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LA – SEMIRINGS In Which (S,.) Is Anti-Inverse Semigroup

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Abstract:

This paper deals with the some results on LA– Semirings in which(s, ▪) is anti-inverse semigroup. In the first case of the LA - Semiring (S, +, .) satisfying the identity a+1 = 1, for all a in S then it is proved that (S, +) is anti-inverse semigroup. It is also proved that if (S, +,▪) is a LA – Semiring satisfying the above identity then (S, +) is an abelian semigroup and sum of two anti inverse elements is again anti inverse element in (S, +) and also proved that (S, +, ▪) is medial semiring. In this second case we consider LA- semiring (S, +, ▪) in which (s, ▪) is anti-inverse semigroup satisfying the identity a+1= a for all a in S then (S, +) is anti inverse semigroup and sum of two inverse elements is again anti inverse element in (S, +). It is also proved that in LA semiring in which (s, ▪) is anti-inverse semigroup then (S, +) is an abelian semigroup and the product of two inverse elements is again inverse element in (s, ▪).

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Keywords — ***LA-Semigroup, LA-semirings, Anti-inverse semigroup***

Introduction:

LA- semirings are naturally developed by the concept of LA- semigroup. The concepts of LA – semigroup was introduced by M.A. Kazim andM. Naseeruddin [1] in 1972. Since then lot of papers has been presented on LA - semigroups like. Mushtaq, Q and Khan [02], M Mustaq, Q. and yousuf, S.M. [03], Qaiser Mushtaq[04]. Anti inverse semigroups are studied by S-Bogdanovic S.Milic V.Pavloric. In this paper mainly we concentrate on the structures of anti inverse semigroup in LA-semirings. We determine some structures of LA-semirings in which the multiplicative structure is anti inverse semigroup.

1.1.1. Definition: A left almost semigroup (LA-semigroup) or Abel-Grassmanns groupoid (AG-groupoid) is a groupoid S with left invertive law: (ab)c = (cb)a for all $a, b, c \in S$

Example:- Let S = {a, b, c} the following multiplication table shows that S is a LA-Semigroup.

▪	a	b	c
a	a	a	a
b	a	a	c
c	a	a	a

1.1.2. Definition: A semiring (S, +, ▪.) is said to be LA-Semiring if

1. (S, +) is a LA-Semigroup
2. (S, ▪) is a LA-Semigroup

Example: Let S = {a, b, c} is a mono semiring with the following tables 1, 2 which is a LA-Semiring

+	a	b	c
a	a	a	a
b	a	a	c
c	a	a	a

1.2.1. Definition: A semi group S is called anti inverse if every element of S is anti inverse element.

Example: Let S = {a, b} then (S, ▪) with following table 1 or 2 or 3 is an inverse semigroup

▪	a	b
a	a	a
b	b	b

▪	a	b
a	a	b
b	b	b

▪	a	b
a	a	b
b	b	a

1.2.2. Theorem: Let (S, +, ▪) be a LA-semiring in which (S, ▪) is anti-inverse semi group and satisfying the identity a+1=1 $\forall a \in S$ then (S, +) is anti-inverse semigroup.

Proof: Let (S, +, ▪) be a LA-Semiring and (S, .) be anti-inverse semigroup satisfying the identity a+1=1, $\forall a \in S$

Let $a \in S$, since (S, ▪) is anti-inverse there exist an element $x \in S$ such that x a x = a.

Consider x+a+x = x+a.1+x

= x + a(1 + xa) + x

= x + (a + axa + x)

▪	a	b	c
a	a	a	a
b	a	a	c
c	a	a	a

= x + x + axa + a

= x(1+1) + axa + a

= x.1 + (ax+1)a

= x + 1.a

= x + a

= axa + a

= (ax + 1)a

= 1.a

= a

∴ x+a+x = a

Similarly a+x+a = x

∴ (S, +) is a anti-inverse semigroup

1.2.3. Theorem : Let (S, +, ▪) be a LA-semiring and (S, ▪) be anti-inverse semi group then the product of two anti-inverse elements is also anti-inverse element in (S, ▪)

Proof : Let (S, +, ▪) be a LA-semiring and (S, ▪) be an anti-inverse semi group.

