euler (NE) equations of motion for this link in the fixed inertial frame, *OXY*, are written as [11]:

$$\mathbf{M}_i \dot{\mathbf{t}}_i + \mathbf{C}_i \mathbf{t}_i = \mathbf{w}_i . \quad (1)$$

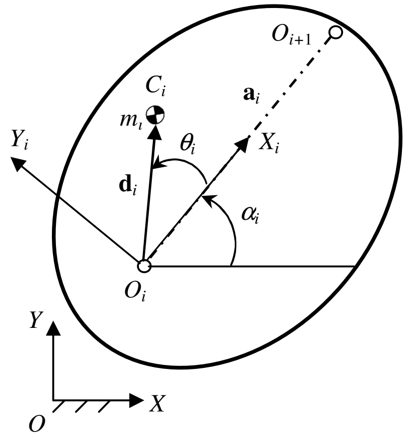


Figure 1. The *i*th rigid link moving in *XY* plane

In Eq. (1), 3 vectors, $\mathbf{t}_i$, $\dot{\mathbf{t}}_i$ and $\mathbf{w}_i$ are twist, twist-rate, and wrench vectors of the *i*th link with respect to O_i, respectively, i.e.,

$$\mathbf{t}_i = \begin{bmatrix} \omega_i \\ \mathbf{v}_i \end{bmatrix}; \quad \dot{\mathbf{t}}_i = \begin{bmatrix} \dot{\omega}_i \\ \dot{\mathbf{v}}_i \end{bmatrix} \quad \text{and} \quad \mathbf{w}_i = \begin{bmatrix} n_i \\ \mathbf{f}_i \end{bmatrix} \tag{2}$$

where, ω_i and $\mathbf{v}_i$ are the scalar angular velocity about the axis perpendicular to the plane of motion and the 2-vector of linear velocity of the origin O_i, respectively. Accordingly, $\dot{\omega}_i$ and $\dot{\mathbf{v}}_i$ are time derivatives of ω_i and $\mathbf{v}_i$, respectively. Also, the scalar, n_i, and the 2-vector, $\mathbf{f}_i$, are the resultant moment about O_i and the resultant force at O_i, respectively. In Eq. (1), the 3×3 matrices, $\mathbf{M}_i$ and $\mathbf{C}_i$ are defined as:

$$\mathbf{M}_i = \begin{bmatrix} I_i & -m_i d_i \sin(\theta_i + \alpha_i) & m_i d_i \cos(\theta_i + \alpha_i) \\ -m_i d_i \sin(\theta_i + \alpha_i) & m_i & 0 \\ m_i d_i \cos(\theta_i + \alpha_i) & 0 & m_i \end{bmatrix};$$
$$\mathbf{C}_i = \begin{bmatrix} 0 & 0 & 0 \\ -\omega_i m_i d_i \cos(\theta_i + \alpha_i) & 0 & 0 \\ -\omega_i m_i d_i \sin(\theta_i + \alpha_i) & 0 & 0 \end{bmatrix} \tag{3}$$

Now, the points on the link, O_i and O_{i+1} are defined at the joints connecting preceding and succeeding links. The body fixed frame, $O_i X_i Y_i$, is then defined in such a way that the axis X_i is aligned from O_i to O_{i+1}. The shortest distance between O_i and O_{i+1} is defined as link length. The parameters d_i and θ_i are polar coordinates of the mass center as shown in Fig. 1.

