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A STUDY ON THE EFFECT OF TRUNCATING THE DISCRETE COSINE TRANSFORM (DCT) COEFFICIENTS FOR IMAGE COMPRESSION

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ABSTRACT

Image compression is essential to reduce the cost of storage and transmission of large image files. Image compression methods are classified into lossy and lossless compression methods. In lossless compression, the image data in the decompressed image is exactly the same as that of the original image, whereas, in the lossy compression, the data in the decompressed image is very close that of the original. For most applications, lossy compression is used. Lossy compression achieves a higher compression than the lossless compression. Discrete Cosine Transform (DCT) is one of the methods used for lossy compression. In DCT, the image is divided into small blocks, usually of 8X8 pixel size, and the DCT transform is applied to that. The transformed image also contains 8X8 coefficients. The information in the first few coefficients carries most information about the image and the remaining carry less information. Therefore, the coefficients that carry the least information can be dropped and the remaining can be stored as a compressed image. This results in the reduction of file size. In this article, an experimental study is made on the image quality of the reconstructed image by truncating a certain number of DCT coefficients.

KEYWORDS: *Image Compression, Discrete Cosine Transform, Truncation of DCT*

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1. INTRODUCTION

Use of images is quite common in almost in all subjects of study, from simple images captured in mobile phones to high resolution images captured in satellites. All images captured are to be stored or transmitted to a different location over a network. When image files are huge and the number images stored are large, such as medical images for diagnostic purposes in hospitals, the storage required increases considerably and the cost of storage increases. For transmitting such large image files, the cost of transmission is also increases. To reduce the storage cost and transmission cost, methods are devised to reduce the size of the image files.

The compression methods make use of features found in an image to compress them. Few of the properties used for compression are [1]:

- **Coding Redundancy:** In this, the bits required to represent pixel can be varied instead of a fixed length representation.

- **Spatial and Temporal Redundancy:** In this, the intensity value of a pixel is correlated with the neighboring pixel that can be exploited to reduce thefile size.
- **Irrelevant Information:** In this technique, the limitation of the human eye to recognize all colors in the image is exploited. It is also called psycho-visual redundancy.

All the compression techniques make use of one or more of the above property in their technique to reduce the file size. An image compression system consists of a coder (compressor) and decoder (decompressor) as shown in Figure 1

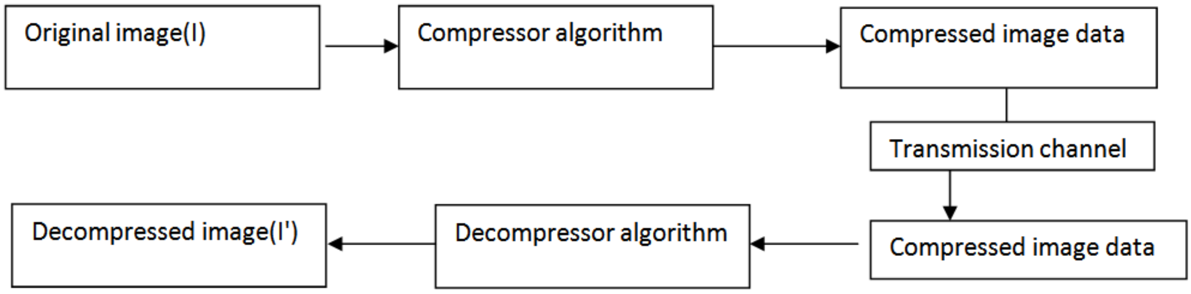


Figure 1: Schematic Diagram of an Image Compression System

Image compression falls into two categories: i) Lossless compression and ii) Lossy compression. In a lossless compression, the original image data (I) and the decompressed image data(I') are exactly the same. In a lossy compression system, the original image data (I) and the decompressed image data (I') will not be the same. The decompressed image data will be very close to that of the original image data.

Run length encoding (RLE), Huffman coding, prefix codes, Golomb code, Shanon- Fano code, arithmetic coding, Lembell-Zive(LZ) coding are few examples of lossless compression[2]. Lossless compressors can make only a limited compression of the images.

Vector quantization, (VA), block truncation coding (BTC), discrete cosine transform (DCT) coding, discrete wavelet transform(DWT) coding, Walsh-Hadamard Transform(WHT), Harr-Tranform (HT)are few examples for lossy compression[2]. Lossy compressors are able to achieve a high compression. Popular and widely used compressors like JPEG[3] are a system, where two or algorithms are combined to achieve a varying degrees of compression.[2]

Orthogonal transforms like DCT are widely used either as a single algorithm or as a combination with other compressors in multistage compressors like JPEG [3]. In this paper, a study on the effect of truncating the number of DCT coefficients for different levels of compression is done. The remaining part of the paper is organized as follows. In section 2 a brief theory on DCT is given. In section 3 the proposed study is given. In section 4 the results and discussion are presented. In section 5 the conclusions are given.

