[Ibraheem *, Vol.6 (Iss.2): February, 2018] ISSN- 2350-0530(O), ISSN- 2394-3629(P)
(Received: Jan 20, 2018 - Accepted: Feb 28, 2018) DOI: 10.5281/zenodo.1199133

INTERNATIONAL JOURNAL OF RESEARCH – GRANTHAALAYAH
A knowledge Repository

Science

THE COMMUTATIVITY OF PRIME NEAR RINGS

Afrah Mohammad Ibraheem [*1]
[*1] Department of Mathematics, College of Education, Al-Mustansiriyah University, Iraq

Abstract

Let N be a near-ring, and σ be an automorphisms of N. An additive mapping d from a near-ring N into itself is called a reverse σ-derivation on N if d (xy) = d(y) x + σ(y) d(x), holds for all x, y∈ N. In this paper, we shall investigate the commutativity of N by a reverse σ-derivation d satisfied some properties, when N is a prime ring.

Keywords: Prime Near Ring; Reverse Derivation; Reverse Σ-Derivation; Commutativity.

Mathematics Subject Classification: 16W25, 16Y30, 16U80.

Cite This Article: Afrah Mohammad Ibraheem. (2018). "THE COMMUTATIVITY OF PRIME NEAR RINGS." *International Journal of Research - Granthaalayah*, 6(2), 339-345. https://doi.org/10.5281/zenodo.1199133.

1. Introduction

Near-rings are one of the generalize structures of rings. A near-ring N is a ring $(N, +, .)$, where + is not necessarily abelian and with only one distributive law. A left near-ring (resp. right near-ring) is called a zero-symmetric left near-ring (resp. a zero-symmetric right near-ring) if $0x = 0$ (resp. $x0 = 0$), for all $x \in N$. A near-ring N called a prime near-ring if $xNy = 0$ implies $x = 0$ *or* $y = 0$, *for all* $x, y \in N$. The multiplicative center Z of N will denote, $Z = \{ x \in N: xy = yx$ *for all* $y \in N \}$. The symbol $[x,y]$ will denote the commutator $xy - yx$, *for all* $x,y \in N$, and note that important identities $[x, yz] = y[x,z] + [x,y]z$ *and* $[xy, z] = x[y, z] + [x, z]y$ satisfied for all $x,y,z \in N$. An additive mapping $d: N \rightarrow N$ is called derivation if $d(xy) = xd(y) + d(x)y$, or equivalently (cf.[7]) that $d(xy) = d(x)y + xd(y)$, *for all* $x,y \in N$. The derivation d will called commuting if $[d(x), x] = 0$, for all $x \in N$. The study of commutativity of prime near-rings by using derivations was initiated by H. E. Bell and G. Mason in 1987 [2], and Yilun Shang [8] satisfying the commutativity of prime near rings N if there exist $k, l \in N$ such that N admits a generalized derivation D satisfying either $D([x,y]) = xk\,[x,y]xl$ for all $x, y \in N$ or $D([x,y]) = -xk\,[x,y]xl$ for all $x, y \in N$.In [5] A. A. M. Kamal generalizes some results of Bell and Mason by studying the commutativity of 3-prime near-rings using a σ-derivation instead of the usual derivation, where σ is an automorphism on the near-ring. Bresar and Vukman in 1989 [3] have introduced the notion of a reverse derivation as an additive mapping d from a ring R into itself satisfying $d(xy) = d(y)x + yd(x)$, *for all* $x,y \in R$. Samman and Alyamani [6] studied the reverse derivations on semi prime

ISSN- 2350-0530(O), ISSN- 2394-3629(P)
DOI: 10.5281/zenodo.1199133

rings. C.Jaya S. R., G.Venkata B.Rao and S.Vasantha Kumar in [4] studied generalized reverse derivation of a semi prime ring R and proved that if f is a generalized reverse derivation with a derivation d, then f is a strong commutativity preserving and R is commutative. Afrah M.Ibraheem in [1] used the notion of reverse derivations on a prime Γ-near ring M to study the commutativity conditions of M, when U be a non-zero invariant subset of M. In this paper, we shall prove that a prime near-ring which admits a nonzero reverse σ-derivation satisfying certain conditions must be a commutative ring. Throughout the paper N will denote a zero symmetric near-ring with multiplicative center Z.

