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Optimal Design of DC Electromagnets Based on Imposed Dynamic Characteristics

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In this paper is proposed a method for computing of optimal geometric dimensions of a DC electromagnet, based on the imposed dynamical characteristics. For obtaining the optimal design, it is built the criterion function in an analytic form that may be optimized in the order to find the constructive solution. Numerical simulations performed in Matlab software confirm the proposed work. The presented method can be extended to other electromagnetic devices which frequently operate in dynamic regime.

***Keywords**: design, optimization, DC electromagnet, dynamic characteristics.*

1. Introduction

The design methodology of electromagnets based on steady-state characteristics cannot met the desired optimal solution, if in operation are dominate the dynamic regimes. In this case, the parameters selection based on steady-state characteristics may be used as a first approximation of entire computing algorithme stage.

The computation is based on the toughest operating conditions. One of these may be considerate the assuring of the necessary speed on the entire domain. In this conditions, it is considerate the dynamic characteristic of displacement of mobile armature, due to the fact that this characteristic determines the time motion of mobile part of electromagnet and the kinetic energy accumulated during displacement.

Speed variations determines the contraelectromotive voltage which appears in electromagnetic winding during the moving of mobile armature.

The speed variation low determine both constructive characteristics and the response action of construction on the electromagnet.

The link between speed and displacement is determined from concrete conditions, which can lead to different imposed requirements on the design of electromagnet. For rapid time action electromagnets, it is desired to obtain a minimal value of time action. In the case when the electromagnet is used as a driving element in a connection devices designed for s high frequency of connections, the main requirement is the ensuring of the speed of the displacement elements in the moment of contacts coupling, which must not exceeded an allowed value from wear and operation resistance.

On the design of an electromagnet is necessary that speed dx/dt to be expressed as a function of displacement x of mobile armature. The problem of approximation of the dependences of displacement and speed as a time function is presented in many works [1, 3].

2. Modeling of dynamic characteristics

Thus, for the electromagnet with normal reaction time, the displacement-time dependence, described according to relationship:

$$x = bt^2 , \tag{1}$$

while, for that electromagnets with rapid time reaction, the dependence is:

$$x = bt^4 , \tag{2}$$

where b=constant.

If is adopted the last dependence (2), then $dx/dt=4bt^3$, and the dependence between speed and displacement is described by:

$$v = \frac{dx}{dt} = 4\sqrt[4]{bx^3} . \tag{3}$$

The time action of mobile armature is described by:

$$t_a = \sqrt[4]{\frac{x_t}{b}} , \tag{4}$$

where x_t represents the total displacement.

In the case when is imposed the final value of armature speed $v_{fi}=(dx/dt)_{fi}$, the speed dependence becomes:

$$b = \left[\frac{1}{4}\left(\frac{dx}{dt}\right)_{f_i}\right]^4 \frac{1}{x_t^3} = \left(\frac{v_{f_i}}{4}\right)^4 \frac{1}{x_t^3} , \tag{5}$$

and the time action becomes:

$$t_a = \frac{4x_t}{\left(\frac{dx}{dt}\right)_{f_i}} = \frac{4x_t}{v_{fi}} . \tag{6}$$

If is necessary the ensuring of an imposed value of time action, then $\tau=x_t/t^4_{ai}$, and the speed $v_f=(dx/dt)_f=4x_t/t_{ai}$.

In this case, when is tracking the design of an electromagnet by imposing time action, t_{ai}, and the final value of speed $v_{fi}=(dx/dt)_{fi}$, the value of b is determined from the minimum conditions of function:

$$f(b)=\left(1-\frac{\sqrt[4]{\frac{x_t}{b}}}{t_{a_i}}\right)^2+\left(1-\frac{4\sqrt[4]{bx_t^3}}{v_{fi}}\right)^2, \tag{7}$$

which are expressing the sum of the square of relative errors of time and speed of displacement armature, beside imposed values.