2. DISCRETE COSINE TRANSFORM (DCT)

The discrete cosine transform was originally proposed by Ahmed et al[4]. In this method, a block of the image is taken from the input image and the discrete transform is applied on it. Since an image is represented in two dimensional (2D) array, 2D discrete cosine transform is used to transform an image.

The 2D Discrete Cosine Transform

The 2D discrete cosine transform for an image f(x,y) of size nxm pixel is given by

$$F(u,v) = \left(\frac{2}{n}\right)^{-\frac{1}{2}} C(u)C(v) \sum_{u=0}^{n-1} \sum_{v=0}^{m-1} f(x,y) \cos\left[\frac{\pi(2x+1)u}{2n}\right] cos\left[\frac{\pi(2x+1)v}{2m}\right] \tag{1}$$

$$where\ C(u), C(v) = \begin{cases} 2^{-\frac{1}{2}} & for\ u,v = 0 \\ 1 \\ \text{Otherwise} \end{cases}$$

The DCT coefficients carry the information about the pixels in the in the image. The coefficient in the top left corner carries the maximum information and the

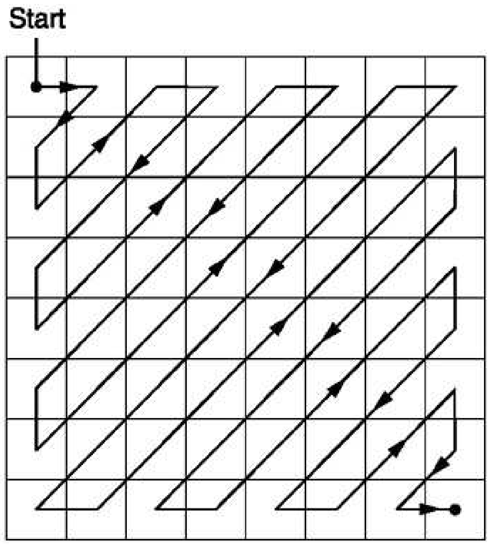


Figure 2: The Zigzag Arrangement of DCT Coefficients in Decreasing Order of Weighteage

Remaining coefficients, arranged in a zigzag manner as shown in Figure 2[2], carry information in the decreasing order. Therefore, to achieve compression, the coefficients starting from the end can be dropped depending on the quality required for the decompressed image.

The data obtained after dropping the coefficients starting at the end of the zigzag path form the compressed image. It is this process that helps to achieve the compression. Usually, it has been shown that a block size of 8X8 gives the best performance for compressing an image using DCT. Figure 3 shows pixel values of an 8X8 image block.

$$Original = \begin{bmatrix} 154 & 123 & 123 & 123 & 123 & 123 & 123 & 136 \\ 192 & 180 & 136 & 154 & 154 & 154 & 136 & 110 \\ 254 & 198 & 154 & 154 & 180 & 154 & 123 & 123 \\ 239 & 180 & 136 & 180 & 180 & 166 & 123 & 123 \\ 180 & 154 & 136 & 167 & 166 & 149 & 136 & 136 \\ 128 & 136 & 123 & 136 & 154 & 180 & 198 & 154 \\ 123 & 105 & 110 & 149 & 136 & 136 & 180 & 166 \\ 110 & 136 & 123 & 123 & 123 & 136 & 154 & 136 \end{bmatrix}$$

Figure 3: Original Image Block

By applying the DCT on the image block given in Figure 3, the DCT coefficients D are obtained as:

$$D = \begin{bmatrix} 162.3 & 40.6 & 20.0 & 72.3 & 30.3 & 12.5 & -19.7 & -11.5 \\ 30.5 & 108.4 & 10.5 & 32.3 & 27.7 & -15.5 & 18.4 & -2.0 \\ -94.1 & -60.1 & 12.3 & -43.4 & -31.3 & 6.1 & -3.3 & 7.1 \\ -38.6 & -83.4 & -5.4 & -22.2 & -13.5 & 15.5 & -1.3 & 3.5 \\ -31.3 & 17.9 & -5.5 & -12.4 & 14.3 & -6.0 & 11.5 & -6.0 \\ -0.9 & -11.8 & 12.8 & 0.2 & 28.1 & 12.6 & 8.4 & 2.9 \\ 4.6 & -2.4 & 12.2 & 6.6 & -18.7 & -12.8 & 7.7 & 12.0 \\ -10.0 & 11.2 & 7.8 & -16.3 & 21.5 & 0.0 & 5.9 & 10.7 \end{bmatrix}$$