2. Preliminary Results

To prove our results we start with the following definition and lemmas:

Definition 2.1:

Let N be a near-ring, and σ is an automorphism on N. An additive mapping d from N into itself is called a reverse σ-derivation on N if satisfying $d(xy)= d(y)x + \sigma(y)d(x)$, for all $x,y\in N$.

Lemma 2.2:

Let d be an arbitrary additive automorphism of N. Then $d(xy) = \sigma(y)d(x)+d(y)x$ for all $x,y\in N$ if and only if $d(xy) = d(y)x+\sigma(y)d(x)$, for all $x, y\in N$. Therefore d is a reverse σ-derivation if and only if $d(xy) = d(y)x+\sigma(y)d(x)$.

Proof: Suppose

$d(xy) = \sigma(y)d(x)+d(y)x,$
for all $x,y\in N$. Since
$(x+x)y = xy + xy$,
$d((x+x)y) = d(xy + xy)$
$d((x+x)y)) = \sigma(y)d(x+x)+ d(y)(x+x).$

$$= \sigma(y)d(x)+ \sigma(y)d(x)+ d(y)x+ d(y)x\ldots \qquad (1),$$

for all $x,y\in N$. And,
$d(xy+xy) = d(xy)+ d(xy)$

$$= \sigma(y)d(x)+ d(y)x+ \sigma(y)d(x)+ d(y)x\ldots \qquad (2),$$

for all $x,y\in N$. From (1) and (2), we get
$\sigma(y)d(x)+ d(y)x= d(y)x+ \sigma(y)d(x),$

So,
$d(xy) = d(y)x+ \sigma(y)d(x),$
for all $x,y\in N$. The converse is similarly.

Lemma 2.3:

Let N be a prime near-ring, and d be a nonzero reverse σ-derivation of N. If $d(N)\subset Z(N)$ then N is a commutative ring.

Proof

Let $d(x)\in Z(N)$, for all $x\in N$. Then

$d(x)z = zd(x)$… (1)

Replacing x by xy *in* (1), we have

$(d(y)x+ \sigma(y)d(x))\ z = z\ (d(y)x+ \sigma(y)d(x))$,

Then

$\sigma(y)d(x)z - z\sigma(y)d(x))= - d(y)xz+ zd(y)x$,.

$= - d(y)xz+ d(y)zx$… (2),

for all $x, y\in N$. Replacing $\sigma(y)$ by $d(x)$ in (2) and using (1), we get

$d(y)\ (-xz+ zx)= d(y)[-x, z]$

$= d(y)[z, x] = 0$… (3),

for all $x, y, z \in N$. Replacing z by zy in (3) and using (3) again, we get

$d(y)\ z\ [y,x]= 0$,

for all $x, y, z \in N$. Since N is a prime, and $d\neq 0$, we have

$[y, x]= 0$,

for all $x, y\in N$. Therefore N is commutative.

Lemma 2.4:

Let N be a prime near-ring with center Z, and let d be a nonzero reverse σ-derivation of N, then $d(Z)\subset Z$.

Proof: For any $z\in Z$ *and* $x\in N$, we have

$d(xz) = d(zx)$.

$d(xz)= d(z)x+ \sigma(z)d(x)$

$= \sigma(z)d(x)+ d(z)x$,

by lemma 2.2. If we replace $\sigma(z)$ by z, we get

$d(xz)= zd(x)+ d(z)x$… (1),

for all $x, z\in N$.

$d(zx)= d(x)z+ \sigma(x)d(z)$… (2),

for all $x, z\in N$. From (1) and (2), we get

$d(z)x = \sigma(x)d(z)$,

and since σ is automorphism, we have
$d(z)x = xd(z)$,

for all $x, z \in N$, Therefore $d(z) \in Z$, this complete proof.

Lemma 2.5:

Let d be a nonzero reverse σ-derivation of a prime near-ring N, and $x \in N$.
If $xd(N)= 0$ or $d(N)x= 0$, then $x= 0$.

