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Study of the Deformation of the Flexible Wheel by Mechanical Generators with Two Deformation Waves

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The correct operation and durability of the harmonic gear transmissions depends to a large extent on the precision of the execution, the deformation law and the durability of the flexible toothed wheel, the most requested element of the transmission. In this paper there is presented a comparative study of the deformation state of the short flexible toothed wheel of a double harmonic gear transmission for the case of four types of mechanical generators with two deformation waves: with cam, 2 rolls, 4 rolls and 2 eccentric discs. By analyzing the deformation laws of the flexible wheel for the 4 cases of generator waves, one can achieve a better understanding of how the harmonic engagement process takes place.

Keywords: *double harmonic transmission, flexible wheel, generator wave, deformation law, displacement*

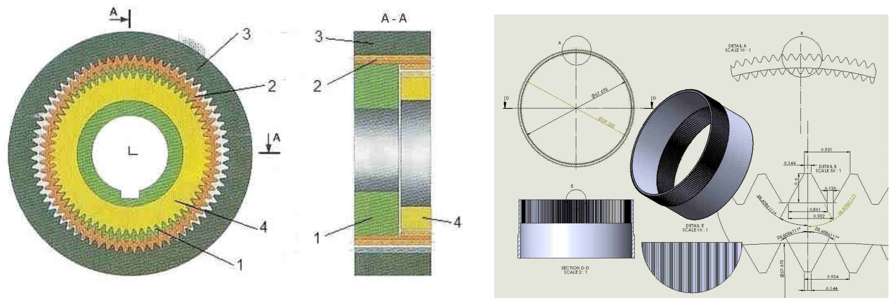
1. Introduction

Double harmonic gear transmission investigated (Figure 1, a) it contains in its structure an element called the short flexible toothed wheel, which is the strongest element requested to fatigue [1, 2, 3, 4, 5, 6, 8, 13].

This transmission is used in the following functional composition: 1 - is the wave generator, as the driving element; 2 – the flexible toothed wheel as an intermediate element; 3 - the rigid wheel as a fixed element; 4 - the rigid mobile wheel as a driven element.

The flexible toothed wheel (Figure 1, b) has the shape of a flexible circular tube with thin wall, which is open at both ends and is provided at each end with a toothed crown (at one end an outer one, and at the other end an inner one).

The correct operation and durability of the double harmonic gear transmission is largely influenced by the law of deformation of the flexible toothed wheel, as well as the precision of its execution [7, 10, 12].



a. b.

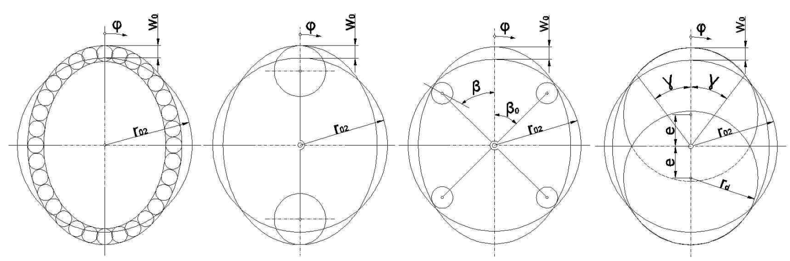
Figure 1. Double harmonic gear transmission

By choosing an appropriate deformation law of the flexible wheel, the optimal conditions of harmonic engagement can be ensured, through the entry of the outer teeth of the flexible wheel (2) into the gaps between fixed rigid wheel teeth (3 - in the diametrically opposed areas immediately adjacent to the high axis of symmetry of the wave generator), respectively of the inner teeth of the flexible toothed wheel in the gaps between the mobile rigid wheel teeth (4 - in areas near the small axis of symmetry of the wave generator).

The paper presents a comparative analysis of the laws of deformation of the flexible toothed wheel for the cases of four types of mechanical generators with two deformation waves: with cam, 2 rolls, 4 rolls and 2 eccentric discs.

2. The deformation laws of the flexible toothed wheel

In the vast majority of cases of use in practice of the harmonic gear transmission the most frequently used are constructive variants with mechanical generators with two deformation waves. In this context, it is necessary to study the deformation state of the flexible element produced by these types of wave generators.



a. b. c. d.

Figure 2. Mechanical generators with two deformation waves

In Figure 2 are presented the four types of investigated mechanical wave generators with two deformation waves: with cam (Figure 2, a), with 2 rolls (Figure 2, b), with 4 rolls (Figure 2, c), and with 2 eccentric discs (Figure 2, d).

Figure 3 shows the photo with three mechanical generators with two deformation waves (with cam, 2 eccentric discs and 2 rolls) manufactured in the Faculty of Engineering and Management of Resita. The four-roller construction can easily be achieved by mounting two more rollers on the wave generator with 2 rolls.