2.2. Modified equations of motion for equimomental system of point-masses

To formulate an optimization problem to minimize shaking force and shaking moment, the rigid links are modeled as dynamically equivalent systems of point-masses referred to equimomental systems. The rigid link and the system of point-masses will be dynamically equivalent (equimomental) if they have same mass, same center of mass and same inertia tensor with respect to same coordinate frame [22]. Hence, a set of dynamically equivalent system of rigidly connected n point-masses, m_{ij}, located at l_{ij}, θ_{ij}, as shown in Fig. 2 must satisfy the following conditions:

$$\sum_j m_{ij} = m_i \tag{4}$$

$$\sum_j m_{ij} l_{ij} \cos(\theta_{ij} + \alpha_i) = m_i d_i \cos(\theta_i + \alpha_i) \tag{5}$$

$$\sum_j m_{ij} l_{ij} \sin(\theta_{ij} + \alpha_i) = m_i d_i \sin(\theta_i + \alpha_i) \tag{6}$$

$$\sum_j m_{ij} l_{ij}^2 = I_i \tag{7}$$

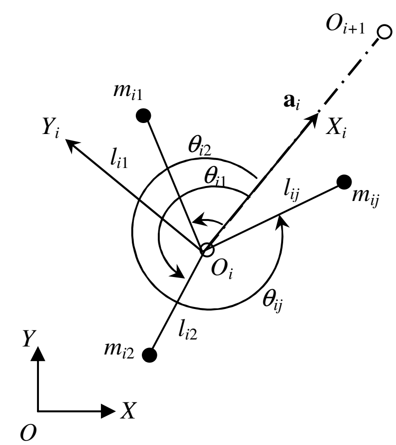


Figure 2. Equimomental system of point-masses for *i*th link

where m_i and I_i are the mass of the ith link and its mass moment of inertia about O_i. The first subscript i denotes the link number, and the second subscript j represents the point-mass. The NE equations of motion, Eq. (1), are now rewritten for the equimomental system of point-masses using equimomental conditions, Eqs. (4)-(7). It can be shown that the form, Eq. (1), does not change except the elements of matrices, $\mathbf{M}_i$ and $\mathbf{C}_i$, which are given as:

$$\mathbf{M}_i = \begin{bmatrix} \sum_j m_{ij} l_{ij}^2 & -\sum_j m_{ij} l_{ij} \mathrm{S}(\theta_{ij}+\alpha_i) & \sum_j m_{ij} l_{ij} \mathrm{C}(\theta_{ij}+\alpha_i) \\ -\sum_j m_{ij} l_{ij} \mathrm{S}(\theta_{ij}+\alpha_i) & \sum_j m_{ij} & 0 \\ \sum_j m_{ij} l_{ij} \mathrm{C}(\theta_{ij}+\alpha_i) & 0 & \sum_j m_{ij} \end{bmatrix}; \; \mathbf{C}_i = \begin{bmatrix} 0 & 0 & 0 \\ -\omega_i \sum_j m_{ij} l_{ij} \mathrm{C}(\theta_{ij}+\alpha_i) & 0 & 0 \\ -\omega_i \sum_j m_{ij} l_{ij} \mathrm{S}(\theta_{ij}+\alpha_i) & 0 & 0 \end{bmatrix} \tag{8}$$

In Eq. (8), C and S are abbreviations for cosine and sine functions, respectively. There are $3k$ parameters, m_{ij}, θ_{ij}, l_{ij} for j=1, 2,…,k if k point-masses are defined for the ith link. For a mechanism of n moving links, there will be a total $3kn$ point-mass parameters. All or some of these can be taken as the design variables in optimization formulation discussed in the next section.

3. FORMULATION OF OPTIMIZATION PROBLEM

Without loosing generalization, the problem for minimizing shaking force and shaking moment in a planar five-bar mechanism is now formulated on the basis of the dynamics presented in previous section. For minimizing the inertia forces by redistributing the link masses, mass and inertia properties of moving links are represented by the dynamically equivalent systems of point-masses. The point-mass parameters are treated as the design variables. The five-bar mechanism under consideration is shown in Fig. 3. The links are numbered as #0, #1, #2, #3 and #4, where link #0 represents the frame to which link #1 and link #4 are connected. All joints are revolute type. The joints are numbered as 1, 2, 3, 4 and 5 while a_0, a_1, a_2, a_3 and a_4 represent the link lengths. The fixed inertial frame, *OXY*, is located at joint 1, between link #1 and the frame #0.