By proposing the speed characteristic, it is proposed the dynamic characteristic and force equation described by:

$$F=m\frac{d^2x}{dt^2}+a\frac{dx}{dt}+kx+F_r, \tag{8}$$

where: m is the mass of mobile armature and drive mechanism, F is the force developed by electromagnet, a is the viscous coefficient, k is the equivalent module of elasticity and F_r is the rezistente force which is not a dependence of the displacement x of mobile armature.

If we set the supply voltage at a fixe value, the dynamic characteristics of electromagnet are determined by the inductance dependence of mobile armature (for the case of a un-saturated magnetic system) and winding resistance.

Taking into account the above considerations, the optimal design of an electromagnet can be divided into two steps. On the first step, are known the dependences of speed and force by displacement and mass, which are used in the order to build the dynamic characteristic of electromagnet. At this strep, the objective is to find the low variation of inductance and resistance which ensure the desired characteristics.

On the second step, are determinted the geometric dimensions of magnetic system by an optimization criterion, which can achive the low variation of inductance and resistence determined on the first step of design.
In both steps are searched parameters that leads to minimal square errors, computed from parameters initial imposed and the resulted one.

3. Determining of inductance low variation

The inductance L and resistance R will be considerate separatly, being linked by time constant T=L/R. In this case, the resistance will be considerate by having a constant value, and the inductance as a function of displacement x, L(x).

The equation of the electric circuit of electromagnet is described by:

$$u=Ri+L\frac{di}{dt}+i\frac{dL}{dt}, \tag{9}$$

which may be wrote in the next form:

$$I = i + T\frac{di}{dt} + i\frac{dT}{dt}, \qquad (10)$$

where I=U/R is the current value on steady-state regimes, which must be known, and may varies.

Taking into consideration that L=L(x), it results T=T(x), and the equation (10) becomes:

$$I = i + T\frac{di}{dt} + i\frac{dT}{dx}\frac{dx}{dt} = i\left(1 + \frac{dT}{dx}\frac{dx}{dt}\right) + T\frac{di}{dt}. \qquad (11)$$

The force developed by the electromagnet is described by relationship:

$$F = \frac{1}{2}i^2\frac{dL}{dx}, \qquad (12)$$

or in a adequate form:

$$F = \frac{i^2}{2}\frac{u}{I}\frac{dT}{dx}. \qquad (13)$$

Due to the fact that inductance, time constant, respectively, depends on the mobile armature position, it is necessary to establish an analytic function for approximation of this dependence.

For a DC electromagnet with culisant armature, the dependence T(x) is approximated by a polinomyal function:

$$T(x) = a_0 + a_1x + a_3x^3 + a_{17}x^{17}. \qquad (14)$$

This approximation is checked on the design of the electromagnets with basculant armature, based on the imposed dynamic characteristics.
From relationships (13) and (14), it is obtained:

$$i = \sqrt{\frac{2FI}{u(a_1 + 3a_3x^2 + 17a_{17}x^{16})}}. \qquad (15)$$

Taking into consideration relationships (14) and (15), the equation (11) becomes:

$$I = \sqrt{\frac{2FI}{u(a_1 + 3a_3x^2 + 17a_{17}x^{16})}}\left[1 + \left(a_1 + 3a_3x^2 + 17a_{17}x^{16}\right)\frac{dx}{dt}\right] +$$

$$\frac{\left(a_1 + 3a_3x^2 + 17a_{17}x^{16}\right)\frac{dF}{dt} - \left(6a_3x + 27{,}2a_{17}x^{15}\right)\frac{dx}{dt}F}{2\sqrt{F\left(a_1 + 3a_3x^2 + 17a_{17}x^{16}\right)^3}}. \qquad (16)$$

$$\sqrt{\frac{2I}{u}}\left(a_0 + a_1x + a_3x^3 + a_{17}x^{17}\right) = I(a_0, a_1, a_3, a_{17})$$