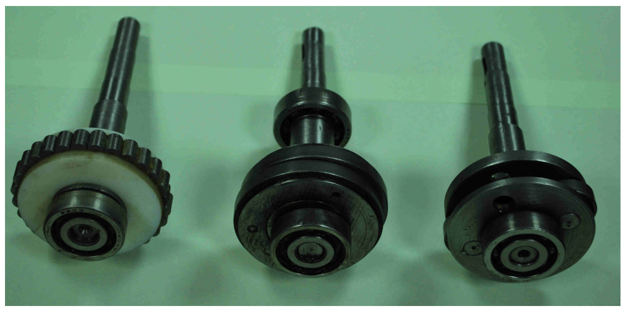


Figure 3. Photo of mechanical generators with two deformation waves

The following are the relations for calculating the deformations of the flexible toothed wheel, which defines the static image of its deformation in the front section where the wave generator operates.

- Wave generator with cam:

$$w = w_0 \cdot \cos 2\varphi$$
$$v = -(w_0 / 2) \cdot \sin 2\varphi \tag{1}$$
$$\theta = (3\,w_0 / 2r_{02}) \cdot \sin 2\varphi$$

- Wave generator with 2 rolls:

$$w = [w_0 / (\pi/2 - 4/\pi)] \cdot [\sin\varphi + (\pi/2 - \varphi) \cdot \cos\varphi - 4/\pi]$$
$$v = -[w_0 / (\pi/2 - 4/\pi)] \cdot [(\pi/2 - \varphi) \cdot \sin\varphi - 2 \cdot \cos\varphi - (4/\pi) \cdot \varphi + 2] \tag{2}$$
$$\theta = [w_0 / r_{02}\,(\pi/2 - 4/\pi)] \cdot [2 \cdot \cos\varphi + (4/\pi) \cdot \varphi - 2]$$

- Wave generator with 4 rolls:

for: $0 \le \varphi \le \beta$:

$$w = [w_0 / (A - 4/\pi)] \cdot [A \cdot \cos\varphi + \varphi \cdot \sin\beta \cdot \sin\varphi - 4/\pi]$$
$$v = -[w_0 / (A - 4/\pi)] \cdot [A \cdot \sin\varphi + \sin\beta\,(\sin\varphi - \varphi \cdot \cos\varphi) - (4/\pi)\,\varphi]$$
$$\theta = \{w_0 / [r_{02} \cdot (A - 4/\pi)]\} \cdot [(4/\pi) \cdot \varphi - 2\sin\beta \cdot \sin\varphi]$$

for: $\beta \le \varphi \le \pi/2$: (3)

$$w = [w_0 / (A - 4/\pi)] \cdot [B \cdot \sin\varphi + (\pi/2 - \varphi) \cdot \cos\beta \cdot \cos\varphi - 4/\pi]$$
$$v = -[w_0 / (A - 4/\pi)] \cdot [-B \cdot \cos\varphi + (\pi/2) \cdot \cos\beta \sin\varphi - \cos\beta \cdot (\cos\varphi + \varphi \cdot \sin\varphi) - (4/\pi) \cdot \varphi + 2]$$
$$\theta = \{w_0 / [r_{02}\,(A - 4/\pi)]\} \cdot [2\cos\beta \cdot \cos\varphi + (4/\pi) \cdot \varphi - 2]$$

where: $A = \sin\beta + (\pi/2 - \beta) \cdot \cos\beta$; $B = \cos\beta + \beta \cdot \sin\beta$

- Wave generator with 2 eccentric discs:

for: $0 \le \varphi \le \gamma$:

$$w = [w_0 / (A_1 - B_1)] \cdot (A_1 \cdot \cos\varphi - B_1)$$
$$v = -[w_0 / (A_1 - B_1)] \cdot (A_1 \cdot \sin\varphi - B_1 \cdot \varphi)$$
$$\theta = \{w_0 / [r_{02} (A_1 - B_1)]\} \cdot B_1 \cdot \varphi$$

for: $\gamma \le \varphi \le \pi/2$: (4)

$$w = [w_0 / (A_1 - B_1)] \cdot [(1 + \sin^2\gamma) \cdot \sin\varphi + (\pi/2 - \varphi) \cdot \cos\varphi - 2\sin\gamma - B_1]$$
$$v = -[w_0 /(A_1 - B_1)] \cdot [(\pi/2 - \varphi) \cdot \sin\varphi - (2 + \sin^2\gamma) \cdot \cos\varphi - (2\sin\gamma + B_1) \cdot \varphi + 2(\cos\gamma + \gamma \cdot \sin\gamma)]$$
$$\theta = \{w_0 / [r_{02} (A_1 - B_1)]\} \cdot [2\cos\varphi + (2\sin\gamma + B_1) \cdot \varphi - 2(\cos\gamma + \gamma \cdot \sin\gamma)]$$

where: $A_1 = \pi/2 - \gamma - \sin\gamma \cdot \cos\gamma$; $B_1 = (4/\pi) \cdot [\cos\gamma - (\pi/2 - \gamma) \cdot \sin\gamma]$

The meanings of the notations in the relations presented are the following: w – the radial displacement; v – the tangential displacement; θ – the angle of rotation of the normal; w_0 – the maximum radial deformation, w_0 = 0,3 mm; r_{02} – the medium fiber radius of the undeformed flexible wheel, r_{02} = 29,3 mm; φ – angular parameter of the wave generator; β – the angle of action of the rollers, β = (25°...35°); γ – the angle of action of the discs, γ = (20°...40°). In the case of the wave generator with 4 rolls, the angle of action of the generator rollers β is not the same as the roll positioning angle β_0, ($\beta \cong \beta_0 + 5°$).

