



## **A Practical Method to Increase the Frequency Readability for Vibration Signals**

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*Damage detection and nondestructive evaluation of mechanical and civil engineering structures are nowadays very important to assess the integrity and ensure the reliability of structures. Thus, frequency evaluation becomes a crucial issue, since this modal parameter is mainly used in structural integrity assessment. The herein presented study highlights the possibility of increasing the frequency readability by involving a simple and cost-effective method.*

**Keywords:** *frequency estimation, power spectrum, polynomial curves*

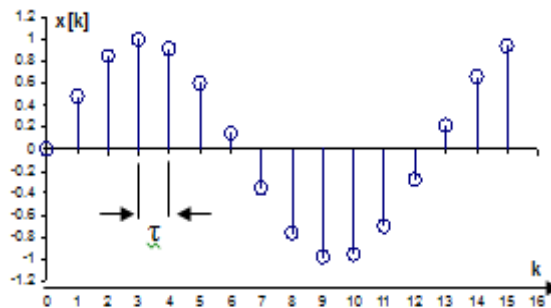
### **1. Introduction**

Frequency estimation of real-life signals is becoming an important issue in various technical domains, among which is worth to mention structural health monitoring [1]. This emerging domain bases on analysis of vibration signals, most of them acquired for rapid damping structures [2]-[4]. A short acquisition time results due to the damping, which makes frequency evaluation difficult [5]. Several methods are used to improve the frequency readability, from complicated time- and resources-consuming procedures to simple algorithms [6]-[7].

In this paper a frequency estimation method is introduced and the achieved results are compared with that obtained from standard frequency evaluation in order to find the accuracy of the proposed method.

### **2. Problems in standard frequency evaluation**

Multidimensional signals are in general linear combination of sinusoids. Frequency evaluation makes use of this property, it being in fact the decomposition of the signal in harmonic components. This is usually made by the Fourier Transform or derivate procedures, i.e. Discret Fourier Transform (DFT), Power Spectrum (PS), Power Spectral density (PSD) and so on.

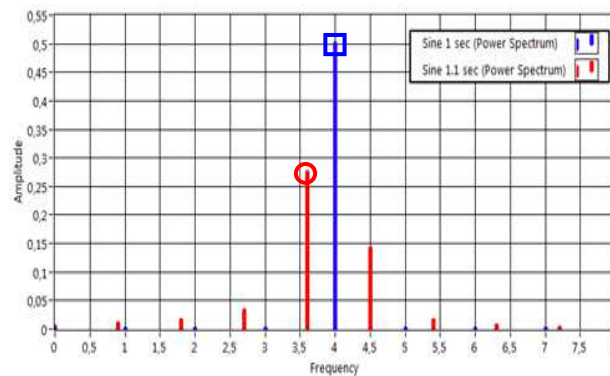


**Figure 1.** Digital signal.

The problem arising from this procedure is the sinusoids' frequencies. These have values fixed by the time of analysis [5], the difference between two consecutive sinusoids being in the relation:

$$\Delta f = \frac{j}{(N-1)\tau} = \frac{1}{T_s} \quad (1)$$

where  $\Delta f$  is the frequency resolution,  $j$  is the number of the sinusoid placed at the  $j$ -th spectral line,  $N$  is the total number of samples in the signal,  $\tau$  is the sampling time and  $T_s$  is the signal length in time.



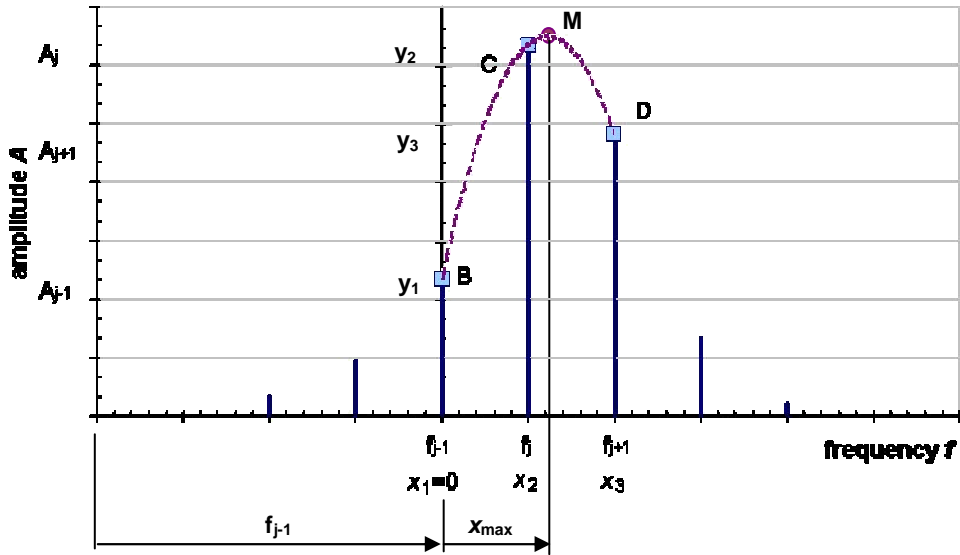
**Figure 2.** Power spectral representation of a harmonic signal for two analysis time lengths.