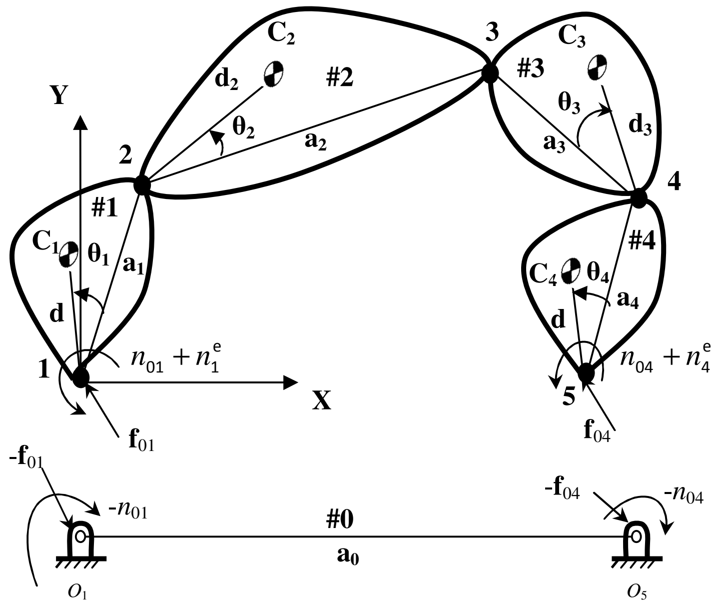


Figure 3. Five-bar mechanism detached from its frame

3.1. Identification of design variables

A system of *k* equimomental point-masses is used for each link and the corresponding point-mass parameters are taken as the design variables. Therefore, the 3*k*-vector of design variables for the *i*th link includes point-mass and their locations, and is defined as:

$$\mathbf{DV_1} = [m_{i1} \quad l_{i1} \quad \theta_{i1} \quad m_{i2} \quad l_{i2} \quad \theta_{i2} \quad ..... \quad m_{ik} \quad l_{ik} \quad \theta_{ik}]^{\mathrm{T}} \tag{9}$$

Hence, the design variable 3*kn*-vector, **DV**, for mechanism having *n* moving links can be defined as:

$$\mathbf{DV} = [\mathbf{DV}_1^{\mathrm{T}} \quad \mathbf{DV}_2^{\mathrm{T}} \quad ..... \quad \mathbf{DV}_n^{\mathrm{T}}]^{\mathrm{T}} \tag{10}$$

3.2. Objective function and constraints

For a mechanism in motion, shaking force is the vector sum of the inertia forces, whereas the shaking moment about any point is the sum of the inertia couples and the moment of the inertia forces about that point [12]. In the current problem, the external forces like gravity and dissipative forces are not considered. Once all the joint reactions are determined, the shaking force and shaking moment at and about joint 1 are presented as:

$$\mathbf{f}_{sh} = -(\mathbf{f}_{01} + \mathbf{f}_{04}) \text{ and } n_{sh} = -(n_1^e + \mathbf{a}_0 \text{ x } \mathbf{f}_{04}) \tag{11}$$

In Eq. (11), $\mathbf{f}_{01}$ and $\mathbf{f}_{04}$ are the reaction forces of the ground on the links 1 and 4, respectively, while n_1^e is the driving torque applied at joint 1. $\mathbf{a}_0$ is the vector from O_1 to O_5. Considering the RMS values of the shaking force, $f_{sh,rms}$, and the shaking moment, $n_{sh,rms}$, the optimization problem is proposed as:

$$\text{Minimize } Z = w_1 f_{sh,rms} + w_2 n_{sh,rms} \tag{12}$$

$$\text{Subject to } \quad m_{i,min} \le \sum_j m_{ij} \le m_{i,max} \text{ for } i = 1, 2, 3,4 \text{ and } j = 1, 2,\ldots,k \tag{13}$$

where w_1 and w_2 are the weighting factors whose values may vary depending on an application. For example, w_1=1.0 and w_2=0 if objective is to minimize the shaking force only and vice-versa. These weighting factors can also be taken as the design variables to get the most approprate values for a multi-objective optimization problem. The minimum mass and inertia, $m_{i,min}$ and $I_{i,min}$, of ith link can be defined according to its force bearing capabilities and link material properties. The solution of this optimization problem finds the values of the design variables that minimize the objective function Z.