Let a, b are two elements in (S, ▪) then there exist x, y in S such that xax = a, yby = b

Consider yx ab yx = byb xaby yaxa

= by(bxa) byaxa

= by ax(b by a)xa

= by(ax a) (by b)xa

= by xy xa

= byx (yxa)

= by(xax)y

= (bya)y

= ayby

= ab

$\therefore$ yx ab yx = ab

Similarly we can prove that ba xy ba = xy

Hence the product of two anti-inverse elements is again anti-inverse element in (S, ▪)

1.2.4. Theorem: Let (S, +, ▪) be a LA-Semiring and (S, ▪) be anti-inverse semigroup then (S, ▪) is an abeliean semigroup.

Proof: Let (S, +, ▪) be a LA-semi ring and (S, ▪) is an anti-inverse semigroup.

From the above theorem for any $a, b \in S$, there exist $x, y \in S$ such that

yx a byx = ab

y xa (byx) = ab

y xa xyb = ab

y(ayb) = ab

y b y a = ab

ba = ab

Hence (S, .) is an abelieon semigroup.

12.5. Theorem: Let (S, +, ▪) be a LA-semiring in which (S, .) is an anti-inverse semigroup and satisfying the identity a+1 = 1 for all $a \in S$ then (i) (S, +) is an abeliean semigroup.

(ii) The sum of two anti-inverse elements is again anti-inverse element in (S, +)

Proof : Let (S, +, ▪) be a LA-Semiring in which (S, ▪) is a LA semi group and satisfying the identity a+1= 1 $\forall a \in S$. Then by the theorem 3.2.2 (S, +) is anti-inverse semigroup.

Let $a, b \in S$ then there exists $x, y \in S$ such that x+a+x = a, y+b+y = b and a+x+a = x, b+y+b = y

Consider
y+x+a+b+y+x
=b+y+b+x+a+b+y+a+x+a
=b+y+(b+x+a)+b+y+a+x+a
=b+y+a+x+(b+(b+y)+a)+x+a
=b+y+(a+x+a)+(b+y+b)+x+a
=b+y+x+(y+x+a)
=b+y+(x+a+x)+y
= (b+y+a)+y
= a+y+b+y
= a+b

$\therefore$ y+x+a+b+y+x=a+b ….(1)

Similarly we can prove that b+a+x+y+b+a = x+y

$\therefore$ a+b is an anti-inverse element in (S, +)

Therefore the sum of two anti-inverse elements is again anti-inverse element in (S, +).

To show that (S, +) is an abelieen semigroup.

From equation (1) a+b = y+x+a+(b+y+x)

= y+(x+a+x)+y+b

= y+(a+y+b)

= (y+b+y)+a

= b+a

$\therefore$ a+b = b+a

Hence (S, +) is an abeliean semi group

1.2.7. Theorem : Let (S, +, ▪) be a LA-Semiring which satisfies the identity a+1 = 1 $\forall a \in S$. If (S, ▪) is an anti-inverse semigroup then (S, +, .) is a medial semiring.

Proof: Let (S, +, ▪) be a LA-Semiring satisfying the identity a+1 = 1, $\forall a \in S$ Let (S, ▪.) be an anti-inverse semigroup with

From the theorems 3.2.4 and 3.2.5 we have (S, ▪) and (S, +) are abeliean semigroups.

Let $a, b, c, d \in (S, .)$ then
abcd = a(bc)d

= a(cb)d

abcd = a cb d

$\therefore$ (S, ▪) is a medial semigroup

Similarly (S, +) is also a medial semigroup

Hence (S, +, ▪) is a medial semiring

1.2.8. Theorem: Let (S, +, ▪) be a LA-semiring in which (S, ▪) is an anti-inverse semigroup and satisfying the identity a+1 = a $\forall a \in S$ then (S, +) is an anti-inverse semigroup.

Proof: Let (S, +, ▪) be a LA-semiring satisfying the identity a+1 = a, $\forall a \in S$. Let (S, .) be an anti-inverse semigroup.

Since (S,▪) is an anti-inverse semigroup then for any $a \in S$ therexist $x \in S$ such that xax = a and axa = x

Consider x+a+x = x+a+x

= x+xax+x

= x(1+ax)+x

= xax+x

= (xa+1)x

= xa.x = xax = a

x+a+x = a

Similarly we can prove that a+x+a = x

Hence a is an anti-inverse element is (S, +)

Therefore (S, +) is an anti-inverse semigroup.

1.2.9. Theorem: Let (S, +, ▪) be a LA-semiring in which (S, .) is an anti-inverse semigroup and satisfying the identity a+1 = a, $\forall a \in S$ then the sum of two anti-inverse elements is also anti inverse element in (S, +).

Proof: Proof is similar to theorem 3.2.5

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