Figure 4: Coefficients Obtained by Applying DCT on the Image in Figure 3

The compressed data, when inverse DCT is taken gives the decompressed image. The 2D inverse DCT is given by:

$$f(x,y) = \left(\frac{2}{n}\right)^{-\frac{1}{2}} C(x)C(y) \sum_{x=0}^{n-1} \sum_{y=0}^{m-1} F(u,v) \cos\left[\frac{\pi(2u+1)x}{2n}\right] cos\left[\frac{\pi(2v+1)y}{2m}\right] \quad (2)$$

$$where\ C(x), C(y) = \begin{cases} 2^{-\frac{1}{2}} & for\ x,y = 0 \\ 1 & \\ \text{Otherwise} & \end{cases}$$

3. PROPOSED STUDY

Compression using DCT is achieved by reducing the number of bits required to represent the image using some technique by making use of the properties of the DCT coefficients. As mentioned earlier, the DCT coefficients when arranged in a zigzag manner as given in Figure 2, carry information in the decreasing order of significance. The first coefficient carry the maximum information and the last one the least. In this study, it is proposed to drop the certain number of coefficients out of 64 DCT coefficients for compression, decompress and estimate the image quality measure Peak Signal to Noise Ratio (PSNR).

3.1. Dropping the DCT Coefficients

In this approach, P number of DCT coefficients is retained and drop 64-P coefficients in a block of 8X8 elements. The truncated D with P number of coefficients forms the compressed data. To truncate the coefficients, a mask M of the same size as that of the image block, i.e., 8X8 size with all coefficients having value 1 as shown in eqn. (3) is created.

$$M = \begin{pmatrix} 1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1 \end{pmatrix} \quad (3)$$

To retain P number of DCT coefficients in D, P number of elements in eqn.(4) that are counted in the zigzag order given in Figure 2 are set as 1, and the remaining 64-P coefficients as 0. Then a convolution of M with D is made to give the truncated DCT coefficients. For example, to retain only 28 coefficients, the mask M is set as :

$$M28= \begin{pmatrix} 1&1&1&1&1&1&1&0 \\ 1&1&1&1&1&1&0&0 \\ 1&1&1&1&1&0&0&0 \\ 1&1&1&1&0&0&0&0 \\ 1&1&1&0&0&0&0&0 \\ 1&1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

The compressed data CD is obtained as :

$$CD= Mxx.*D \tag{4}$$

Where, the ". " indicates the element by element operation. The CD forms the compressed data for that block. The compression of the image is done by dividing the whole image into non-overlapping 8X8 pixel blocks and storing the CD for each block as compressed image CI.

To get back the (decompressed) image, the data in CI is taken. It is again divided into 8X8 size as it was stored and an inverse DCT is applied to each block as given in eqn.(2). By taking the inverse DCT for all such blocks in CI, the decompressed image DI is obtained as :

$$DI= L^{-1}DCT(CI) \tag{5}$$

Where, L^{-1} is the inverse operator.

Quality Metrics

The quality of the decompressed image is measured using the Peak Signal to Noise Ratio (PSNR). Suppose the input image is I(x,y) and the decompressed image is I' (x,y), then the squared error is given by :

$$SE(x,y)= (I(x,y) - I'(x,y))^2 \tag{6}$$

The mean squared error MSE is computed as:

$$MSE= \Sigma\Sigma\ SE(x,y) /mxn, \tag{7}$$

Where, m and n are the number of rows and columns in the image respectively.

The PSNR is computed as:

$$PSNR = 10 \log_{10} \left(\frac{MAX_I^2}{\sqrt{MSE}} \right) \tag{8}$$

Where, MAX_I is the maximum intensity level in that image. For a grayscale, 8-bit image, it is 255. Higher the value of PSNR, higher will be quality of the decompressed image.