Proof: Let assume that,
$x\, d(n)= 0$… (1),

for all $n \in N$. Replacing n by mn in (1), we have
$x\, d(n)m+ x\, \sigma(n)d(m)= 0$… (2),

for all $x, n, m \in N$. By using (1) in (2), and since σ is automorphism, we have
$x\, N\, d(m)= 0$,

for all $x, m \in N$, and since N is a prime, and $d(N) \neq 0$, we have $x= 0$.
Similarly, we can prove $x= 0$, if $d(N)x= 0$.

3. The Commutativity of Prime Near Ring *N*

In this section we give conditions under which a prime near ring N must be commutative ring.

Theorem 3.1:

For a prime near ring N, let d be a nonzero reverse σ-derivation of N, such that $[x, d(x)]= 0$, for all $x \in N$, then N is commutative.

Proof: Let
$[x, d(x)]= 0$… (1),

for all $x \in N$. Replacing $d(x)$ by $yd(x)$ in (1) and using (1) again, we have
$[x, y]\, d(x) = 0$… (2),

for all $x, y \in N$. Replace y by zy in equ.(2) and using (2), we get,
$[x ,z]\, y\, d(x) = 0$,

for all $x, y, z \in N$.Since N is a prime, we have either $[x, z]=0$ or $d(x)= 0$.

Since $d(x) \neq 0$, for all $x \in N$, then we have $[x, z]= 0$, it follows that $x \in Z(N)$ for each fixed $x \in N$, and by lemma 2.4, we get $d(x) \in Z(N)$, that's $d(N) \subset Z(N)$. Then by lemma 2.3, we get N is commutative.

(Received: Jan 20, 2018 - Accepted: Feb 28, 2018) DOI: 10.5281/zenodo.1199133

Theorem 3.2:

Let N be a prime near ring, and d be a nonzero reverse σ-derivation of N. If $[d(y), d(x)]= 0$, for all $x, y\in N$, then N is commutative.

Proof: Given that

$[d(y), d(x)]= 0$… (1),

for all $x, y\in N$. Replacing y by yx in (1), we get,
$[d(x)y+ \sigma(x)d(y), d(x)]= 0,$

By using (1) again, we get
$d(x) [y, d(x)] + [\sigma(x), d(x)] d(y)= 0$… (2),
for all $x, y\in N$. Replacing y by zy, where $z\in Z(N)$ in equ.(2), we get,

$d(x)z[y, d(x)]+ d(x)[z, d(x)]y+ [\sigma(x), d(x)]d(y)z+ [\sigma(x), d(x)]\sigma(y)d(z)= 0$… (3),

for all $x,y,z\in N$. Since σ is automorphism, and by using (2) in (3), we get
$[\sigma(x), d(x)] y d(z)= 0,$

for all $x, y, z\in N$. Since N is a prime, we have either
$[\sigma(x), d(x)]= 0$, or $d(z)= 0$.

Since $d(z)\neq 0$, we have
$[\sigma(x), d(x)]= 0$… (4),

for all $x\in N$. Replacing $\sigma(x)$ by x in (4), and by using the similar procedure as in Theorem 3.1, we get, N is commutative.

Theorem 3.3:

Let N be a prime near ring, and d be a nonzero reverse σ-derivation of N. If $[x, d(y)] \in Z(N)$, for all $x, y\in N$, then N is commutative.

Proof: Assume that
$[x, d(y)] \in Z(N),$

for all $x, y\in N$. Hence for all $n\in N$,
$[[x, d(y)], n] = 0$… (1).

Replacing x by $xd(y)$ in (1), and using (1) again, we get
$[x, d(y)] [d(y), n] = 0$... (2),

for all $x, y, n\in N$.

Replacing x by nx in (2), and using (2) again, we get
$[n, d(y)]\ x\ [d(y), n] = 0$... (3),

for all $x, y, n \in N$. Since N is a prime, we have either
$[n, d(y)] = 0$... (4),

for all $y, n \in N$, or
$[d(y), n] = 0$... (5),

for all $y, n \in N$. If we replacing $d(y)$ by $md(y)$ in (4) and (5), and using them again, we get
$[n, m]\ d(y) = 0$

or $[m, n]\ d(y) = 0$,

for all $y, n, m \in N$. By using lemma 2.5 in two cases, we have

$[n, m]=0$, and $[m, n]=0$,

for all $n, m \in N$. Therefore, N is commutative.