4. SOLUTIONS AND RESULTS

After formulating the balancing problem as an optimization problem, it can be solved by using either conventional or evolutionary optimization algorithms. The gradient based conventional algorithms use the gradient information of the objective function with respect to the design variables. Starting with an initial guess point, these methods converge on the optimum solution near to the starting point and thus produce local optimum solution.

Genetic algorithm is evolutionary search and optimization algorithm based on the mechanics of natural genetics and natural selection [24]. This algorithm evaluates only the objective function and genetic operators - selection, crossover and mutation are used for exploring the design space. One can specify the initial population, bounds and nonlinear constraints for the variables in this algorithm. After selection operation, crossover and mutation operators are used to form the new population. This process is repeated till the convergence criteria is satisfied [25]. The drawbacks of the GAs are that (1) they require a large amount of calculation and (2) there is no absolute guarantee that a global solution is obtained. These drawbacks can be overcome by using parallel computers and by executing the algorithm several times or allowing it to run longer [26]. The flow chart of the proposed method is shown in Fig. 4.

The proposed method is applied for the balancing of a five-bar mechanism [17]. In [17], only shaking force is minimized through conventional optimization method, i.e., non-linear constraint optimization. The center of mass parameters of moving links were chosen as the design variables. The natural orthogonal complement method was used for dynamic analysis of the mechanism. However, the resulting effect on shaking moment and driving torque was not considered. For the same numerical problem, both shaking force and shaking moment are simultaneously minimized in this paper using the global optimization method, i.e., genetic algorithm.

As shaking force and shaking moment are of different units, these quantities need to be dimensionless for adding them in a single objective function. For this, the mechanism parameters are made dimensionless with respect to the parameters of the driving link and shown in Table 1. For this example, the driving link, i.e. link 1, rotates with a constant speed of 100 rad/sec.

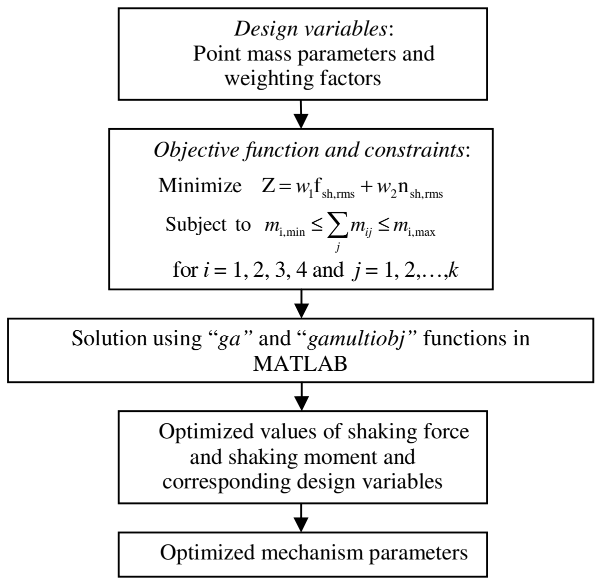


Figure 4. Flowchart of proposed method

To demonstrate the effectiveness and accuracy of the method, here each link is represented by three equimomental point-masses. To reduce the dimension of the problem, out of nine variables, m_{ij}, l_{ij}, θ_{ij}, for j=1, 2, 3, for the ith link, five parameters are assigned as: θ_{i1}=0; θ_{i2}=2π/3; θ_{i3}=4π/3 and l_{i2}=l_{i3}=l_{i1}.

The other four point-mass parameters and weighing factors, namely, m_{i1}, m_{i2}, m_{i3}, l_{i1}, w_1 and w_2 are brought into the optimization scheme. A MATLAB program was developed using the equimomental conditions, Eqs. (4)-(7), for finding the dynamically equivalent point-masses for each link. The resulting point-masses and their locations are shown in Table 2.