Having a look onto figure 2 one can remark that different pick values are indicated for the two analysis cases, obviously associated with different frequencies. This shows that even for harmonic signals the standard frequency evaluation method is not suitable if short or medium analysis time is involved.



#### 4. Frequency estimation by involving several amplitudes from the spectra

The first analyzed case concerns the frequency estimation from three points located around the evaluated frequency, as shown in figure 3. Note that the difference between any two consecutive frequency values indicated at the spectral lines is  $x_i - x_{i-1} = \Delta f$ , which actually the frequency resolution is. For simplicity, the abscissa of the first considered point  $x_1$  (that is frequency  $f_{j-1}$ ) is taken zero; this is possible by the shifting the vertical axis with  $f_{j-1}$  to the  $j-1$ -th spectral line. The next two points are thus located at  $\Delta f$  respectively  $2\Delta f$ . The three amplitudes  $A_{j-1}$ ,  $A_j$  and  $A_{j+1}$  confer the ordinates  $y_1$ ,  $y_2$  and  $y_3$ . This is illustrated in figure 3.



**Figure 3.** Power spectral representation of a sinusoid with highlighting the points used in frequency evaluation and the vertical axis shift.

The three points have therefore the coordinates:  $B(0, y_1)$ ,  $C(x_2, y_2)$  and  $D(x_3, y_3)$ . The point with biggest amplitude is always centrally located. From eq.(3), the coefficients result as:

$$c_0 = y_1 \quad c_1 = \frac{-y_3 + 4y_2 - 3y_1}{2\Delta f} \quad c_2 = \frac{y_3 - 2y_2 + y_1}{2\Delta f^2} \quad (4)$$

The regression curve will indicate a pick  $M(x_{\max}, y_{\max})$  between the two biggest amplitudes that is, in the case of figure 3, between points C and D. The abscissa  $x_{\max}$  is used to correct the read frequency and attain a value closer to the real frequency. This is made by adding  $x_{\max}$  to  $f_{j-1}$  which represents the vertical axis shift. The corrected frequency results as:

$$f_{corr} = f_{j-1} + x_{\max} \quad (5)$$

The  $x_{\max}$  coordinate of point M is found at the location where the derivative of the regression function  $P(x)$  is null. For

$$P'(x) = c_1 + 2c_2x = 0 \quad (6)$$

which has the solution

$$x_{\max} = -c_1 / 2c_2 \quad (7)$$

An exemplification for a signal having the real frequency  $f_{\text{real}} = 4\text{Hz}$  is presented below. The signal has an amplitude  $A=1$  and was created for a sampling rate  $r_s = 1000\text{Hz}$ . Two cases differing the number of samples is presented. In the first analyzed case the number of samples is  $N_I = 11300$ , and on two spectral lines the amplitudes have close values. For the second case the number of samples is  $N_{II} = 11600$ ; here one point has the amplitude much higher as the other two points. For both cases, the resulted time length, frequency resolution and position of the first 6 spectral lines are presented in table 1.

**Table 1.** Signal parameters for the two considered cases

$N$	$T_s$	$\Delta f$	$f_0$	$f_1$	$f_2$	$f_3$	$f_5$	$f_6$
11300	1.13	0.8849	0	0.8849	1.7699	2.6548	3.5398	4.4247
116200	1.16	0.8620	0	0.8620	1.7241	2.5862	3.4482	4.3103

Table 2 presents the coordinates for the three points used to plot the polynomial curve. The frequency-amplitude pairs are indicated by coordinates  $x_i$  respectively  $y_i$ . Note that  $x_1 = 0$ , thus the free term of the polynomial function is  $y_1$ . These points are used to calculate the coefficients  $c$  which are indicated in table 3, together with the  $x$ -coordinate of point M, namely  $x_{\max}$ .

Figures 4 and 5 illustrate the regression curves for the two analyzed cases.