In the least equation F, x, dx/dt and dF/dt are known from the given time functional characteristics. The unknown quantities are a_0, a_1, a_3, a_{17} whose determination is used the function:

$$g(a_0, a_1, a_3, a_{17}) = \left[1 - \frac{I_1(a_0, a_1, a_3, a_{17}, x_1)}{I}\right]^2 + \left[1 - \frac{I_2(a_0, a_1, a_3, a_{17}, x_2)}{I}\right]^2 + ... + \left[1 - \frac{I_n(a_0, a_1, a_3, a_{17}, x_n)}{I}\right]^2 \leq \varepsilon_g, \quad (17)$$

where: I_1, I_2,... , I_n are the values of the current, established by equation (16) as a function of approximation coefficients and position of mobile armature; ε_q is the imposed error.

From condition (17) it is determinate the approximation coefficients a_0, a_1, a_3, a_{17}, and sow the dependence T(x), necessary for computing the parameters of electromagnet.

The optimal geometric dimensions of electromagnet are determinate from the minimum conditions of function:

$$h(y, z, ..., v) = \sum_{i=1}^{n} \left[1 - \frac{T_i(y, z, ..., v, x_i)}{T(x_i)}\right]^2, \quad (18)$$

where: n represents the number of the positions of mobile armature; $T_i(y,z,... , v,x_i)$ is the constant time for position x_i of armature, determined as a function of geometric dimensions y,z,... ,v; $T(x_i)$ is the time constant corresponding to position x_i, determined from the first step by relationship (14).

The complete optimization problem is solved by adding adequate restrictions. Thus, it is necessary that magnetic flux density to not exceed a fixed value in the most saturated element, according to relationship:

$$B(y, z, ..., v) = B_i. \quad (19)$$

Another method for determining of optimal dimensions of electromagnet, is based on the steady-state computing and a one of the dynamic characteristic.
In calculus are given the values of the attraction force F and air gap δ. These quantities allow the selection of magnetic flux density on the air gap, and based on the value of those, the values of currents solenation NI and transversal cross-section of ferromagnetic core.

Voltage equations can be written in the next form:

$$NI = Ni + \frac{N^2}{R} \sigma_s(t) \frac{d\Phi_\delta}{dt}, \quad (20)$$

where σ_s is the total factor of air gap dispersion, and Φ_δ is the magnetic flux in air gap.

The attraction force developed by electromagnet is described by:

$$F = \frac{(NI)^2}{2} \frac{d\Lambda}{dx} = \frac{\Phi_\delta^2}{2\mu_0 S_\delta}. \quad (21)$$

The magnetic dispersion factor σ_s describe the correlation between the permeances that corresponds to the useful magnetic flux in air gap $\Lambda_s(x)$, and that of magnetic dispersion $\Lambda_\delta(x)$:

$$\sigma_s = 1 + \frac{\Lambda_s}{\Lambda_\delta(x)}, \tag{22}$$

In computing stage, as initial quantities, will be adopted: NI, S, $N^2/R=(NI)^2/P$, the dependence $\sigma_s(x)$, the dynamic characteristics, as example the time dependence of motion speed of mobile armature.

Based on given dependence dx(t)/dt, will be determinate x(t) and $d^2x(t)/dt^2$. After this, from equation (8) and (21), it will be determined the dependences Φ_δ, $d\Phi_\delta(t)/dt$.

By using the dependences $\sigma_s(x)$ and x(t) it can be obtained de dependence $\sigma_s(t)$.

From equation (20) it can be determined:

$$Ni = NI - \frac{N^2}{R}\sigma_s(t)\frac{d\Phi_\delta}{dt}. \tag{23}$$

For determining dimensions which corresponds to the imposed variation low of dispersion factor $\sigma_s(x)$, it is minimized the function:

$$p = \sum_{i=1}^{m}\left[1 - \frac{\sigma_{s_i}(y,z,\ldots,v,x_i)}{\sigma_s(x_i)}\right]^2, \tag{24}$$

where: m is the number of the points on curve of x(t), $\sigma_{si}(y,z,\ldots,v,x_i)$ is the dispersion factor for position x_i of armature, determined as a function of geometric dimensions y,z,… , v; $\sigma_s(x_i)$ is the dispersion factor corresponding to position x_i, determined from dependences $\sigma_s(x)$ imposed.