Table 1. Dimensionless parameters of standard mechanism

Link	Length a_i	Mass m_i	Moment of inertia Io_{zzi}	Center of mass distance d_i	Center of mass location θ_i
1	1	1	0.3333	0.5	0
2	5	5	41.6667	2.5	0
3	5	5	41.6667	2.5	0
4	2	2	2.6667	1	0
0	2	-		-	-

Table 2. Point-mass parameters

Link	m_{i1}	m_{i2}	m_{i3}	l_{i1}
1	0.9107	0.0447	0.0447	0.5774
2	4.5534	0.2233	0.2233	2.8868
3	4.5534	0.2233	0.2233	2.8868
4	1.8214	0.0893	0.0893	1.1547

Considering $m_{i,min}= 0.75\, m_i^0$, $m_{i,max}= 2\, m_i^0$ where m_i^0 is original mass of the *i*th link, the optimization problem as explained in Eqs. (12)-(13) is solved using "*ga*" function in *Genetic Algorithm and Direct Search Toolbox* of MATLAB [27]. The original values of point-mass parameters are taken as the initial population and the algorithm was run for 100 generations. The comparison of original values with optimum values of the shaking force and shaking moment obtained using genetic algorithm are presented in Table 3 and Fig. 5. The optimized link parameters are found by using the equimomental conditions presented in Eqs. (4)-(7) and shown in Table 4.

Table 3. RMS values of dynamic quantities of standard and optimized mechanisms

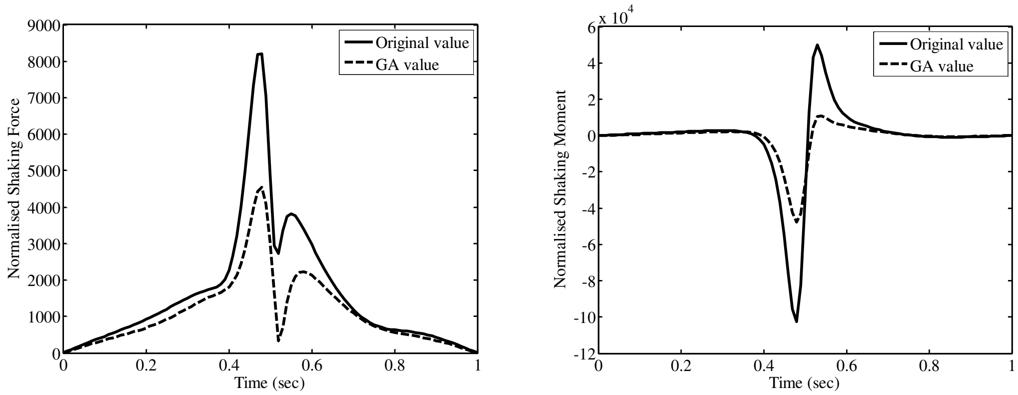
	RMS values of dimensionless dynamic quantities	
	Shaking force	Shaking moment
Standard value	2388	21913
Genetic algorithm	1603 (-32.87%)	11214 (-48.82%)

The values in the parenthesis denote percentage increment/decrement with respect to corresponding RMS values of the standard mechanism

Table 4. Dimensionless parameters of balanced mechanism

Link	Length a_i	Mass m_i	Moment of inertia Io_{zzi}	Center of mass distance d_i	Center of mass location θ_i
1	1	1.8783	2.1803	0.6767	356.00
2	5	4.3356	15.6911	1.1160	16.10
3	5	4.6127	9.6128	1.1681	358.12
4	2	3.3909	0.0812	0.0839	331.92
0	2	-	-	-	-

By using the genetic algorithm, the reduction of 32.87% and 48.82% were found in the values of shaking force and shaking moment, respectively.