**Table 2.** Coordinates of the points considered to find the polynomial curve

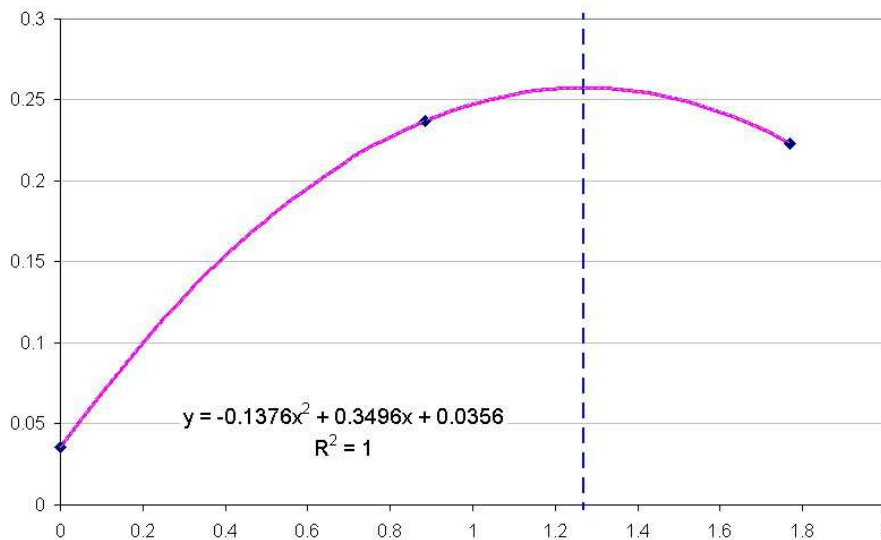
Case number	$x_1$	$x_2$	$x_3$	$y_1$	$y_2$	$y_3$
1	0	0.8849	1.7699	0.03559	0.237207	0.223348
2	0	0.862069	1.724138	0.129233	0.35403	0.022186

**Table 3.** Coefficients of the polynomial curves and the abscissa of point M

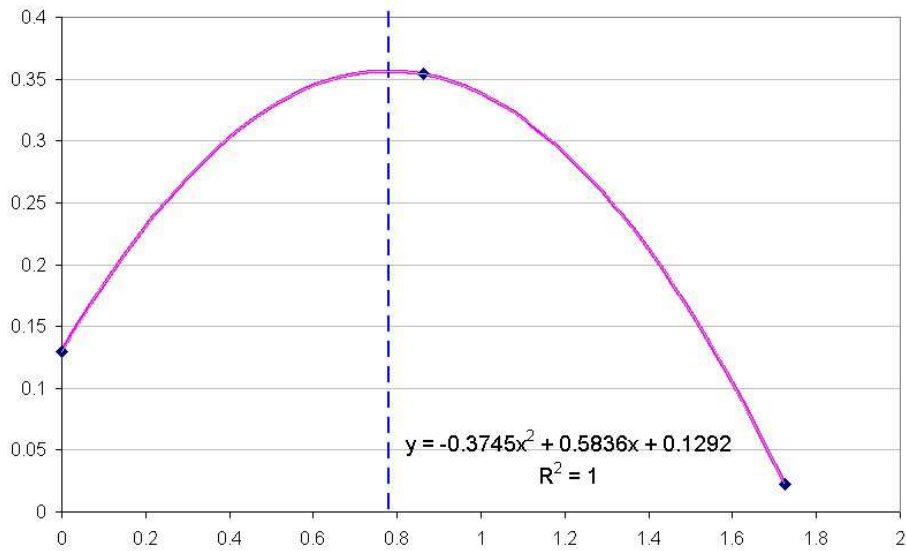
Case number	$c_0$	$c_1$	$c_3$	$x_{\max} = -c_1 / 2c_2$
1	0.03559	0.349571	-0.13757	1.270515
2	0.1292	0.5836	-0.37451	0.779177

Applying now eq. (5) to find the corrected frequency one obtain:

- for case 1, the corrected frequency is  $f_{corr} = 2.6548 + 1.2705 = 3.9253\text{Hz}$  is improved comparing to  $f_{read} = 3.5398\text{Hz}$  or  $f_{read} = 4.4247\text{Hz}$  ;
- for case 2, the corrected frequency is  $f_{corr} = 3.4482 + 0.7791 = 4.2273\text{Hz}$  in stand of  $f_{read} = 4.3103\text{Hz}$  .



**Figure 4.** Polynomial function used to find the local maximum – case 1



**Figure 5.** Polynomial function used to find the local maximum – case 2

The frequencies evaluated involving the standard procedure in comparison with that obtained using the proposed method is presented in table 4. It is shown that the method definitely improves the frequency readability, but the level of improvement depends on the relative position (in terms of amplitudes) of the involved points.

**Table 4.** Frequency evaluated by standard procedure and involving the method proposed in this paper

Case number	$f_{read}$ [Hz]	$f_{real}$ [Hz]	Error [%]	$f_{corr}$ [Hz]	Error [%]
1.1	3.5398	4	-11.505	3.9253	-1.867
1.2	4.4247		10.617		
2	4.3013	4	7.532	4.2274	5.685

Good results are achieved in the case that two points have closely-big amplitudes the correction, the error being reduced from around 10% to less than 2%. In contrast, for the case when one point has much bigger amplitude as the other two, the correction is insignificant, from 7.5% to 5.5%.

#### 4. Conclusion

A method to increase the frequency readability is presented in this paper, and the results are compared with that obtained from standard evaluation. It is found that the proposed method increases the frequency readability, diminishing the maximum achieved error from 11% to 5%.

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