It should be emphasized that the relations of calculating the deformations of the flexible element in the front section where the wave generator, presented above, shall also be valid for the front section at the opposite end, provided that the maximum radial deformation value w_0 substitution with w'_0 (w'_0 = 0,27 mm).

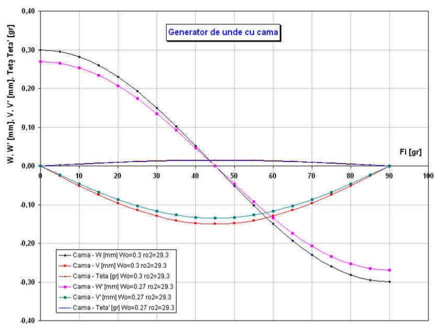
The optimal materialization of the elliptical law of deformation of the flexible toothed wheel can be achieved by the convenient choice of the type of the wave generator and its constructive form [9, 11].

3. The results of the analysis of the state of deformation of the flexible toothed wheel

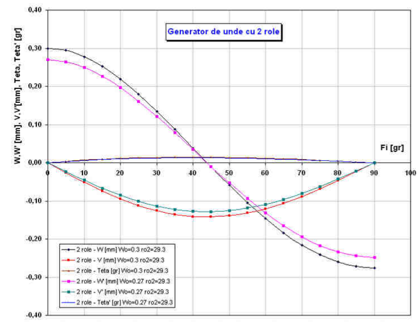
In order to perform a more detailed comparative analysis of the deformation states of the two front sections of the flexible toothed wheel, a special program for calculating the elastic deformations, written in the Visual Basic language of Excel, was developed and run.

Radial and tangential displacement diagrams of points on dynamic reference fiber of the statically deformed flexible toothed wheel, in the two front sections at the ends of the wheel: $w = w(\varphi)$ and $v = v(\varphi)$, respectively $w' = w'(\varphi)$ and $v' = v'(\varphi)$, as well as variation in the normal rotation: $\theta = \theta(\varphi)$, respectively $\theta' = \theta'(\varphi)$, for all four constructive types of mechanical generators with two deformation waves, are shown in Figure 4.

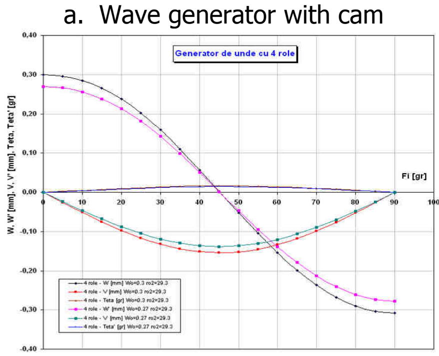
It can be observed that it was sufficient to represent the deformation curves only on the range $\varphi \in [0°, 90°]$, since the generator with two deformation waves presents symmetry from a constructive point of view.



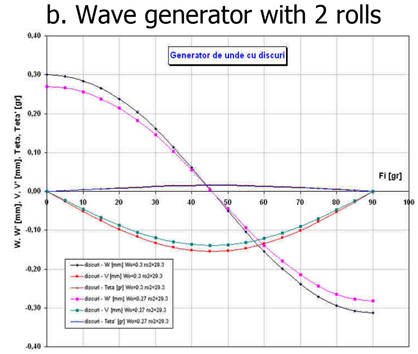
a. Wave generator with cam



b. Wave generator with 2 rolls



c. Wave generator with 4 rolls



d. Wave generator with 2 discs

Figure 4. Deformation curves of the flexible wheel

4. Conclusion

From the analysis of the radial and tangential displacement variation curves, it can be concluded that in all four deformations studied, the corresponding curves retain the same shape in both the front sections of the flexible toothed wheel, but the maximum radial deformation in the free front section of the flexible wheel always has a lower value than that in the front section where the generator acts.

It can be seen from Figure 4, that all four constructive investigated wave generators can provide the elliptical law of deformation of the flexible toothed wheel with a good approximation.

However, the great disadvantage of the roller or eccentric disc generator consists in the fact that outside the contact areas with the flexible toothed wheel it can freely deform, which results in increased tensions in the body of the flexible wheel and lowering the carrying capacity of the harmonic transmission. This disadvantage can be entirely avoided by using the cam-wave generator, which allows the entire contour of the deformed flexible toothed wheel according to the cam profile.

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