4. RESULTS AND DISCUSSIONS

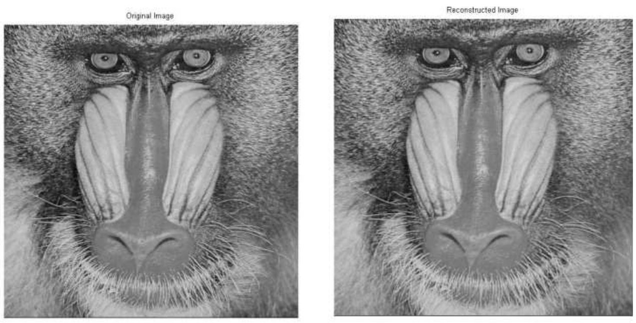
Experiments were carried out by applying the proposed method given in section 3.1 on the standard grayscale images. Standard images Mandrill, Peppers, and Boat were collected from the image database maintained at the University of Southern California [5]. We also used Lena image for this experiment. All the images are of size 512X512 pixels.

The procedure given in section 3.1 was repeated by retaining 4, 6, 10 … 60 DCT coefficients in D. For qualitative assessment of the proposed method, the reconstructed Lena image by retaining only 28 DCT coefficients in the D is shown in Figure 5. Figure 5 (a) shows the original image and the decompressed image is shown in Figure 5 (b)

(a) (b)

Figure 5: Compression by Dropping 28 DCT Coefficients. (a) Original Lena Image (b) Reconstructed image (PSNR=38.61)

A reconstructed Mandrill image with 28 DCT coefficients is shown in Figure 6.



(a) (b)

Figure 6: Compression by Dropping 28 DCT Coefficients. (a) Original Mandrill image (b) Reconstructed image (PSNR=27.2059)

The computed values of PSNR by retaining a different number of DCT coefficients for 4 different images Lena, Mandrill, Peppers and Boat are given in Table 1. A plot of the values given in Table 1 is shown in Figure 7.

Table 1: Computed Values of PSNR by Retaining Limited Number of DCT Coefficients for Lena Image

DCT Coefficients	PSNR			
	Lena	Mandrill	Peppers	Boat
4	27.5982	21.1520	27.4276	25.2678
8	31.1738	22.0901	30.3058	28.1674
12	32.3898	23.4896	32.1093	29.2979
16	34.6190	24.1562	33.7391	31.4653
20	36.2878	25.1038	34.6375	32.7149
24	36.7515	26.4717	35.7987	33.0178
28	38.3871	27.2059	36.8299	34.7689
32	39.7405	27.6937	37.5226	36.5686
36	40.6767	29.2025	38.3394	36.9249
40	41.5929	30.7772	39.0818	37.2402
44	43.1501	31.5544	39.7952	39.6393
48	44.8075	33.0372	40.6388	40.7545
52	46.1729	35.5356	41.5346	41.2549
56	48.5975	37.2216	42.9662	44.1307
60	51.8746	41.9900	45.0097	45.8285

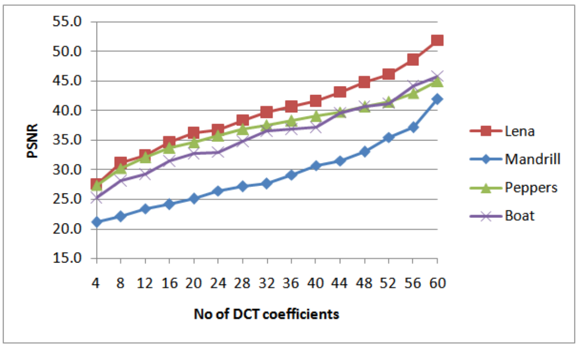


Figure 7: PSNR Values Computed by Retaining Limited Number of DCT Coefficients for Different Images

It is observed from Table 1 and Figure 7 that to get a reasonable quality image with a PSNR value of 35, only 16 DCT coefficients out of 64 is required for Lena. This achieves a compression of 4. However, for Mandrill image even with 52 DCT coefficients, this technique could produce a decompressed image with PSNR value of 35.5 only. It shows that for Mandrill image, this technique did not work efficiently. Even we performed the compression for a block size of 4x4 pixels instead of the standard 8x8 pixels, but the result was poorer than the 8x8 block size.

5. CONCLUSIONS

In this paper, a study on the effect of truncating the number of DCT coefficients in order to compress an image has been done. The quality of the decompressed image was measured in terms of PSNR by dropping a certain number of DCT coefficients. It is found that Lena like images requires only 16 DCT coefficients to compress an image that will produce an image that produces a PSNR of 35 with a compression ratio of 16.Compression by using DCT is very efficient for Lena image than that for the other three images, Mandrill, Peppers and Boat. However, for Mandrill image, this technique required a large number of DCT coefficients to produce a decompressed image with a PSNR value of 35. Therefore, while using JPEG compressor, which involves DCT, one must be careful in selecting the quality factor for images like Mandrill.

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