Theorem 3.4:

Let N be a prime near ring, d be a nonzero reverse σ-derivation of N, and $y \in N$. If $[d(x), y] = 0$ then $d(y)=0$ or $y \in Z(N)$.

Proof: Let
$[x, d(x)] = 0$... (1),

for all $x \in N$. Replacing $d(x)$ by $yd(x)$ in (1) and using (1) again, we have
$[x, y]\ d(x) = 0$... (2),

For all $x, y \in N$. Replace y by zy in equ.(2) and using (2), we get,
$[x, z]\ y\ d(x) = 0$,

For all $x, y, z \in N$. Since N is a prime, we have either
$[x, z]=0$ or $d(x) = 0$.

Since $d(x) \neq 0$, for all $x \in N$, then we have $[x, z] = 0$, it follows that $x \in Z(N)$ for each fixed $x \in N$, and by lemma 2.4 , we get $d(x) \in Z(N)$, that's $d(N) \subset Z(N)$. Then by lemma 2.3, we get N is commutative.

Theorem 3.5:

Let N be a prime near ring, and d be a nonzero reverse σ-derivation of N, such that
$d([x, y]) = [x, d(y)]$, for all $x, y \in N$, then N is commutative.

(Received: Jan 20, 2018 - Accepted: Feb 28, 2018) DOI: 10.5281/zenodo.1199133

Proof: Given that

$d([x, y]) = [x, d(y)] \ldots$ (1),

for all $x, y \in N$. Replacing y by yx in (1) and using (1), we get,

$[x, d(x)]y + [x, \sigma(x)]d(y) = 0 \ldots$ (2),

for all $x, y \in N$. If we replacing $\sigma(x)$ by x in (2), we have

$[x, d(x)]y = 0 \ldots$ (3),

for all $x, y \in N$. Replacing y by $yd(x)$ in (3), we get

$[x, d(x)]\, y\, d(x) = 0,$

for all $x, y \in N$. Since N is a prime, and $d \neq 0$, we have

$[x, d(x)] = 0,$

for all $x \in N$. Then by theorem 3.1, we get, N is commutative.

4. Conclusions

For an automorphism σ on a near ring N, we study the commutativity on N, if N has a non zero reverse σ-derivation d, where d is defined as an additive mapping from N into itself satisfying $d(xy) = d(y)x + \sigma(y)d(x)$, *for all* $x, y \in N$, and introduced some conditions on d to get the commutativity on N when N is a prime near ring.

Acknowledgements

The author would like to thank the referees for useful comments and suggestions.

References

[1] M. Afrah Ibraheem, (2015). “Reverse Derivations on Prime Gamma Near Rings”, Int. J. Pure Appl. Sci. Technol., 26(2), 64-69.
[2] H. E. Bell and G. Mason, (1987). “On derivations in near-rings, Near-rings and near-fields (Tubingen, 1985)”, North-Holland Math. Stud., 137, North-Holland, Amsterdam, 31-35.
[3] M. Bresar and J. Vukman, (1989). “On some additive mappings in rings with involution”, A equations math., 38, 178-185.
[4] C.Jaya Subba Reddy, G. Venkata Bhaskara Rao, S.Vasantha Kumar, (2015). “Generalized Reverse Derivations on Semi prime Rings”, The Bulletin of Society for Mathematical Services, Vol. 15, 1-4.
[5] A. A. M. Kamal, (2001). “σ-derivations on prime near-rings”, Tamkang J. Math. 32, 89-93.
[6] M. Samman, N. Alyamani, (2007). “Derivations and reverse derivations in semi prime rings”, International Mathematical, Forum, 2, No. 39, 1895-1902.
[7] X. K.Wang, (1994). “Derivations in prime near-rings”, Proc. Amer. Math. Soc., 121(2), 361-366.
[8] Yilun Shang, (2015). “A Note on the Commutativity of Prime Near-rings”, Algebra Colloquium, 22(3), 361-366.

*Corresponding author.
E-mail address: afrah.wisam@ uomustansiriyah.edu.iq