(a) Shaking force (b) Shaking moment

Figure 5. Variations of shaking force and shaking moment for complete cycle

Note that the above results are obtained using single objective function mentioned in Eq. (12). Moreover, this problem can be solved by considering the shaking force and shaking moment as two objective functions. The multi-objective optimization, also known as vector optimization, is the procedure used for simultaneous minimization or maximization of more than one objective function. Various nonlinear multi-objective optimization methods are surveyed in [28]. The objective function for the posed problem is defined as:

$$\text{Minimize } Z_m = [f_{sh,rms}, n_{sh,rms}]^T \tag{14}$$

This problem is solved using "*gamultiobj*" function in *Genetic Algorithm and Direct Search Toolbox* of MATLAB. This function, Z_m, finds the minimum using genetic algorithm and creates a set of non-dominated solution set known as Pareto front for objectives, i.e., the shaking force and shaking moment. The values of genetic operators used are:

Selection function	=	Stochastic uniform
elite count	=	2
crossover fraction	=	0.8
migration fraction	=	0.2
function tolerance	=	1×10^{-10}
constraint tolerance	=	1×10^{-10}

All Pareto solutions are optimum as no other solutions in the entire design space is available which is better than these solutions when all the objectives are considered. The Pareto front is shown in Fig. 6 which presents the multiple optimum solutions for the considered problem. The values of the objective function and corresponding design variables associated with each point of this curve are also available in the solution. The optimum values of weighting factors are found as 0.5 each. The optimum values of point-mass parameters are given in Table 5.

Table 5. Optimum point-mass parameters

Link	m_{i1}	m_{i2}	m_{i3}	l_{i1}
1	1.4107	0.1863	0.2813	1.0774
2	3.0743	1.0380	0.2233	1.9024
3	4.0245	0.2233	0.3649	1.4436

4	2.2123	0.0893	1.0893	0.1547

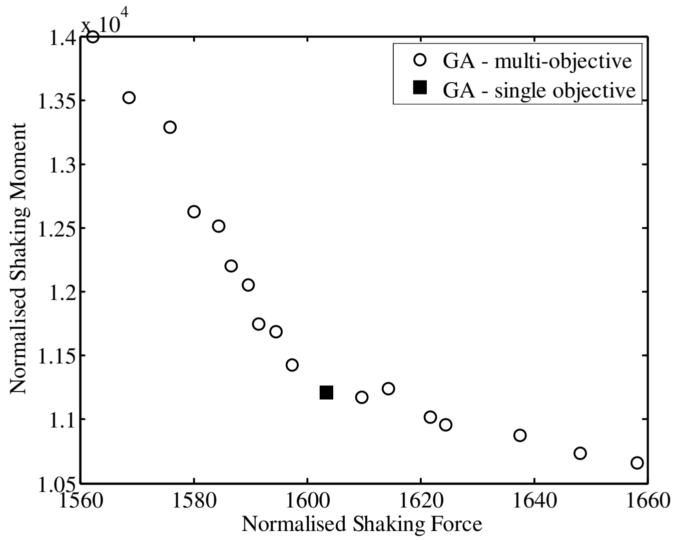


Figure 6. Pareto front

The results obtained by using single and multi-objective optimization methods are shown in Fig. 6. For the optimum solutions lying on Pareto front, the decrease in value of one objective increases value of the other one. Hence, the mechanism designer has several choices to choose the solution as per the specific requirement.

5. CONCLUSIONS

An optimization method for dynamic balancing of five-bar planar mechanisms is presented in this paper using the concept of the equimomental system of point-masses for the rigid body. The dynamic equations of motion are formulated systematically in the parameters related to the equimomental point-masses. Using these equations, the optimization problem is formulated for the minimization of the shaking force and shaking moment as single objective and multi-objective function. With optimum weighting to the objectives, 32.87% and 48.82% reduction is achieved in shaking force and shaking moment, respectively. The problem is also formulated as a multi-objective optimization problem for which Pareto front provides better insight over the combinations of shaking force and shaking moment. The formulation presented in this paper is simple, easy to implement and it can be applied for multi-loop planar and spatial mechanisms also.

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