For the illustration of the presented method, it has been considerate a case study of a culisant armature DC electromagnet (Figure 1), whose permeances $\Lambda_s(x)$ and $\Lambda_\delta(x)$ are computed by [3]:

$$\Lambda_\delta(x) = \mu_0\left[\frac{S}{\delta_{max} - x(t)} + 2.32r_1\right], \tag{25a}$$

$$\Lambda_s(x) = \frac{\pi\mu_0 l}{\ln\left[\frac{r_3}{r_1} + \sqrt{\left(\frac{r_3}{r_1}\right)^2 - 1}\right]}. \tag{25b}$$

By using the least relationships, the dispersion factor becomes as a function of the main geometric dimensions:

$$\sigma_s(r_1,r_3,l,x) = 1 + \frac{\pi l}{\left[\frac{S}{\delta_{max} - x(t)} + 2.32r_1\right]\ln\left[\frac{r_3}{r_1} + \sqrt{\left(\frac{r_3}{r_1}\right)^2 - 1}\right]}. \tag{26}$$

4. Illustrative example

Based on theoretical background developed on above section, in this pharagraph is presented a case study for optimization of a DC electromagnet.

The main technical data of electromagnet are: NI=1.75 [A], $N^2/R=1.15*10^4$ [Ω^{-1}], $dx/dt=4bt^3$, $b=1.55*10^5$.

The geometric dimensions are depicted on the below figure:

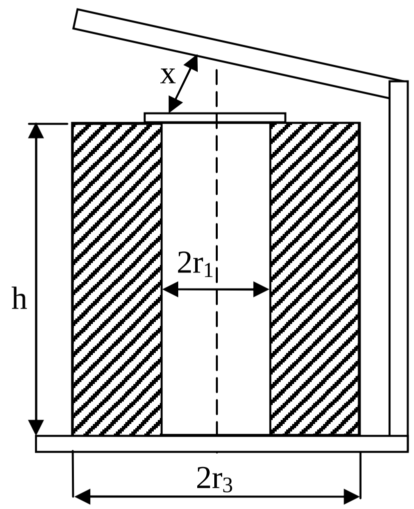
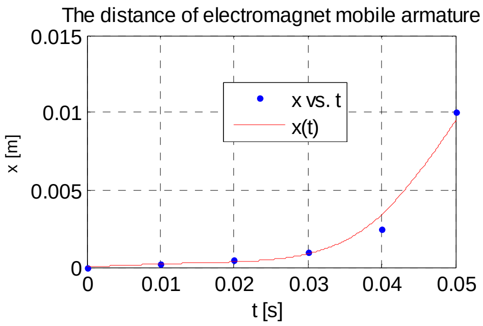


Figure 1. The DC electromagnet structure

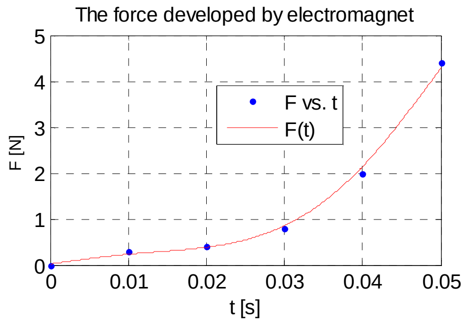
In Figure 2 where represented the dependences x(t), $\Phi_\delta(t)$ and F(t), which where interpolated by spline function. These dependences has been obtained based on speed low variation, which has the relationship $dx/dt=4bt^3$, and on $\sigma_s(x)$ presented on Table 1:

Table 1. The dependence dispersion factor vs distance

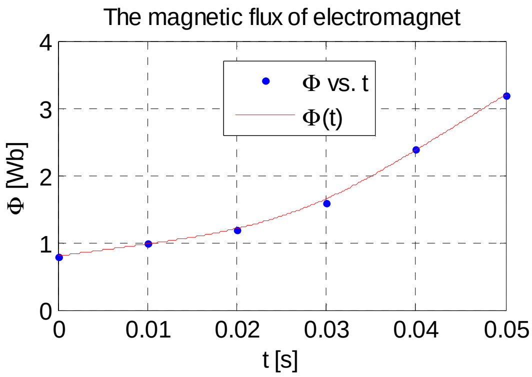
x[m]e-1	0	0.01	0.04	0.06	0.10
$\sigma_s(x)$	2.90	2.00	1.62	1.50	1.10



a. Distance vs time characteristic



b. Force vs time characteristic



c. Magnetic flux vs time characteristic

Figure 2. The dynamic characteristics of DC electromagnet

The criterion function p has been represented on Figure 2. By this graphical representation are determinate the optimal values of geometric dimensions r_3 and h that leads to the minimal values of this function. The final solution may be adopted by tacking into consideration several assumptions. An important aspect may be related on architecture factor, known as aspect ratio that can lead to a proper value of ratio h/l. In other situations, may be tacked into consideration space or technical constraints.

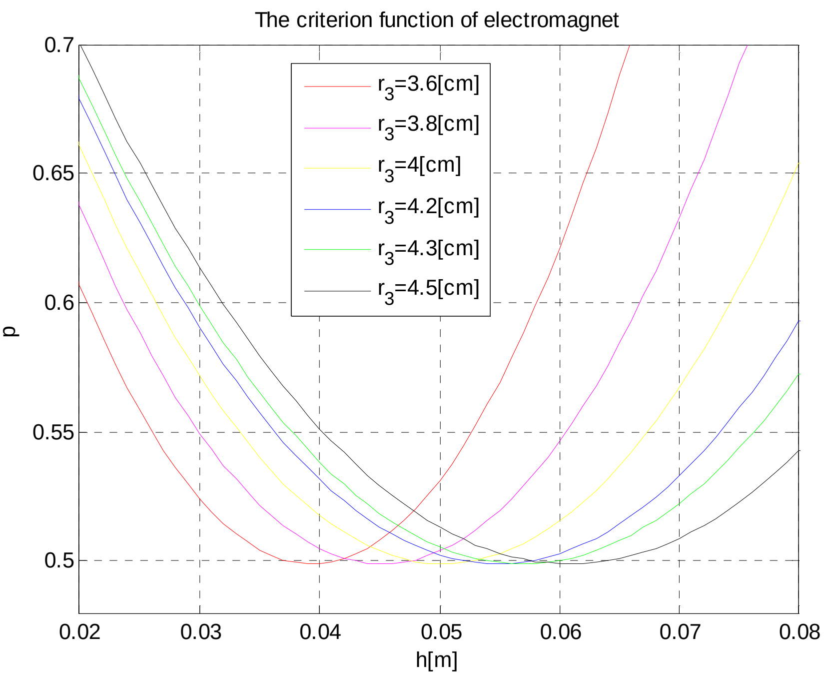


Figure 3. The criterion function for different values of dimension r_3

In the order to find the optimal solution, it is used the gradient method that leads to the next geometric dimensions of the DC electromagnet:

Table 2. Optimization results

r_3 [cm]	3.6	3.8	4	4.2	4.3	4.5
$h(r_3)$[cm]	3.9	4.6	5	5.4	5.7	6.1

4. Conclusion

The optimal design of DC electromagnets based on characteristics leads leads to finding the electromagnetic physical structure that can ensure the developing the desired electromagnetic force on mobile armature.

The method developed on the paper may be used for large area o electromagnets, where can be computed, in a preliminary stage, the dynamic characterstics developed by electromagnets based on the dispersion factor.

An important aspect developed on the paper is described by the analytic form of the performed criterion used in the preliminary stage of the design of DC electromagnets.

Future work involve the developing of AC electromagnets optimization by using the same procedure.

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